MMSE ESTIMATION OF OFDM SYMBOL TIMING AND CARRIER FREQUENCY OFFSET IN TIME-VARYING MULTIPATH CHANNELS

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ABSTRACT

In this paper, we present a new algorithm for blind estimation of the symbol timing and frequency offset in a time-varying frequency-selective Rayleigh fading multipath channel for OFDM systems. It exploits the intrinsic structure of OFDM signals and only relies on second-order moment without knowledge of probability distribution function of received signals. Under minimum mean-square-error (MMSE) sense, the proposed estimators are totally optimum and easily implemented. Furthermore, we expand the estimation range of the frequency offset estimator and improve the timing estimator to be independent of the frequency offset. A more generalized channel model is considered in this paper. It is characterized by its power delay profile and time-varying scattering function. The channel has high reliability for real mobile environment.

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is one of the most promising techniques for achieving high-speed wireless data communications. Recently, this technique has received great interest in satellite and terrestrial digital audio broadcasting (DAB), digital terrestrial TV broadcasting (DVB-T), and broadband indoor wireless systems [1], [2].

OFDM system is far more sensitive to synchronization errors than single carrier system. In order to operate correctly, an OFDM receiver calls for accurate compensation for the symbol timing and carrier frequency offset in the input signal [3]. Most estimators of the timing and frequency offset proposed for OFDM are data aided. They use a known bit pattern or a pilot signal to estimate the timing or frequency [4]. On one side, this quality ensures reliable operation of the synchronizer, but on the side, it has an impact on spectrum efficiency. Recently, a few nondata aided (i.e., blind) estimation techniques have been proposed in an additive white Gaussian noise channel or frequency-selective multipath channel [5], [6]. They exploit only side information concerning the statistics of the information signal to estimate synchronization parameters from the received data. For frequency-selective multipath channel, the transmitted signal that is convoluted with channel impulse response can be modeled as a complex Gaussian process with zero mean by the central limit theorem. Therefore, the likelihood function of received signals is easily achieved in order to estimate symbol timing and carrier frequency offset [7]. However, if the tenable condition of the central limit theorem can't be satisfied, this method based on maximum-likelihood (ML) will become incapable.

In this paper, we present a novel algorithm for the blind estimation of symbol timing and carrier frequency offset in time-varying frequency-selective Rayleigh fading multipath channel. The proposed blind minimum mean-square-error (MMSE) estimators only rely on second-order moment dispense with knowledge of probability function of received signals. Namely they only use the information provided by the autocorrelation function of received signals to minimize a cost function (i.e., mean-square-error) associated with synchronization parameters. An improved approach that can increase estimation range of the frequency offset to entire subcarrier spacing of the OFDM signal is proposed. Firstly, the salient features of this algorithm are that the estimators are globally stable and easy to implement realization; secondly, unlike joint ML estimators, the proposed timing estimator don't depend on frequency offset estimator; finally, performance of the estimators is less influenced by noise than ML estimator. Moreover, we adequately take into account the time variation of the channel within one symbol. The channel is characterized by its power delay profile and scattering function. Unlike conventional OFDM, this generalized channel is more resilient to mobile environment.

2. OFDM SIGNAL MODEL

We consider an OFDM system with N sub-carriers signaling through a time-varying frequency-selective Rayleigh fading channel. The data are modulated in blocks by means of a discrete Fourier transform (DFT).

At the transmitter end, N complex data symbols are modulated onto the N sub-carriers by using Inverse Fast Fourier Transform (IFFT). The last N_g samples of the IFFT outputs are then copied and added to form the guard interval at the beginning of each OFDM symbol. By inserting guard interval in the OFDM symbols, intersymbol interference (ISI) and intercarrier interference (ICI) can be avoided. The baseband-modulated signal $s(n)$ is available after parallel to serial conversion. Thus, $s(n)$ can be expressed as

$$s(n)=\sum_{l=-\infty}^{+\infty}\sum_{k=0}^{N-1} d_k(l)e^{j2pk(n-N_g-lM)/N} g(n-lM) \qquad (1)$$

where $M=N+N_g$. $d_k(l)$ is the data symbol modulating the k th subcarrier during the l th OFDM symbol duration. It may be safely approximated as zero-mean random variables with the correlation

$$E\{d_k(l)d_{k'}^*(l')\}=s_d^2 d(k-k')d(l-l'),$$

$*$ is conjugate operator. $g(\cdot)$ is a rectangular function which can be written as follows:

$$g(n)=\begin{cases}1, & 0\le n\le M-1\\ 0, & elsewhere\end{cases}.$$

At the receiver end, The timing uncertainty in the OFDM signal will be modeled as a time shift $q_e\in[1,M]$, assuming q_e is an integer, and unknown carrier frequency offset, which is caused by the Doppler effect and inherent instabilities of the transmitter and receiver carrier frequency oscillators. It is accounted for by a frequency shift e . The received signal is sampled as

$$y(n)=x(n-q_e)e^{j2pen/N}+v(n) \qquad (2)$$

where $v(n)$ is a white complex Gaussian noise with zero mean and variance s_v^2, and independent of the signal $x(n)$. $x(n)$ can be expressed as

$$x(n)=\sum_{q=0}^{Q-1} h(n,q)s(n-q) \qquad (3)$$

where $h(n,q)$ is the channel impulse response of q th tap at time n . In the radio propagation channel, the presence of reflecting and scatterers in the channel creates a constantly changing environment that dissipates the signal energy in amplitude, phase, and time. These effects result in multiple versions (multipath) of the transmitted signal that arrive at the receiving antenna. If we assume that the channel is the wide sense stationary uncorrelated scattering (WSSUS), it may be model as a tapped delay line channel [8], where the length Q of the tapped delay line and the power distribution of each tap are determined by the duration of the power delay profile, the scattering function that is determined by the Doppler frequency describes time-varying behavior of each tap. In this paper, we assume that the power delay profile is exponential distribution, the length Q of the tapped delay line is less than the length N_g of guard interval, and the inverse Fourier transform of Doppler spectrum is the zero-order Bessel function of the first kind. Each tap $h(n,q)$ is independently generated by low-pass filtering of a white complex Gaussian process. By above assumptions, the autocorrelation of the channel impulse response can be expressed as

$$\begin{aligned}&E\{h(m,q)h^*(n,q)\}\\ &=c_1J_0(2pf_dT(m-n)/N)e^{-c_2q/Q}\end{aligned} \qquad (4)$$

where c_1 is normalization constant, c_2 is scale constant, and the channel will approximate to Jakes Model [8] with increasing c_2 . $J_0(\cdot)$ is the zero-order Bessel function of the first kind, f_d is Doppler frequency in hertz, and T is period of symbol.

3. MMSE ESTIMATION OF OFDM SYMBOL TIMING AND CARRIER FREQUENCY OFFSET

We now derive the MMSE estimators of symbol timing and frequency offset based on the autocorrelation function of received signal. Firstly, the autocorrelation function of $s(n)$ is obtained by

$$\begin{aligned}R_s(n,t)&=E\{s(n)s^*(n-t)\}\\ &=s_d^2\sum_{l=-\infty}^{+\infty}\sum_{k=0}^{N-1}e^{j2pkt/N}g(n-lM)g(n-lM-t) \qquad (5)\\ &=\begin{cases}Ns_d^2\sum_{l=-\infty}^{+\infty}g^2(n-lM), & t=0\\ Ns_d^2\sum_{l=-\infty}^{+\infty}g(n-lM)g(n-lM\pm N), & t=\pm N\\ 0, & t=otherwise\end{cases}\end{aligned}$$

Secondly, since each tap is uncorrelated and the channel is independent of $s(n)$, the autocorrelation function of $x(n)$ becomes

$$\begin{aligned}R_x(n,t)&=E\{x(n)x^*(n-t)\}\\ &=\sum_{q=0}^{Q-1}c_1J_0(2pf_dTt/N)e^{-c_2q/Q}R_s(n-q,t)\end{aligned} \qquad (6)$$

Above calculation process uses the result of (4). Finally, according as (5) and (6), the autocorrelation function of received signal $y(n)$ can be expressed as

$$\begin{aligned}&R_y(n,t)=E\{y(n)y^*(n-t)\}\\ &=\begin{cases}\sum_{l=-\infty}^{+\infty}\sum_{q=0}^{Q-1}c_1Ns_d^2e^{-c_2q/Q}g(n-q_e-q-lM)d(t)\\ \qquad +s_v^2d(t), & t=0\\ e^{j2pe}\sum_{l=-\infty}^{+\infty}\sum_{q=0}^{Q-1}c_1Ns_d^2J_0(2pf_dT)e^{-c_2q/Q}\\ \quad\cdot g(n-q_e-q-lM)g(n-q_e-q-lM\pm N), & t=\pm N\\ 0, & t=otherwise\end{cases}\end{aligned} \qquad (7)$$

From (7), it follows that $R_y(n,t)$ is $M-$periodic in n for every t and only contains information on both synchronization parameters. Without loss of generality, $R_y(n,t)$ can be expressed as a matrix form at $t=N$ as follows

$$\mathbf{R}_y=c'e^{j2pe}\mathbf{AR}_g \qquad (8)$$

where $c'=c_1Ns_d^2J_0(2pf_dT)$, $\mathbf{A}$ is a tridiagonal teopliz matrix of $M\times(M+Q-1)$ as expressed by

$$\mathbf{A}=\begin{bmatrix} e^{c_2\frac{Q-1}{Q}} & e^{c_2\frac{Q-2}{Q}} & \cdots & 1 & 0 & 0 & \cdots & 0 \\ 0 & e^{c_2\frac{Q-1}{Q}} & e^{c_2\frac{Q-2}{Q}} & \cdots & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & e^{c_2\frac{Q-1}{Q}} & e^{c_2\frac{Q-2}{Q}} & \cdots & 1 \end{bmatrix}$$

$$\mathbf{R}_y=\left[R_y(0,N),R_y(1,N),\cdots,R_y(M-1,N)\right]^T$$

$$\mathbf{R}_g=\left[R_g(0,Q-1),\cdots R_g(0,0),R_g(1,0),\cdots R_g(M-1,0)\right]^T,$$

where

$$R_g(n,q)=\sum_{l=-\infty}^{+\infty} g(n-q-\theta_e-lM)g(n-q-\theta_e-lM-N)$$

Notice that equation (8) is independent of noise. From (8), mean-square-error (MSE) function is defined by

$$\Lambda(\theta_e,\varepsilon)=\left\|\hat{\mathbf{R}}_y-c' e^{j2\pi\varepsilon}\mathbf{A}\mathbf{R}_g\right\|^2 \quad (9)$$

where $\hat{\mathbf{R}}_y$ is the estimation of the correlation function $\mathbf{R}_y$. In practice, its entries can be estimated from a finite data record $\{y(n+k)\}_{k=0}^{KM-1}$ of length KM according to

$$R_y(n,t)=\frac{1}{K}\sum_{k=1}^{K-1} y(n+kM)y^*(n+kM-t) \quad (10)$$

The MMSE estimation of θ_e and ε is the argument minimizing $\Lambda(\theta_e,\varepsilon)$. The minimization of the error can be performed in two steps:

$$\min_{\theta_e,\varepsilon}\Lambda(\theta_e,\varepsilon)=\min_{\theta_e}\min_{\varepsilon}\Lambda(\theta_e,\varepsilon)=\min_{\theta_e}\Lambda(\theta_e,\hat{\varepsilon}(\theta_e)) \quad (11)$$

The minimum with respect to the frequency offset ε is obtained by

$$\frac{\partial\Lambda(\theta_e,\varepsilon)}{\partial\varepsilon}=0 \quad (12)$$

This yields the MMSE estimation of ε

$$\hat{\varepsilon}(\theta_e)=\frac{1}{4\pi}\angle\frac{\mathbf{R}_g^T\mathbf{A}^T\hat{\mathbf{R}}_y}{\hat{\mathbf{R}}_y^H\mathbf{A}\mathbf{R}_g} \quad (13)$$

Substituting (13) into (9), the MMSE estimation of θ_e becomes

$$\hat{\theta}_e=\arg\min_{\theta_e}\left\|\hat{\mathbf{R}}_y-c'\sqrt{\frac{\mathbf{R}_g^T\mathbf{A}^T\hat{\mathbf{R}}_y}{\hat{\mathbf{R}}_y^H\mathbf{A}\mathbf{R}_g}}\mathbf{A}\mathbf{R}_g\right\|^2 \quad (14)$$

where $(\cdot)^T$ and $(\cdot)^H$ denote transpose and conjugate transpose operator, respectively.

We notice that the maximum allowed frequency offset follows from (13) as $|\theta_e|<1/4$, i.e., the carrier frequency estimation range is half the subcarrier spacing of the OFDM signal. In order to increase its estimation range, we can calculate module of $\mathbf{R}_y$'s entries. Due to $\mathbf{A}$ and $\mathbf{R}_g$ are real matrix, (8) can be rewritten as

$$\mathbf{R}'_y=c'\mathbf{A}\mathbf{R}_g \quad (15)$$

where $\mathbf{R}'_y=\left[|R_y(0,N)|,|R_y(1,N)|,\cdots,|R_y(M-1,N)|\right]^T$.

Notice that equation (15) is completely independent of the frequency offset ε. Then, the MMSE estimation of θ_e is given by

$$\hat{\theta}_e=\arg\min_{\theta_e}\Lambda'(\theta_e)=\arg\min_{\theta_e}\left\|\hat{\mathbf{R}}'_y-c'\mathbf{A}\mathbf{R}_g\right\|^2 \quad (16)$$

From (8), the phase angle of $\mathbf{R}_y$ is equal to the frequency offset ε multiplied by 2π, i.e.,

$$\hat{\varepsilon}(\hat{\theta}_e)=\frac{1}{2\pi}\angle\sum_{k=0}^{N_g-1} y(n+k+\hat{\theta}_e)y^*(n+k+\hat{\theta}_e+N) \quad (17)$$

where n is an arbitrary time. Now, the estimation range of the carrier frequency offset increases to the entire subcarrier spacing of the OFDM signal.

4. SIMULATIONS

Numerical results are presented to demonstrate the performance of the proposed estimation of timing and frequency offset in OFDM system. The OFDM model selected for simulation in this paper consists of 64-point FFT, a guard interval of 16 samples, i.e., $1/4$ of the useful data interval, and 16-QAM modulation mapping scheme. The signal-to-noise radio (SNR) was defined as $SNR(dB)=10\lg\sigma_s^2/\sigma_v^2$. Each realization consisted of 25 observation frames except for special description. All results were obtained by averaging over 200 independent Monte Carlo trials.

In our simulations, the sixth-order Butterworth filter is used to generate the Doppler spectrum. Q =12, $f_d T=0.02$.

A. Expanded Estimation Range of Frequency Offset

For $\theta_e=68$, $c_2=1$, and $SNR=10dB$, MSE of the frequency offset estimator (13) and (17) is shown in Fig.1., respectively. The estimator (17) has wider estimation range and lower MSE than (13).

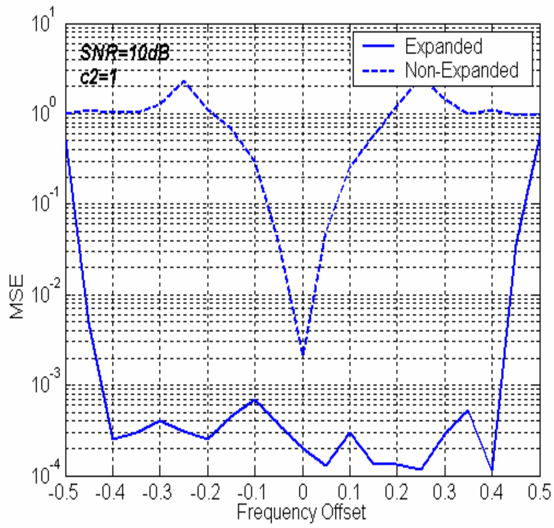

Fig. 1. MSE of the frequency offset estimator (13) (dash line) and (17) (solid line)

B. Effect of Multipath Channel

In a time-varying multipath channel, the received signal includes multiple versions of the transmitted waveform which are attenuated and delayed in time. Thus the channel induces ISI to deteriorate the performance of the estimators.

Fig. 2 shows MSE of timing offset estimator when $c_2 = 1,5$ and 20, respectively. We can see the MSE becomes small with c_2 increased, i.e., the estimation accuracy of timing can be improved with decreasing the degree of multipath, and the performance of the timing estimator is almost independent of the SNR for a fixed c_2.

Fig. 3 shows MSE of frequency offset estimator when $c_2 = 1,5$ and 20, respectively. We compute the MSE after the timing offset is corrected. Like the timing offset estimator, the estimation accuracy of frequency offset can be also improved with decreasing the degree of multipath.

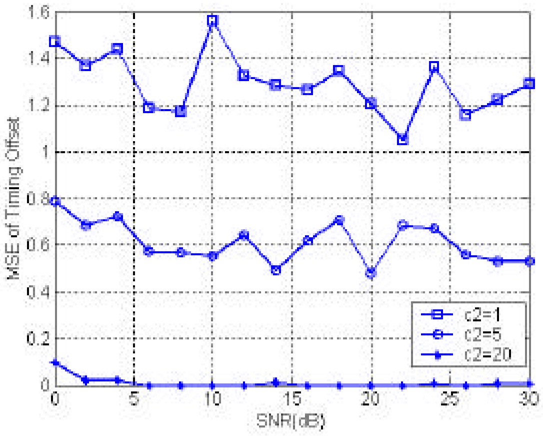

Fig. 2. MSE of symbol timing estimator at $c_2 = 1,5$ and 20, respectively.

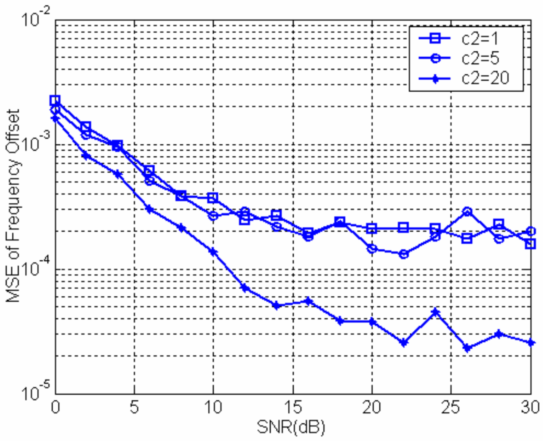

Fig. 3. MSE of frequency offset estimator at $c_2 = 1,5$ and 20, respectively.

5. CONCLUSIONS

We derive novel blind estimators of the symbol timing and frequency offset in a time-varying frequency-selective Rayleigh fading multipath channel for OFDM systems. Since the estimators use the inherent information of the OFDM signals, no additional training sequence is needed. The proposed MMSE estimators that only depend on second-order statistic are that no probabilistic assumptions are made about the received data. Hence, they are globally stable and easy to implement realization. Moreover, we improve the estimators in order to increase the length of the frequency offset estimation range. The symbol timing estimator is independent of the frequency offset. For multipath channel model, the time variation of the channel within one symbol and exponential distribution of the power delay profile are considered in this paper. This generalized channel more adapts to mobile environment.

6. REFERENCES

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