A NEW PILOT ASSISTED FREQUENCY SYNCHRONIZATION FOR WIRELESS OFDM SYSTEMS

Wen Lei, Jianhua Lu, Jun Gu

Dept. of Electronic Engineering, Tsinghua University, Beijing, 100084, China

ABSTRACT

In this paper, a new frequency offset synchronization algorithm is proposed for OFDM systems over time varying multi-path fading channels. This algorithm employs pilot symbols embedded in data symbols to estimate frequency offset. In particular, fine frequency offset estimate is achieved by properly combining frequency- offset estimates from pilots over different frequency locations. Moreover, the effects of time varying channels and the number of sub-carriers are analyzed and the rule for the selection of the number of OFDM symbols used for the synchronization algorithm is given. Simulations are conducted to demonstrate the effectiveness of the proposed scheme.

1. INTRODUCTION

Recently, Orthogonal Frequency Division Multiplexing (OFDM) system for supporting high-speed wireless communications has gained great attention worldwide [1]. Potential applications include digital audio broadcasting, high-speed wireless local area networks and mobile communications. However, OFDM is very sensitive to frequency offset caused by Doppler shift or misalignment between frequency oscillators [2], while in a mobile environment, to achieve an accurate frequency synchronization for an OFDM system is even a critical and challenging problem.

Many frequency synchronization algorithms for OFDM systems have been proposed, including two essential elements: frequency acquisition and frequency tracking. Frequency acquisition is usually realized with repeated training symbols, called as preamble, attached ahead of data packet [3], and frequency offset tracking may be achieved by making use of pilot tones embedded in OFDM data symbols [4,5].

Samir Kapoor presented a pilot-assisted frequency synchronization algorithm, which averages the offset estimates of pilots in different spectral locations to achieve the final frequency offset estimate over time varying channels [5]. But in multi-path fading channels, the signal power fluctuates significantly among pilots in different spectral locations. Thus, simply averaging the frequency offset estimates may result in serious errors.

In this paper, we present a new pilot-assisted frequency offset estimation algorithm based on the use of Weighting Function (WF). In particular, the frequency offsets estimated from pilots in different spectral locations are properly combined together so as to reduce the influence of multi-path fading. Analysis and simulation results show that this WF-based method provides an excellent estimation of frequency offset in wireless systems.

In the next section, pilot assisted MLE frequency offset tracking algorithm is presented. Section III introduces the proposed frequency offset estimation algorithm with WF and analyzes how to select the block length used for frequency offset estimate. In the section IV, simulation results are given to demonstrate the effectiveness of the algorithm. Section V concludes the paper.

2. PILOT ASSISTED MLE ALGORITHM

Pilots are usually embedded in transmitted data symbols to realize frequency offset tracking and channel estimation in OFDM systems. A typical OFDM symbol consists of N sub-carriers, where N_p pilot sub-carriers are inserted. Define Υ be the set of sub-carriers indexes. After transmission, the l-th received symbol can be expressed as

$$r_l[n] = \frac{1}{\sqrt{N}} e^{j\frac{2\pi\varepsilon(n+l(N+G))}{N}} \sum_{k=0}^{N-1} S_l(k) H_l(k) e^{j\frac{2\pi kn}{N}} + \omega_l[n] \tag{1}$$

with $S_l(k) = \begin{cases} P_l(k) & k \in \Upsilon \\ X_l(k) & k \notin \Upsilon \end{cases}$

where $P_l(k)$, $H_l(k)$ and $X_l(k)$ are the transmitted pilot-symbol, the channel's frequency response and the transmitted complex data symbol in the k-th subcarrier of the l-th OFDM symbol, respectively, ε is the frequency offset normalized to the inter sub-carrier spacing, $\omega_l[n]$ denotes the complex additive white Gaussian noise, and G is the length of cyclic prefix, which is appended to all symbols to preserve orthogonality between sub-carriers.

After the DFT, the i-th demodulated pilot tone of the l-th OFDM symbol is given by

$$R_l(i) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} r_l[n] e^{-j\frac{2\pi ni}{N}}, \qquad i \in \Upsilon \tag{2}$$

Substitute (1), we have

$$R_l(i) = \frac{1}{N} \sum_{n=0}^{N-1} e^{j\frac{2\pi\varepsilon(n+l(N+G))}{N}} e^{-j\frac{2\pi ni}{N}} \sum_{k=0}^{N-1} S_l(k) H_l(k) e^{j\frac{2\pi kn}{N}} + \varpi_l(i) \tag{3}$$

For the next OFDM symbol, the i-th demodulated pilot tone is

$$R_{l+1}(i) = \frac{1}{N} e^{j2\pi\varepsilon(1+G/N)} \sum_{n=0}^{N-1} e^{j\frac{2\pi\varepsilon(n+l(N+G))}{N}} e^{-j\frac{2\pi ni}{N}} \sum_{k=0}^{N-1} S_{l+1}(k) H_{l+1}(k) e^{j\frac{2\pi kn}{N}} + \varpi_{l+1}(i) \tag{4}$$

Using only the i-th pilot tone, an approximate of maximum likelihood estimate, $\hat{\varepsilon}_i$, may be constructed with a block of L symbols, that is [3]

$$\hat{\varepsilon}_i = \frac{1}{2\pi(1+G/N)} \tan^{-1}\left[\frac{\operatorname{Im}\sum_{r=l}^{l+L-2} R_r^*(i) R_{r+1}(i)}{\operatorname{Re}\sum_{r=l}^{l+L-2} R_r^*(i) R_{r+1}(i)}\right] \tag{5}$$

And the variance of $\hat{\varepsilon}_i$ in an AWGN channel can be estimated as [3]

$$\operatorname{var}[\hat{\varepsilon}_i] = \frac{1}{(2\pi)^2 (1+G/N)^2 L\gamma_i} \tag{6}$$

where γ_i denotes the SNR of the i-th pilot. Finally, the estimated frequency offset is [5]

$$\hat{\varepsilon} = \frac{1}{N_p} \sum_{i=0}^{N_p-1} \hat{\varepsilon}_i \tag{7}$$

In AWGN channels, this method can achieve good results. But in multi-path fading channels, the frequency offsets estimated from pilots with different SNRs in different spectral locations should have different influences on the final result. Thereby, properly combining different estimates may accomplish improved performance.

3. ESTIMATION ALGORITHM WITH WEIGHTING FUNCTION

3.1 Frequency Offset Estimation

To reduce the influence of frequency selective fading channels, the estimated frequency offsets $\hat{\varepsilon}_i \quad (i = 0, 1, \cdots, N_p - 1)$ from pilots in different spectral locations are weighted with $G_i \quad (i = 0, 1, \cdots, N_p - 1)$, respectively, shown in Fig. 1, and then to obtain a more accurate estimated frequency offset, $\hat{\varepsilon}_p$, by

$$\hat{\varepsilon}_p = \frac{\sum_{i=0}^{N_p-1} G_i \hat{\varepsilon}_i}{\sum_{i=0}^{N_p-1} G_i} \tag{8}$$

Select $G_i \quad (i = 0, 1, \cdots, N_p - 1)$ to minimize the variance of $\hat{\varepsilon}_p$, which is given by

$$\operatorname{var}[\hat{\varepsilon}_p] = E[(\hat{\varepsilon}_p - \varepsilon)^2]$$

$$= E\left[\left(\frac{\sum_{i=0}^{N_p-1} G_i \hat{\varepsilon}_i - \sum_{i=0}^{N_p-1} G_i \varepsilon}{\sum_{i=0}^{N_p-1} G_i}\right)^2\right] = E\left[\left(\frac{\sum_{i=0}^{N_p-1} G_i \Delta_i}{\sum_{i=0}^{N_p-1} G_i}\right)^2\right] \tag{9}$$

where Δ_i is the estimation error from the i-th pilot. With the assumption of uncorrelated fluctuates against pilots in different spectral locations, Δ_i should be independent of each other. Then, with reference to Eqn. (6),

$$\operatorname{var}[\hat{\varepsilon}_p] = E\left[\frac{\sum_{i=0}^{N_p-1} (G_i^2 \Delta_i^2)}{\left|\sum_{i=0}^{N_p-1} G_i\right|^2}\right] = Q \cdot E\left[\frac{\sum_{i=0}^{N_p-1} (G_i^2 \cdot \frac{1}{\gamma_i})}{\left|\sum_{i=0}^{N_p-1} G_i\right|^2}\right] \tag{10}$$

where $Q = \frac{1}{(2\pi)^2 (1+G/N)^2 L}$, and γ_i is the received SNR of the i-th pilot.

Using Cauchy inequality, for $\gamma_i \geq 0$, $i = 0, 1, \cdots, N_p - 1$,

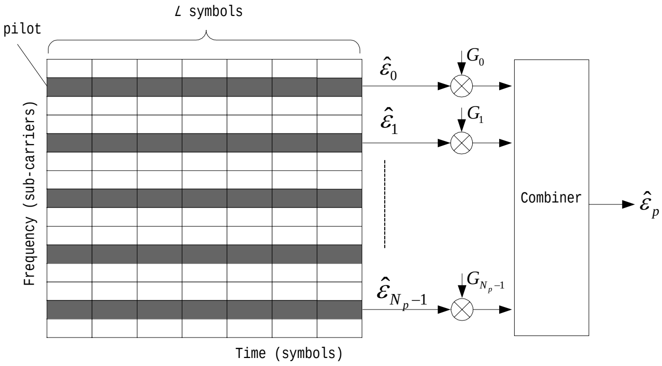

Fig. 1 Algorithm with Weighting Function.

$$\frac{\sum_{i=0}^{N_p-1}\left|G_i\cdot\frac{1}{\sqrt{\gamma_i}}\right|^2}{\left|\sum_{i=0}^{N_p-1}(G_i\cdot\frac{1}{\sqrt{\gamma_i}})\sqrt{\gamma_i}\right|^2}\geq\frac{\sum_{i=0}^{N_p-1}\left|G_i\cdot\frac{1}{\sqrt{\gamma_i}}\right|^2}{\sum_{i=0}^{N_p-1}\left|G_i\cdot\frac{1}{\sqrt{\gamma_i}}\right|^2\sum_{i=0}^{N_p-1}(\gamma_i)} \quad (11)$$

where the relation holds with equality if and only if $G_i=\gamma_i\ (i=0,1,\cdots,N_p-1)$. This condition minimizes the variance of $\hat{\varepsilon}_p$. Hence, an optimally estimated frequency offset is achieved as

$$\hat{\varepsilon}_p=\frac{\sum_{i=0}^{N_p-1}\gamma_i\hat{\varepsilon}_i}{\sum_{i=0}^{N_p-1}\gamma_i} \quad (12)$$

In static channels, the proposed algorithm achieves better performance as more OFDM symbols are included in a block for frequency offset estimate.

However, in time varying fading channels, OFDM symbols included in a block would suffer differently. Hence, the selection of the number of OFDM symbols in a block in time varying fading channels should take the state of channels into condition.

3.2 Effects of Some Factors on The Proposed Algorithm

3.2.1 Block length

In a time varying fading channel, the channel can be assumed to be static over several symbol intervals, which implies that the coherence time T_c (the time domain dual of Doppler spread) of the channel is much longer than the length of the block with *L* symbols, i.e.

$$T_c \gg L(T+T_g) \quad (13)$$

where *T=N/B*, *B* is the bandwidth of the OFDM signal, T_g is the guard interval, which refers to the cyclic prefix in OFDM systems.

For conservative design, the value of T_c is [6]

$$T_c \approx \frac{9}{16\pi f_m} \quad (14)$$

f_m is the maximum Doppler shift given by $f_m=v/\lambda$.

So with the same f_m and as long as the value of *L* meets the inequality (13), the algorithm performance increases with the larger *L*. In other words, with the same *L*, Doppler spread is less, the coherent time of the channel is larger, and the algorithm performance is better.

3.2.2 The Number of Sub-carriers

Taken the number of pilots inserted proportional to the number of sub-carriers, the larger of the number of sub-carrier, the more pilots are inserted. As more pilots involved, the performance of the proposed algorithm increases.

4. SIMULATION RESULTS

To evaluate the performance of the proposed algorithm, a typical OFDM system is considered in simulation. With no special illustration, the transmission parameters are specified in Table 1, and the channel model defined in [6] is employed as the multipath model.

Table 1. Simulation parameters

Bandwidth	10MHz
Carrier frequency	2.4 GHz
The number of sub-carriers	256
RMS delay spread	300ns
Guard Interval	1/5 OFDM Symbol
Normalized Frequency offset	0.01
Modulation	QPSK
The length of Block	3 OFDM symbols
Doppler spread	50Hz
The interval between pilots	14 sub-carriers

Fig. 2 shows the Mean-Squared-Error (MSE) of frequency offset tracking algorithms. The results in Fig. 2 shows that the WF-based algorithm achieves far better performance than the traditional MLE algorithm. From parameters given in Table 1, the value of *L* should be less than 112. Clearly, with $L \ll 112$, the proposed algorithm gets more improvement as symbols included more, but with larger time delay introduced.

Fig. 3 shows the results of the proposed algorithm under different Doppler Spreads. As we show in Section 3.2, the algorithm achieves better performance with less Doppler spread, especially in high SNR.

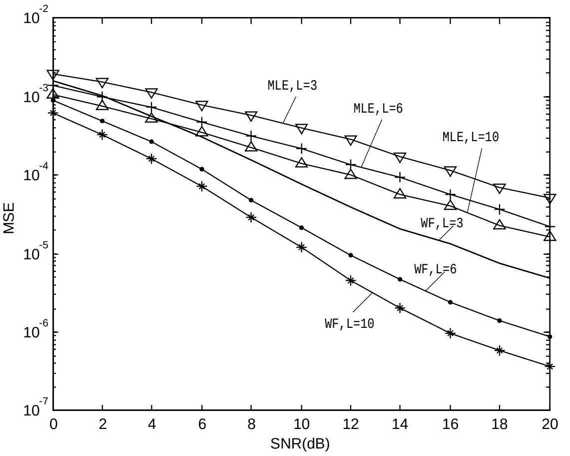

Fig. 2 Comparison of frequency offset estimation with MLE algorithms and that with WF-based algorithms.

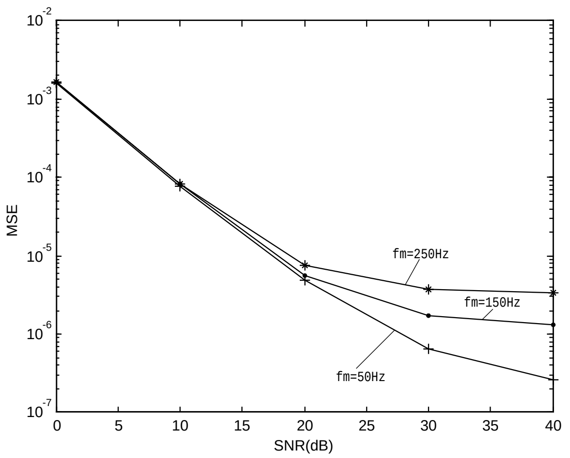

Fig. 3 Average MSE versus SNR for different Doppler spread in the WF-based algorithm.

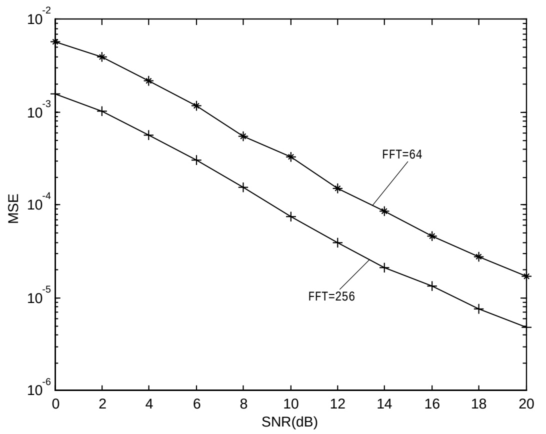

Fig. 4 Average MSE versus SNR for different number of sub-carriers in the WF-based algorithm.

Fig. 4 gives the MSE results of the proposed algorithm under different number of sub-carriers with the same interval between pilots. With the larger number of sub-carriers, the more pilots are involved, and the algorithm performance is better.

5. CONCLUSION

In this paper, a weighting function (WF) based frequency offset tracking algorithm is proposed for OFDM systems in time varying multi-path fading channels. This algorithm exploits pilot tones inserted in data symbols, and achieves an optimal estimation of frequency offset in wireless environment.

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REFERENCES

[1] Richard van Nee, "A new OFDM standard for high rate wireless lan in the 5GHz band, Vehicular Technology Conference", VTC 1999 - Fall. IEEE VTS 50th , Volume: 1 , pp. 258 –262, 1999.

[2] T. Pollet, "The BER performance of OFDM systems using Non-synchronized sampling", Proc. Of GLOBECOM, pp. 253-257, 1994.

[3] P. H. Moose, "A technique for Orthogonal Frequency Division Multiplexing Frequency Offset Correction", IEEE Transactions on communications, 42(10), pp. 2908-2914, 1994.

[4] M.Julia Fernandez-Garcia Garcia, Ove Edtors, Jose M.Paez-Borrallo, "Frequency offset correction coherent OFDM in wireless systems", IEEE Transactions on Consumer Electronics, Vol. 47, No.1, pp. 187-193, 2001.

[5] Samir Kapoor, Daniel J.Marchok, Yih-Fang Huang, "Pilot assisted synchronization for wireless OFDM systems over fast time varying fading channels", IEEE VTC, pp. 2077-2080, 1998.

[6] Theodore S. Rappaport, Wireless Communications Principles and Practice, Publishing House of Electronics Industry, Beijing, 2001.