AN ENHANCED CHANNEL ESTIMATION ALGORITHM FOR OFDM: COMBINED EM ALGORITHM AND POLYNOMIAL FITTING

Xiaoqiang Ma, Hisashi Kobayashi, and Stuart C. Schwartz

Electrical Engineering Department, Princeton University
Princeton, New Jersey 08544-5263
Email: xma, hisashi, stuart@ee.princeton.edu

ABSTRACT

Estimating a channel that is subject to frequency selective Rayleigh fading is a challenging problem in an orthogonal frequency division multiplexing (OFDM) system. In this paper we propose an enhanced channel estimation algorithm that combines the EM-based algorithms proposed previously and a Least Squares polynomial fitting (LSPF) approach. The combined algorithm can efficiently estimate the channel response of an OFDM system operating in an environment with multipath fading and additive white Gaussian noise (AWGN). The algorithm can improve the channel estimate obtained from the EM-based algorithms by polynomial fitting. Simulation results show that the bit error rate (BER) as well as the mean square error (MSE) of the channel can be improved by the algorithm. In particular, with these additional computations and demodulation delay, the MSE can be made smaller than the Cramer-Rao lower bound (CRLB).

1. INTRODUCTION

An efficient and accurate channel estimation procedure is necessary for coherent demodulation in OFDM systems. Many channel estimation algorithms have been reported in the literature [2]-[4]. Recently we proposed three different EM-based channel estimation algorithms (refer to [5]-[7]) that can achieve the CRLB in the high SNR region. In these algorithms the channel is estimated frame by frame by assuming the channel is changing from one frame to another. If we further assume the channel variation is smooth, then we might do a better job by considering several OFDM frames together. Obviously, if the channel does not change during several OFDM frames, we can certainly apply the EM-based algorithm [5]-[7] or just average the channel estimates of those OFDM frames to obtain a better estimate. However, in a mobile environment with fast moving objects this assumption fails. Fortunately, the channel changes smoothly along the time axis in a real mobile environment. This smoothness motivates us to use well designed curve fitting methods (or time-domain smoothing), e.g., least squares polynomial fitting, in order to further improve the channel estimation accuracy. This basic idea is discussed in [8] and [9]. Similar time-frequency polynomial models are adopted in these two referenced papers. Both of them concentrate on the channel frequency response which contains many more components than the corresponding CIR in a typical OFDM system. Thus in their methods, the complexity to establish model coefficients is high. Furthermore, they need a large amount of pilot symbols in the time-frequency grid of the OFDM systems to minimize the model mismatch. This degrades the overall system capacity or spectral efficiency.

The main objective of this paper is to investigate the use of the polynomial fitting method in the time domain for channel estimation of an OFDM system subject to slow time varying frequency selective fading. An EM-based channel estimation algorithm is carried out first to obtain near optimal channel estimates using the information of the current OFDM frame only. Then, polynomial fitting is adopted by gathering channel estimates of several consecutive OFDM frames. By applying this concatenated channel estimation (EM-based algorithms followed by polynomial fitting), better channel estimates can be obtained.

2. EM-BASED CHANNEL ESTIMATION ALGORITHM

The most important step in an EM-algorithm is determining what is known as “complete” data. Different selections of “complete” data result in different algorithms. Three different EM-based channel estimation and signal detection algorithms have been proposed in [5]-[7] by carefully defining different “complete” data for each algorithm. Their advantages and disadvantages are discussed therein. Basically, they have almost the same performance as measured by BER and MSE. The only difference is the rate of convergence and computational burden. The second EM-based algorithm [6] has the fastest convergence speed as seen from the simulations. Thus, we suggest using this algorithm as the first stage channel estimation method. Furthermore, one simplified version that does not need the channel statistics has negligible performance deterioration. The corresponding simplified algorithm is the one adopted.

The system model can be expressed in vector form for a particular OFDM frame as (refer to [6] for details)

$$\underline{Y} = \mathbf{X}\mathbf{W}_L\underline{h} + \underline{N}, \quad (1)$$

where L is the channel delay spread, M is the number of subcarriers and $\mathbf{W}_L$ is the submatrix of the FFT matrix with first L columns. We assume that the channel is static over the period of one frame.

The "incomplete" and "complete" data are defined as $(\underline{Y})$ and $(\underline{Y}, \underline{h})$, respectively. Each iterative process $p = 0, 1, 2, \cdots$, in the EM algorithm for estimating $\underline{X}$ from $\underline{Y}$ consists of the following two steps:

$$\mathbf{E-step}: \quad Q(\underline{X}|\underline{X}_p) = E\left\{\log f(\underline{Y}, \underline{h}|\underline{X})|\underline{Y}, \underline{X}_p\right\} \quad (2)$$

$$\mathbf{M-step}: \quad \tilde{\underline{X}}_{p+1} = \arg\max_{\underline{X}} Q(\underline{X}|\underline{X}_p), \quad (3)$$

Detail derivations and discussion are included in [6].

3. LEAST SQUARES POLYNOMIAL FITTING

A typical mathematical procedure for finding the best fitting curve to a given set of points is to minimize the sum of the squares of the offsets (the residuals) of the points from the curve. The linear least squares fitting technique is the simplest and most commonly applied form of linear regression and provides a solution to the problem of finding the best fitting straight line through a set of points. However, it also introduces large fitting error because of the model mismatch. Polynomial fitting is one of the nonlinear curve fitting methods that is often used due to its good properties. First, it is easy to realize and the computation is low compared to other nonlinear fitting techniques. More importantly, polynomials of different degrees can simulate a variety of curves.

Suppose we try to fit a set of given data with a k^{th} degree polynomial

$$y = a_0 + a_1 x + \cdots + a_k x^k. \quad (4)$$

A set of observed data $\{(x_i, y_i)|1 \leq i \leq n\}$ are given. The objective is to find a k^{th} degree polynomial that has the least offset

$$R = \sum_{i=1}^{n}[y - (a_0 + a_1 x_i + \cdots + a_k x_i^k)]^2. \quad (5)$$

The coefficients of the least squares polynomial fitting can be easily expressed as

$$\underline{a} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\underline{y}. \quad (6)$$

where where $\underline{y} = (y_1, \cdots, y_n)^T$, $\underline{a} = (a_0, \cdots, a_k)^T$ and $\mathbf{X}_{ij} = x_i^{j-1}$.

4. CONCATENATED CHANNEL ESTIMATION

A number of OFDM channel estimation methods have been reported in the literature. Shortcomings of the existing algorithms are (1) a large error floor that may be incurred by a mismatch between the estimated and real CIRs, (2) difficulty in obtaining the correlation function of the channel impulse response and inevitable error due to channel statistics mismatch, and (3) spectrum inefficiency due to the use of a high percentage (typically 20%) of pilot symbols. On the other hand, several kinds of blind channel estimation algorithms have been proposed in order to improve transmission efficiency. These algorithms are based on the statistical nature of the received signals, virtual carrier, and finite-alphabet property of the transmitted signals. However, most of them need a large amount of data to obtain reliable channel estimates. Thus, these blind algorithms can not be used in a high mobility environment (i.e., large Doppler spread). We, on the other hand, have extended the existing pilot-based channel estimation algorithms and reduced the number of pilot symbols by using the EM-algorithm [5]-[7]. These EM-based algorithms are only suitable when the D blocks of the channel are constant. If the channel is not constant, the algorithms developed fail to give reliable channel estimates. Therefore, it is desirable to use them for only one OFDM frame. Consequently, the accuracy of the channel estimates is limited by these frame-based channel estimation algorithms. More precisely, we observe that the CRLB for channel estimates is given by

$$CRLB(\underline{h}) = \frac{2L\sigma^2}{MDA}, \quad (7)$$

where D is the number of frames in which the channel is constant. The larger the D, the smaller the CRLB will be. In a typical mobile environment D can be as small as 1.

In the packet or block OFDM transmission (e.g. IEEE802.11a), one coded data packet always occupies several OFDM frames. Consequently, channel estimation among consecutive OFDM frames is required to further improve channel estimation accuracy. In such applications, one stage of channel estimation may not be accurate enough to meet the system performance requirement. Although iteration between channel estimation and channel decoding is also possible to improve channel estimation accuracy, the computational complexity is prohibitively high due to the Viterbi trellis search in each iteration. We, on the other hand, propose a so called concatenated channel estimation that combines EM-based channel estimation algorithms and LSPF as shown in Fig. 1. This concatenated channel estimation can be extended in an iterative fashion if the output performance is not satisfactory. Furthermore, this concatenated channel estimation idea can be extended to other channel estimation methods.

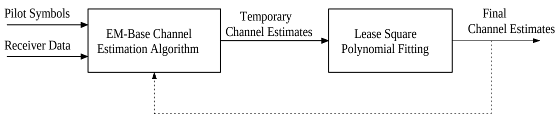


Fig. 1. Concatenated channel estimation algorithm.

5. SIMULATION RESULTS

In our channel estimation problem, $\underline{y}$ is the estimate of the l^{th} tap of CIR at consecutive OFDM frames $(\hat{h}_l(1), \cdots, \hat{h}_l(n))$, x_i denotes the i^{th} time instant and $\underline{a}$ is the polynomial coefficients from the LSPF (6).

The simulation model we use in this paper is the same as that in [6]. As previously stated, we will only show the system performance in which the described EM-based channel estimation algorithm is used in the first stage of the concatenated algorithm. Extensive simulations show that the concatenated algorithms adopting the other two EM-based channel estimation algorithms in the first stage result in similar performance. For the LSPF (the second block), we consider three different scenarios:

1. The LSPF is carried out every 8 OFDM frames.
2. The LSPF is carried out every 16 OFDM frames.
3. The LSPF is carried out every 24 OFDM frames. Only the channel estimates of the middle 8 frames are accepted as the final estimates. Thus, there is overlap between two consecutive LSPFs.

Figs. 2 and 3 illustrate the BER and MSE performance by applying the proposed concatenated channel estimation algorithm in the system described above. The BER improvement due to the LSPF is negligible since the BER after the EM-based channel estimation procedure is already good. However, the MSE improvement due to the LSPF is significant compared with the MSE of the EM-based channel estimation algorithm only. When the SNR is larger than 5dB, a MSE that is lower than the CRLB is achieved. Of course, this MSE improvement is obtained by increasing the demodulation delay. Theoretically, the longer the observed date, the more improvement we can get. In practice, we should specify the transmission delay according to the system requirement. For LSPFs with longer observed data sequence, higher degree polynomials should be considered to minimize the model mismatch. However, the complexity also becomes higher. Thus, a tradeoff must be carefully considered according to different system parameters and requirements.

Fig. 4 shows a realization of the amplitude of the first channel tap h_1. The curve of initial channel estimates is a step function, which results in a large deviation from the real channel. The curve of EM-based channel estimates oscillates around the real channel. The channel mismatch is higher when the deep fades occur. By filtering out spikes in the estimates of the EM-based algorithm, the curve of channel estimates after applying the LSPF is very smooth. It is also fairly close to the real channel response. It demonstrates the effectiveness of the concatenated channel estimation algorithm.

There are two underlying reasons for the MSE improvement. The obvious one is that the concatenated algorithm uses additional information from other OFDM frames which are located close to the current OFDM frame. The more important reason is that the channel estimates obtained by the EM-based algorithms can be approximated as

$$\underline{\hat{h}}_{EM}(t) = \underline{h}(t) + \underline{n}_{EM}(t), \quad (8)$$

where $\underline{\hat{h}}_{EM}(t)$ is the channel estimate obtained from the EM-based algorithms at time t, $\underline{h}(t)$ is the true CIR and $\underline{n}_{EM}(t)$ is the residual estimation error. After EM-based iterations, $\underline{n}_{EM}(t)$ can be considered as independent Gaussian noise for different OFDM frames, as is justified in Fig. 5(a) and 5(b). The empirical distribution is close to the Gaussian distribution with zero mean and variance σ^2_{EM}. The analytical curve is the Gaussian density function with zero mean and variance equal to the MSE after the EM-based channel estimation procedure.

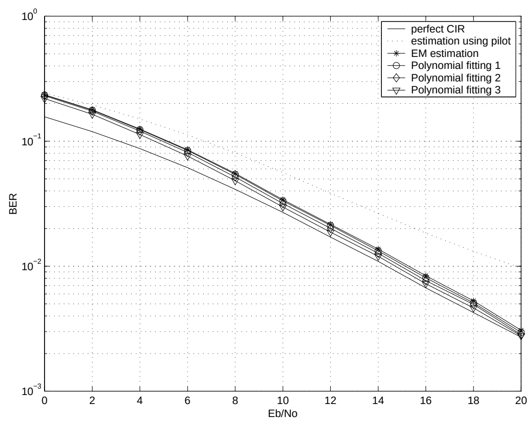


Fig. 2. BER of channel estimates for OFDM systems EM-based algorithm concatenated with LSPF.

Therefore, the LSPF can reduce the channel estimation error by taking advantage of the Gaussian-like errors. If there is a common residual error (bias) in all the channel estimates used to do LSPF, this residual error can not be cancelled. This can happen if we take channel estimate of the previous frame as the the channel estimate of the current frame. Polynomial interpolation will cause a similar problem. Thus, in order to make the concatenated channel estimation idea work properly, the first stage should yield Gaussian-like estimation errors. EM-base algorithms exactly provide such estimates through their iterative proce-

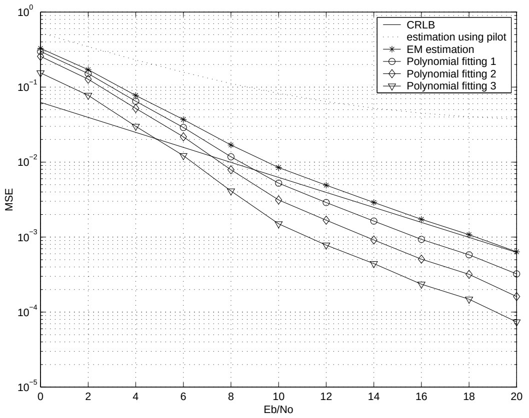


Fig. 3. MSE of channel estimates for OFDM systems EM-based algorithm concatenated with LSPF.

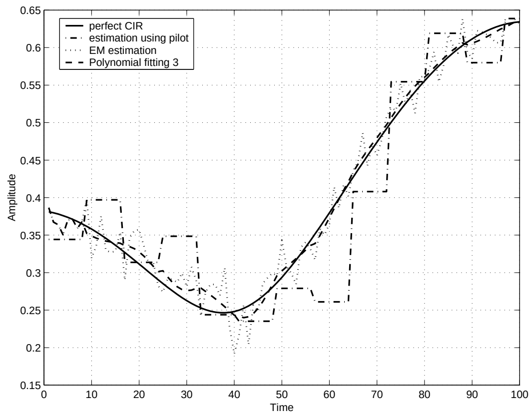
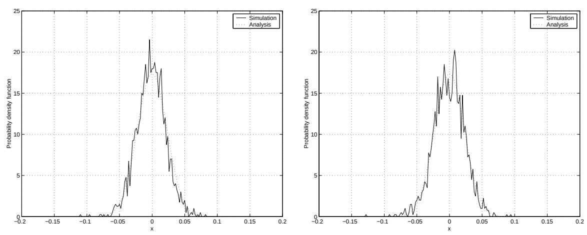


Fig. 4. The realization of the amplitude of h_1 when SNR equals to 10dB.



(a) PDFs of the estimate of $\Re(h_1)$

(b) PDFs of the estimate of $\Im(h_1)$

Fig. 5. Simulated and analytical PDFs of the estimate of $\Re(h_1)$ and $\Im(h_1)$ when SNR equals to 10dB.

dures. Another merit of EM-base algorithms is the good MSE performance that is close to the CRLB. Otherwise, a MSE lower than the CRLB could not be achieved. That is why the concatenated channel estimation algorithm with the various EM-based algorithms in the first stage will yield similar final performance after applying the LSPF.

6. CONCLUSION

In this paper we proposed a concatenated channel estimation algorithm for OFDM systems in order to enhance system performance. In particular, EM-based channel estimation algorithms followed by LSPF is proposed and tested. Simulation results show a significant MSE improvement, compared to the EM-based algorithm alone. The proposed concatenated channel estimation method can be used in other applications such as single carrier systems with frequency domain channel estimation. In order to make the concatenated channel estimation work, Gaussian-like estimation errors should be generated from the first stage.

Note: Additional details in a longer paper and larger figures can be found at http://www.ee.princeton.edu/˜xma/fit.pdf.

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