EFFICIENT CHANNEL ESTIMATION BASED ON DISCRETE COSINE TRANSFORM

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ABSTRACT

Channel impairment caused by multi-path reflections can deeply degrade the transmission efficiency in wireless communication systems. Based on the property of the channel frequency response and the concept of interpolation, in this paper a DCT-based pilot-aided channel estimator for orthogonal frequency division multiplexing is proposed. This approach can mitigate the aliasing effect in the DFT-based channel estimator when there is non-sample-spaced path delay. Compared with DFT-based estimator, DCT-based estimator significantly improves the performance with a comparable complexity. In addition, a noise reduction scheme is introduced and combined with the estimator. In implementation, the DCT-based estimator has the advantages of utilizing mature fast DCT algorithms and compatible FFT algorithms, which is favorable to other matrix-based channel estimation methods.

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a highly efficient and popular technique for high bit-rate data transmission over wireless communication channels. It has been adopted in wireless LAN and MAN standards IEEE 802.11a and 802.16, and the European digital audio broadcasting (DAB) and digital video broadcasting (DVB) standards.

In wireless communication channels, multi-path is a very common and severe problem. It causes inter-symbol interference (ISI) in the signal stream and this may degrade the transmission efficiency. OFDM can easily avoid this problem by inserting guard interval (GI). Besides ISI, multi-path also causes frequency-selective fading. If coherent demodulation is adopted, the effect of amplitude and phase fluctuation should be mitigated. One typical solution is to perform channel estimation, followed by channel equalization. Generally, there are two types of approaches for channel estimation. One is blind type of algorithms [1] and the other is pilot-aided type of algorithms [2]. Although the pilot-aided algorithms waste a little more bandwidth than the blind algorithms, their performance is usually better than that of the blind case. Pilot-aided approach has been adopted in many standards such as 802.11a and many others. Therefore, in this paper we will focus on the pilot-aided case.

The optimal interpolation filtering in Minimum Mean Square Error (MMSE) sense for channel estimation [3], [4] needs the information of channel statistic and the associated computation complexity is very high. This may be hard to implement in practice. The approach of DFT-based interpolation [5] can theoretically achieve ideal lowpass interpolation, and has the advantages of low complexity by employing FFT algorithms. This technique works well when multi-path delays are integer multiples of the sampling time. However, this hardly happens in practical transmission environment. When the condition is not satisfied, performance of the DFT-based algorithm may degrade considerably. This is because the equivalent channel impulse response will be a disperse version of the original shorter one [6]. As a result, the DFT-based interpolation process will be based on the aliased data of the disperse impulse response.

In this paper, for the consideration of better channel interpolation result and lower aliasing error, we will propose a DCT-based channel estimation method, as detailed below.

2. CHANNEL ESTIMATION BASED ON DFT INTERPOLATION

2.1. OFDM System Model

We assume that an OFDM symbol contains N sub-carriers, and the OFDM symbol duration is T. Then the sampling period will be T/N and the sub-carrier spacing is $1/T$. The transmitted signal can be expressed as:

$$s(t) = \sum_{i=-\infty}^{\infty}\sum_{n=0}^{N-1} D_{i,n}\theta_{i,n}(t) \quad (1)$$

where $D_{i,n}$ is the data on the n-th sub-carrier in the i-th OFDM symbol and

$$\theta_{i,n}(t) = e^{\frac{j2\pi n}{T}(t-T_g-iT_c)}[u(t-iT_c)-u(t-(i+1)T_c)] \quad (2)$$

where T_g is the guard time interval, $T_c = T_g + T$ is the total symbol duration, and $u(t)$ is the unit step function.

A multi-path channel can be characterized as:

$$h(t,\tau) = \sum_{i=0}^{L-1} \alpha_i(t) \cdot \delta(\tau - \tau_i) \quad (3)$$

where $\alpha_i(t)$ is the time-varying gain and τ_i is the delay time for the i-th path. L is the total number of the paths. Usually, the magnitude of $\alpha_i(t)$ is modeled Rayleigh distributed, and the variation is associated with Doppler frequency f_d , $f_d = f_c v / c$, where f_c is the carrier frequency, v is the vehicle speed and c is the velocity of light.

The received OFDM signal passing through the AWGN time-varying multi-path channel can be expressed as

$$r(t) = \sum_{i=0}^{L-1} \alpha_i(t) \cdot s(t-\tau_i) + n(t) \quad (4)$$

where $n(t)$ is the white Gaussian noise. After sampling the signal and removing guard interval, the equivalent channel frequency response is (assuming $\alpha_i(t)$ is constant over one OFDM symbol)

$$H_{s,k} = \sum_{i=0}^{L-1} \alpha_{s,i} \cdot e^{-\frac{j2\pi\tau_i k}{T}} \quad (5)$$

where $H_{s,k}$ is channel frequency response corresponding to the k-th sub-carrier of the s-th symbols, and $\alpha_{s,i}$ is the gain of the i-th path during the s-th symbol period. The received signal on the k-th sub-carrier of the s-th symbol can be expressed as

$$Y_{s,k} = D_{s,k} \cdot H_{s,k} + N_{s,k} \quad (6)$$

The corresponding impulse response is [6]

$$h_{s,n} = \frac{1}{\sqrt{N}} \sum_{i=1}^{L-1} \alpha_{s,i} e^{-j\frac{\pi(n+(N-1)\lambda_i)}{N}} \frac{\sin(\pi\lambda_i)}{\sin(\pi(\lambda_i - n)/N)} \quad (7)$$

where $h_{s,n}$ is the n-th tap of channel impulse response during the s-th symbol and $\lambda_i = \tau_i / T_s$, where T_s is the sampling period. By this equation, when non-integer λ_i exists, the power will leak to all taps $h_{s,n}$, as shown in Fig. 1.

2.2 DFT-based channel estimation [7]

Assume that M pilots are evenly assigned to M sub-carriers out of total N sub-carriers at a spacing of N/M sub-carriers, where N/M is an integer. The DFT-based channel estimation algorithm begins with the least-square (LS) estimation of the pilot sub-carriers.

$$\hat{H}_{p,m} = Y_{p,m} / p_m \quad (8)$$

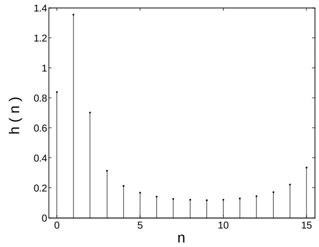

Fig.1. The equivalent impulse response for the continuous channel $h(t) = \delta(t) + \delta(t-0.5Ts) + \delta(t-1.4Ts)$

where $Y_{p,m}$ is the received signal at the m-th pilot subcarrier and p_m is the pre-assigned pilot value for the m-th pilot subcarrier. Then $\hat{H}_{p,m}$ is multiplied by some linear-phase shift as shown below

$$\hat{H}_{p',m} = \hat{H}_{p,m} \cdot e^{j\pi\frac{m\beta}{MT}} \quad (9)$$

where β is the minimum integer greater than all the path delays. The operation amounts to a corresponding time shift of the impulse response. It would make the power of the impulse response much more concentrate around t=0, while the impulse response values in the middle time positions would be smaller. This will facilitate zero insertion in those positions, and lead to a more effective up-sampling result of the channel frequency response, than the case without phase adjustment, as detailed below.

First the M-point impulse response is obtained by

$$\{\hat{h}_p\} = IFFT\{\hat{H}_{p'}\} \quad (10)$$

Next the zero-insertion impulse response is formed by inserting (N-M) zeros in the middle time indices:

$$\hat{h}_{N,n} = \begin{cases} \hat{h}_{p,n} & n \le M/2-1 \\ 0 & M/2 \le n \le N-M/2-1 \\ \hat{h}_{p,n-N+M} & otherwise \end{cases} \quad (11)$$

Then the interpolated channel frequency response is solved after performing FFT on $\hat{h}_N$.

$$\{\hat{H}_{sh}\} = FFT\{\hat{h}_N\} \quad (12)$$

Finally, the actual estimated channel frequency response is obtained by canceling the phase shift operations performed in the beginning stage of the algorithm:

$$\hat{H}_n = \hat{H}_{sh,n} \cdot e^{-j\pi\frac{n\beta}{NT}} \quad (13)$$

3. THE PROPOSED ALGORITHM

As mentioned before, there will be leakage in channel impulse response, when the path delays are non-integer multiples of the sampling period. It is obvious that DFT-based interpolation is not suitable for channel estimation under this condition. This is because the leakage will cause severe aliasing, when the mentioned DFT-based method is used. [7] proposed a windowed DFT-based approach to improve the performance. However, this approach must sacrifice some bandwidth. Next, we will propose a DCT-based interpolation algorithm to mitigate the aliasing problem. DCT is a well-known technique extensively used in image processing. DCT can reduce the high frequency component in the transform domain compared with DFT. The reason is that when given a sequence of N-point data, DFT conceptually treats it as a periodic signal with a period of *N* points. Hence, there is a tendency of noticeable high-frequency components, due to signal discontinuity in between consecutive period boundary. In contrast, DCT conceptually extends the original *N*-point data sequence to 2*N*-point sequence by doing mirror-extension of the *N*-point data sequence. As a result, the waveform will be smoother and more continuous in the boundary between consecutive periods. Correspondingly, high frequency components will be reduced. This benefits interpolation process. The proposed DCT-based channel estimation algorithm is detailed below.

3.1 The new DCT-based channel estimation algorithm

First, we also use LS estimation to get the channel frequency response on the pilot sub-carriers. After that, we perform DCT

$$\hat{h}_{c,k} = w_k \sum_{m=0}^{M-1} H_{p,m} \cos\frac{\pi(2m+1)k}{2M}, \qquad k = 0,\dots,M-1$$
$$w_k = \frac{1}{\sqrt{M}},\ k=0;\ w_k = \sqrt{\frac{2}{M}},\ k \neq 0 \tag{14}$$

The next step inserts zeros in the DCT domain. However, different from DFT-based interpolation, zeros must be inserted at the end of $\hat{h}_c$ as

$$\hat{h}_{N,k} = \begin{cases} \hat{h}_{c,k} & k \le M-1 \\ 0 & otherwise \end{cases} \qquad k = 0,\dots,N-1 \tag{15}$$

Here IDCT can't be directly performed on $\hat{h}_N$ to get the channel frequency response due to the following reason. Compared with DFT, DCT has a shift in the time domain data. Due to this characteristic, the value of the original low-rate data would not remain the same after interpolation process by employing IDCT. Therefore, the interpolation result would be poor. The solution is to use extendible IDCT (EIDCT) [8]. Based on EIDCT, we can get the interpolated channel estimation as

$$\hat{H}_n = \sum_{k=0}^{M-1} w_k \hat{h}_{N,k} \cos\left(\left(\frac{n}{N} + \frac{1}{2M}\right)\pi k\right) \qquad n = 0,\dots,N-1 \tag{16}$$

Alternatively, since the transform is derived from the concept of DFT, we can get the same result by first doing mirror-duplication to get doubled-length data and then applying the DFT-based interpolation.

One may argue that we can exchange the DCT and IDCT processes in the interpolation, then the time shift problem will not occur. Indeed, this is true. However, if we adopt this approach, another problem similar to DFT-based interpolation will be introduced. In the *M*-point DCT transform (14), its value is always zero at $k=M$. Therefore, if we treat the original data as DCT transform domain signal, the estimated channel frequency response after interpolation will decay to zero outside the last pilot sub-carrier. As it turns out, this would lead to degradation of performance at the edge of spectrum.

3.2 Combining a noise reduction scheme

When the delay time of each path is close to zero, the white Gaussian noise can be effectively reduced in the DCT domain. If the path delays are all small, the channel frequency response will be smoother (with less high frequency components). As such, in the DCT domain, the power in the high frequency region can be viewed as noise, and we can eliminate it by setting the value of high frequency to zero. The method works better in the DCT domain than in the DFT domain [5]. Especially, it is most effective when the pilot power is not much larger than the noise power. When the pilot power is limited to a lower level, for low-power consideration, this method can improve performance. The whole operations are detailed below.

After DCT operations, the accumulated power counting from the first index can be calculated. The value is compared with a threshold to determine the region occupied mostly by noises. One way to define the threshold is using percentage of total power, e.g. 90% of total power. After the index is determined, all the impulse response values after this index are set to zero as

$$\hat{h}_{cc,k} = \begin{cases} \hat{h}_{c,k} & 0 \le k \le b \\ 0 & b < k \le M-1 \end{cases} \tag{17}$$

where *b* is the index of threshold.

Note that regardless of DCT-based or DFT-based approaches, the delay spread must be smaller than ($M \cdot T_s$). Otherwise, the estimation will be error prone. This can be explained by the concept of down sampling. The frequency responses at the pilot sub-carrier frequencies are the down sampling version of the complete channel frequency response at all *N* sub-carrier frequencies. Hence, if the delay spread is equal to or larger than ($M \cdot T_s$), then the aliasing of channel impulse will occur. There is no way to recover the aliased impulse response.

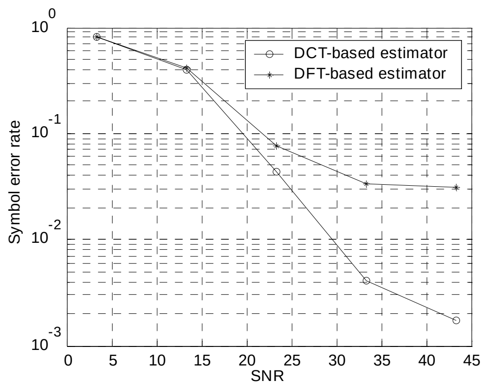

Fig.2. The SER performance with DCT-based estimator compared with DFT-based estimator

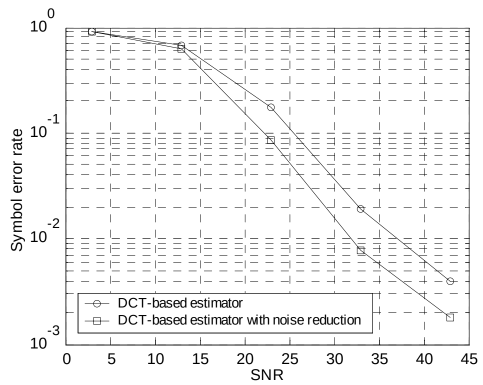

Fig.3. The SER performance of DCT-based estimator with noise reduction

4. SIMULATION RESULT

In this section, we present the simulation result of the DCT-based estimator and compared it with DFT-based approach. The multi-path Rayleigh fading channel is simulated by Jakes' model. And each path gain follows the exponential power delay profile.

$$E[|\alpha_i(t)|^2] = e^{-\mu\cdot\tau_i} \qquad (18)$$

We assume the channel has 4 paths and the set of delay spread is {0, 3.5*Ts*, 7.3*Ts*, 10.4*Ts*}. Meanwhile, we choose μ such that the average power of last path will be 20dB less than first path.

The number of total sub-carriers is 1024. 32 pilots are evenly inserted into the sub-carriers, and the first pilot is put on the first sub-carrier. Assume the transmission bandwidth is 5MHz. Then the sub-carrier spacing is 4.883KHz, and the sampling period is 0.2 μs . The Doppler spread is fixed at 50Hz, such that $f_d T \approx 0.01$. The modulation scheme on each sub-carrier is 16QAM. The guard time interval is 32 sample periods. As for the value assigned to pilot, the outmost constellation point in 16QAM is chosen. Fig.2 shows the simulation result. It is obvious that DCT-based approach noticeably has higher performance especially in high SNR, than the DFT counter part.

We also simulate the case when the proposed new algorithm method combines with a noise reduction scheme as mentioned before. In this case, the set of delay spread is assumed {0, 0.5*Ts*, 2.2*Ts*, 3.1*Ts*}. As explained in section 4, the delay values cannot be too far away from zero. Also we change the pilot value from the outmost constellation point in 16QAM to the innermost point to reduce the pilot power. The threshold is set to 90% of the total power. Fig. 3 depicts the simulation result.

5. CONCLUSION

A DCT-based pilot-aided channel estimator of OFDM system in the multi-path fading channel with non-integer sample-spaced path delay has been proposed in this paper. It achieves significant improvement over the DFT-based approach. It can be realized by the mature, low-complexity fast DCT algorithms in the literature. It is much lower than many other well-known matrix-based estimators. For the case of small path delay spreads and pilots with low power level, we also propose an effective noise reduction method to improve the performance.

6. REFERENCES

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