A CHANNEL ESTIMATOR FOR OFDM TRANSMITTER DIVERSITY SYSTEMS WITHOUT A CYCLIC PREFIX

King F. Lee

Multimedia Architecture Lab
Motorola Labs
Schaumburg, Illinois 60196
USA

Douglas B. Williams

School of Electrical and Computer Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332-0250
USA

ABSTRACT

OFDM transmitter diversity systems that do not required a cyclic prefix have recently been proposed as means to improve bandwidth efficiency. For these systems, knowledge of the channel parameters is required at the receivers for interference cancellation, diversity combining, and decoding. In this paper, we propose a low complexity pilot-symbol-assisted channel estimator for OFDM transmitter diversity systems without a cyclic prefix. The pilot symbols are constructed to be periodic within an OFDM symbol interval in order to avoid ISI and ICI and to be non-overlapping in frequency to allow for the simultaneous sounding of the multiple diversity channels. The time-varying channel responses are then tracked by interpolating a set of estimates obtained through periodically transmitted pilot symbols. The effectiveness and limitations of the proposed estimator are verified by computer simulations.

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) transmitter diversity techniques have been proposed [1–5] for high data rate wireless communications in frequency-selective channels. Most of the proposed OFDM transmitter diversity systems require a cyclic prefix to be added to the transmitted symbols to avoid intersymbol interference (ISI) and interchannel interference (ICI). The addition of the cyclic prefix causes bandwidth expansion if a desired data rate is to be maintained, or a reduction in data rate if the transmission bandwidth is fixed. For many high data rate systems, the addition of a cyclic prefix can cause more than a 15% bandwidth expansion, which is a very significant loss of a valuable resource. In [5], bandwidth efficient iterative space-time and space-frequency block-coded OFDM (ISTBC-OFDM and ISFBC-OFDM) transmitter diversity techniques that do not require a cyclic prefix were proposed. Without a sufficiently long cyclic prefix, the subchannels of these OFDM systems are distorted by ISI and ICI, complicating the channel estimation process. Here, we propose a pilot-symbol-assisted (PSA) channel estimator that eliminates the ISI and ICI caused by the lack of the cyclic prefix during the pilot symbol duration, making it suitable for ISTBC-OFDM, ISFBC-OFDM, and other OFDM transmitter diversity systems without a cyclic prefix.

2. OFDM TRANSMITTER DIVERSITY SYSTEMS

In conventional OFDM systems, the use of a cyclic prefix transforms the frequency-selective fading channel into multiple perfectly decoupled flat fading subchannels. The OFDM transmitter diversity systems in [1–4] all rely on this special property of OFDM with a cyclic prefix in the precoding, decoding, and channel estimation processes to achieve impressive diversity gain in frequency-selective fading channels. A block diagram of a general two-branch OFDM transmitter diversity system with a cyclic prefix is shown in Fig. 1. Assuming the length of the cyclic prefix is greater than or equal to the order of the channel impulse responses (CIRs) and the CIRs remain constant during the entire OFDM symbol interval KT_S, it can be easily shown that the demodulated signal is given by [3]

$$\mathbf{Y}\left(n\right) = \mathbf{\Lambda}_1\left(n\right)\mathbf{X}_1\left(n\right) + \mathbf{\Lambda}_2\left(n\right)\mathbf{X}_2\left(n\right) + \mathbf{Z}\left(n\right), \quad (1)$$

where $\mathbf{X}_1(n)$ and $\mathbf{X}_2(n)$ are the two transmitter diversity encoded data vectors, $\mathbf{\Lambda}_1(n)$ and $\mathbf{\Lambda}_2(n)$ are two diagonal matrices whose elements are the DFTs of the respective CIRs, and $\mathbf{Z}(n)$ is the DFT of the channel noise. The system is ISI and ICI free, and all the subchannels are decoupled, which greatly simplifies the channel estimation processes.

For OFDM transmitter diversity systems without a cyclic prefix, the demodulated signal is significantly different. A block diagram of the ISFBC-OFDM system is shown in Fig. 2. Although not shown here, the ISTBC-OFDM system has a similar structure [5]. A key difference between Fig. 1 and Fig. 2 is that the modulated signals are transmitted without a cyclic prefix. As a result, the demodulated signal is distorted by ISI and ICI. A channel estimation technique that is robust to ISI and ICI is needed for these newly proposed bandwidth efficient systems.

3. CHANNEL ESTIMATION FOR OFDM TRANSMITTER DIVERSITY SYSTEMS WITHOUT A CYCLIC PREFIX

A number of channel estimation techniques for OFDM transmitter diversity systems have previously been proposed [6–8]. However these channel estimation techniques are not suitable for OFDM transmitter systems without a cyclic prefix. In the absence of a sufficiently long cyclic prefix, the subchannels of these OFDM systems are distorted by ISI and ICI. The highly desirable decoupled relationship in (1), which the decision-directed MMSE channel estimator in [6], the decision-directed EM-based channel estimator in [7], and the PSA channel estimator in [8] all depend on, is no

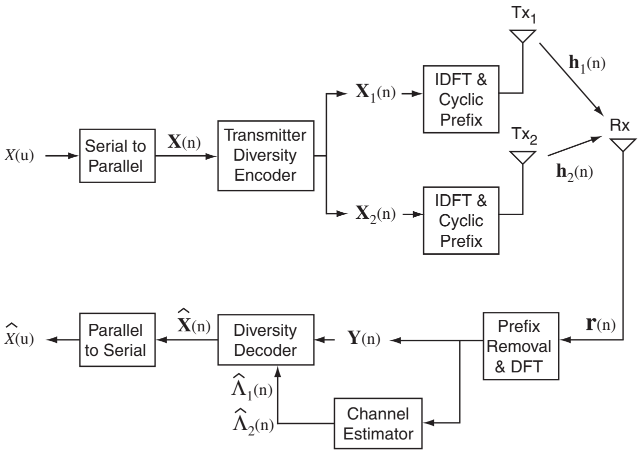

Fig. 1. Block diagram of a two-branch OFDM transmitter diversity system utilizing a cyclic prefix.

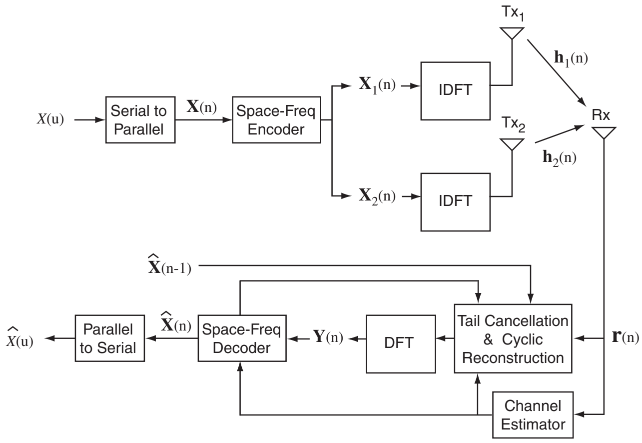

Fig. 2. Block diagram of the ISFBC-OFDM transmitter diversity system.

longer valid. Therefore, neither the decision-directed channel estimators in [6, 7] nor the PSA channel estimator in [8] is directly applicable to OFDM transmitter diversity systems without a cyclic prefix. With the decision-directed approach, in addition to minimizing the interference among the multiple transmitted signals, the channel estimator would also have to eliminate the ISI and ICI caused by the lack of the cyclic prefix during the data transmission mode. Hence, any decision-directed approach is unlikely to yield an effective channel estimator for OFDM transmitter diversity systems without a cyclic prefix. On the other hand, with the PSA channel estimator, the ISI and ICI caused by the lack of the cyclic prefix only need to be eliminated during the pilot mode, which is generally an easier problem to be solved. Therefore, we propose a modification to the PSA channel estimator in [8], making it suitable for OFDM transmitter diversity systems without a cyclic prefix.

First, note an interesting property of any length-K sequence $s(m)$ with only even harmonics; i.e., all the odd frequency bins are zero; is that the sequence $s(m)$ is periodic in $K/2$. That is, $s(m) = s(m + K/2)$ for $0 \leq m \leq K/2 - 1$. The first half of the sequence is in effect the cyclic extension of the second half of the sequence and, therefore, can be used just like a length $K/2$ guard interval for the second half of the sequence [9]. The PSA channel estimator developed in [8] can be extended to work with OFDM transmitter diversity systems without a cyclic prefix by using pilot sequences with the above cyclic property.

Define a length-K chirp sequence as

$$C\left(k\right) = e^{j\frac{\pi k^2}{K}}, \quad 0 \leq k \leq K - 1\,. \tag{2}$$

Let $PS_m(n, k)$ denote the kth tone of the pilot symbol transmitted from the mth transmit antenna during the block instant n. The pilot symbols are constructed as

$$PS_m\left(n, k + 2(m-1)\right) = \begin{cases} (-1)^m \sqrt{M}\, C\left(k + 2(m-1)\right), & \text{if } (k)_{2M} = 0; \\ 0, & \text{otherwise}, \end{cases} \tag{3}$$

where $C(k)$ is the chirp sequence as defined in (2), M is the number of transmitters, $(k)_{2M}$ denotes k modulo $(2M)$, $1 \leq m \leq M$, $0 \leq k \leq K - 1$, and $1 \leq m + k \leq K$. Fig. 3 shows the pilot symbol patterns for a typical two-branch OFDM transmitter diversity system without a cyclic prefix.

Notice that the pilot symbols in Fig. 3 satisfy the following properties:

1. The pilot symbols transmitted from different transmitters occupy different frequency bins. This property enables the avoidance of interference among pilot symbols from different transmitters and is the same property as that implemented for the channel estimator in [8].
2. The pilot symbols transmitted from the same transmitter have nonzero values only on even subcarriers. This property ensures that the time domain pilot sequence is periodic in $K/2$ so that the first half of the sequence can serve as the guard interval for the second half of the sequence.

At the receiver, the last $K/2$ samples of the received signal vector $\mathbf{r}_{PS}(n)$ are assigned to the vector $\mathbf{y}(n)$ as

$$y(n, k) = \begin{cases} r_{PS}\left(n, k + \frac{K}{2}\right), & \text{for } 0 \leq k \leq \frac{K}{2} - 1\,; \\ r_{PS}\left(n, k\right), & \text{for } \frac{K}{2} \leq k \leq K - 1\,, \end{cases} \tag{4}$$

where the subscript PS denotes the received signal during the pilot mode. The resulting vector $\mathbf{y}(n)$ is simply the cyclic extension of the received signal after the removal of the guard interval. The vector $\mathbf{y}(n)$ is then demodulated with a DFT to yield the input signal $\mathbf{Y}(n)$ to the channel estimator. With the pilot symbols constructed as in (3), the cyclic property is ensured during the pilot mode and each symbol in $\mathbf{Y}(n)$ contains only the pilot contribution from one transmitter. The complex gain of the $(k+2(m-1))-\text{th}$ subcarrier from the $m-\text{th}$ transmitter can be estimated by

$$\widetilde{\Lambda}_m\left(n, k + 2(m-1)\right) = \begin{cases} \dfrac{Y\left(n, k + 2(m-1)\right)}{PS_m\left(n, k + 2(m-1)\right)}, & \text{if } (k)_{2M} = 0\,; \\ 0, & \text{otherwise.} \end{cases} \tag{5}$$

Notice that the nonzero estimate

$$\widetilde{\Lambda}_m\left(n, k + 2(m-1)\right) = \Lambda_m\left(n, k + 2(m-1)\right) + W\left(n, k + 2(m-1)\right),$$

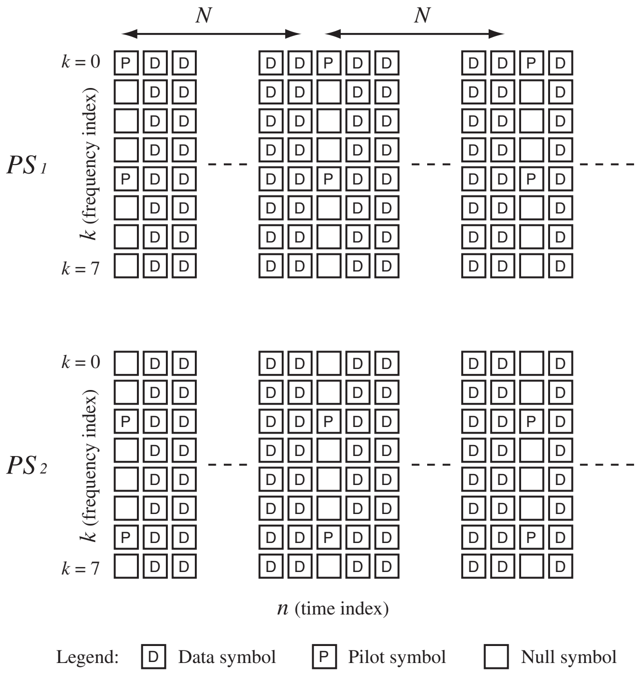

Fig. 3. Pilot symbol patterns for an OFDM transmitter diversity system without a cyclic prefix where $K = 8$ and $M = 2$.

where $\Lambda_m\left(n, k+2(m-1)\right)$, is the actual complex gain of the $(k+2(m-1))-\mathrm{th}$ subcarrier from the $m\mathrm{th}$ transmitter, and $W\left(n, k+2(m-1)\right)$ is a zero-mean complex Gaussian random variable with variance $\sigma_W^2 = \frac{\sigma_Z^2}{2M}$.

The diagonal elements of $\widehat{\mathbf{\Lambda}}_m(n)$ are, in effect, samples of the frequency response of the channel between the $m\mathrm{th}$ transmitter and the receiver. Let $\widetilde{\mathbf{h}}_m(n)$ be the IDFT of the diagonal of $\widetilde{\mathbf{\Lambda}}_m(n)$. In the absence of noise, $\widetilde{\mathbf{h}}_m(n)$ is related to the actual CIR $\mathbf{h}_m(n)$ by [10]

$$\widetilde{h}_m(n,k) = \frac{1}{2M}\sum_{l=0}^{2M-1} h_m\left(n, \left(k + \frac{K}{2M}l\right)_K\right) e^{j\frac{\pi m}{M}l}. \quad (6)$$

Notice that $\widetilde{\mathbf{h}}_m(n)$ is the sum of circularly shifted images of $\mathbf{h}_m(n)$. The images in (6) are the direct result of sampling in the frequency domain. To avoid aliasing in the time domain, the condition $K \geq 2M(L+1)$ must be satisfied. To remove the images, $\widetilde{\mathbf{h}}_m(n)$ is passed through a length $L+1$ rectangular window of gain M to yield the temporal estimate $\widehat{\mathbf{h}}_m(n)$ at the pilot instant as

$$\widehat{h}_m(n,k) = \begin{cases} h_m(n,k) + \xi(n,k), & 0 \leq k \leq L\,; \\ 0, & L+1 \leq k \leq K-1\,. \end{cases} \quad (7)$$

The DFT of $\widehat{\mathbf{h}}_m(n)$ yields the estimate of the channel parameters

$$\widehat{\mathbf{\Lambda}}_m(n) = \mathbf{\Lambda}_m(n) + \mathbf{\Xi}(n)\,, \quad (8)$$

where the elements of the noise vector $\mathbf{\Xi}(n)$ have a variance of $\sigma_W^2 \frac{2M(L+1)}{K}$. Since $2M\left(L+1\right) < K$ in general, in addition to removing the images the windowing operation also reduces the variance of the noise by a factor of $\frac{2M(L+1)}{K}$. These temporal estimates at the pilot instants, $\widehat{\mathbf{h}}_m(n)$, are then passed through a third order least-square interpolation filter [8] to provide the estimated channel parameters during the data transmission mode. A block diagram of the proposed PSA channel estimator for a two-branch OFDM transmitter diversity system without a cyclic prefix is shown in Fig. 4.

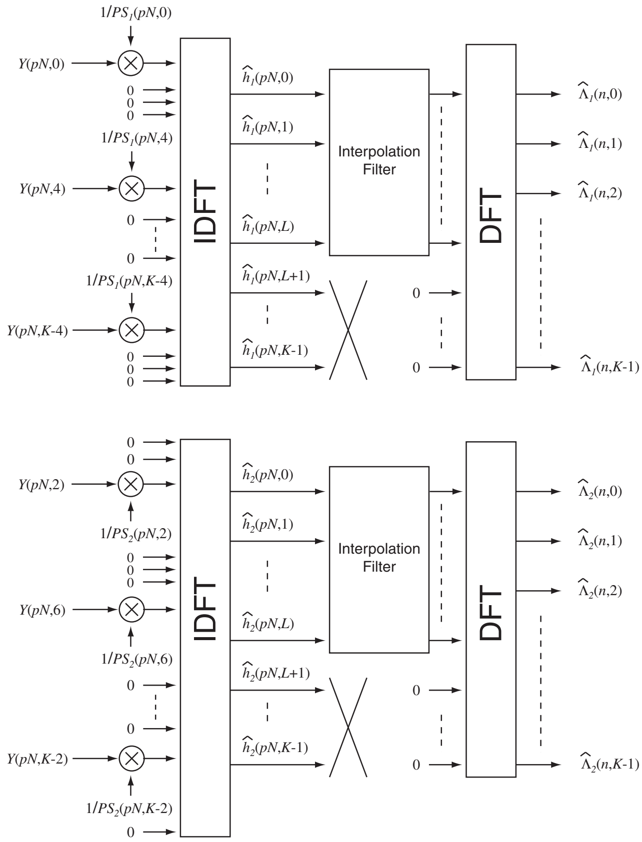

Fig. 4. Block diagram of the proposed PSA channel estimator for a two-branch OFDM transmitter diversity system without a cyclic prefix.

The performance of the proposed PSA channel estimator for OFDM transmitter diversity systems without a cyclic prefix has been evaluated by simulations. The simulations used $K = 128$ and $N = 20$ for ISTBC-OFDM and $K = 256$ and $N = 10$ for ISFBC-OFDM. Simulation results of the average BER after 2 iterations (i=2) for a two-branch ISTBC-OFDM system with ideal channel parameters and with channel parameters estimated by the proposed PSA channel estimator with a third order least-square interpolator are shown in Fig. 5. Simulation results for the ISFBC-OFDM system are shown in Fig. 6.

At low SNR and with estimated channel parameters, both the ISTBC-OFDM and the ISFBC-OFDM systems have about 2 dB performance degradation from the corresponding systems using ideal channel parameters. At high SNR, the BER performance of the ISTBC-OFDM system with estimated parameters approaches that with the ideal parameters. The ISFBC-OFDM system, however, still exhibits a slight degradation with estimated parameters, especially in faster fading environments ($f_D = 100\mathrm{Hz}$). The

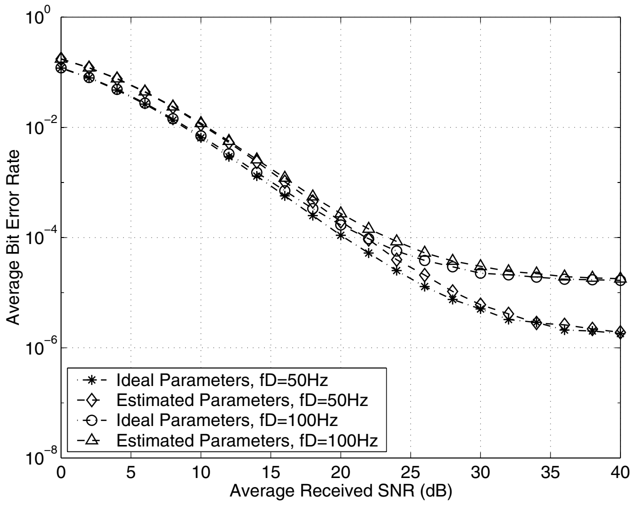

Fig. 5. Performance comparison of ISTBC-OFDM systems with ideal channel parameters and channel parameters estimated with a 3rd order least-square interpolation filter.

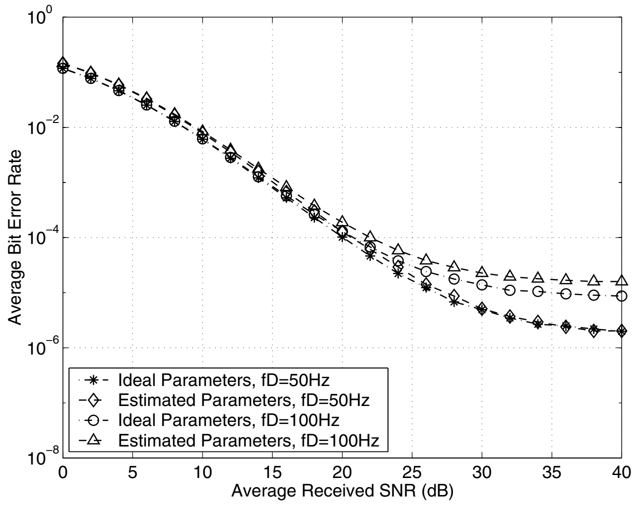

Fig. 6. Performance comparison of ISFBC-OFDM systems with ideal channel parameters and channel parameters estimated with a 3rd order least-square interpolation filter.

ISFBC-OFDM system seems to be more sensitive to channel estimation error at faster fading environments than the ISTBC-OFDM system. The cause of this difference in sensitivity to channel estimation between the two systems is a subject for further investigation.

4. SUMMARY

A low complexity, bandwidth efficient, pilot-symbol-assisted channel estimator for OFDM transmitter diversity systems without a cyclic prefix has been presented in this paper. The operation of the proposed channel estimator has been verified with the ISTBC-OFDM and ISFBC-OFDM systems by computer simulations. For ease of presentation, this paper has focused on systems with two transmit antennas ($M = 2$) and a single receive antenna. It should be noted that the proposed approach is also applicable to systems with a larger number of transmit antennas ($M > 2$), and can be easily extended to systems with multiple receive antennas by replicating the proposed channel estimator at each receive antenna branch.

5. REFERENCES

[1] D. Agrawal, V. Tarokh, A. Naguib, and N. Seshadri, "Space-time coded OFDM for high data-rate wireless communication over wideband channels," in *Proc. IEEE VTC*, Ottawa, Ont., Canada, May 1998, vol. 3, pp. 2232–2236.

[2] Y. Li, J. C. Chuang, and N. R. Sollenberger, "Transmitter diversity for OFDM systems and its impact on high-rate data wireless networks," *IEEE J. Select. Areas Commun.*, vol. 17, no. 7, pp. 1233–1243, July 1999.

[3] K. F. Lee and D. B. Williams, "A space-time coded transmitter diversity technique for frequency selective fading channels," in *Proc. IEEE Sensor Array and Multichannel Signal Processing Workshop*, Cambridge, MA, March 2000, pp. 149–152.

[4] K. F. Lee and D. B. Williams, "A space-frequency transmitter diversity technique for OFDM systems," in *Proc. IEEE GLOBECOM*, San Francisco, CA, Nov.-Dec. 2000, vol. 3, pp. 1473–1477.

[5] K. F. Lee and D. B. Williams, "Bandwidth efficient OFDM transmitter diversity techniques," in *Proc. IEEE ICASSP*, Orlando, FL, May 2002, vol. 3, pp. 2325–2328.

[6] Y. Li, N. Seshadri, and S. Ariyavisitakul, "Channel estimation for OFDM systems with transmitter diversity in mobile wireless channels," *IEEE J. Select. Areas Commun.*, vol. 17, no. 3, pp. 461–471, March 1999.

[7] Y. Xie and C. N. Georghiades, "An EM-based channel estimation algorithm for OFDM with transmitter diversity," in *Proc. IEEE GLOBECOM*, San Antonio, TX, Nov. 2001, pp. 871–875.

[8] K. F. Lee and D. B. Williams, "Pilot-symbol-assisted channel estimation for space-time coded OFDM systems," *EURASIP Journal on Applied Signal Processing*, vol. 2002, no. 5, pp. 507–516, May 2002.

[9] D. Kim and G. L. Stüber, "Residual ISI cancellation for OFDM with applications to HDTV broadcasting," *IEEE J. Select. Areas Commun.*, vol. 16, no. 8, pp. 1590–1599, Oct. 1998.

[10] N. J. Fliege, *Multirate digital signal processing : multirate systems, filter banks, wavelets*, Wiley, Chichester, New York, 1994.