DELAY-SPREAD ESTIMATION USING CYCLIC-PREFIX IN WIRELESS OFDM SYSTEMS

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ABSTRACT

A novel cyclic-prefix based delay-spread estimation technique for wireless OFDM systems is proposed. The technique uses change of gradient of a correlation function as the strategy to detect delayed arrival paths. Estimation of the symbol timing and frequency synchronization information is also inherent in the technique. The proposed technique can be computationally efficiently implemented using the Viterbi search algorithm. Numerical results demonstrate the accuracy of the technique in estimating the relative timing (delay) and magnitude (power) of delayed paths in a multipath environment. The delay-spread information provided by the proposed technique can be adaptively used to improve the accuracy of the frequency domain channel response interpoaltion process in OFDM systems.

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has become a prime candidate for future high-data-rate wireless communication systems due to its features such as multipath immunity, bandwidth efficiency and resistance to narrowband interference [1]. The presence of multiple arrival paths (multipath) in wireless scenario causes frequency selective fading across OFDM subcarriers. Channel estimation and frequency domain equalization (1-tap equalization or tone-by-tone division) are incorporated at the OFDM demodulator to compensate for this channel fading [2]. In pilot-symbol-aided channel estimation techniques, frequency domain interpolation (Wiener filtering) of the channel response in incorporated. However, the accurate frequency domain interpolation of the channel response requires the knowledge of *delay-spread* of the channel, which is normally a-priori unknown. Therefore, it is customary to set the delay-spread parameter (RMS delay spread) in the channel interpolator to a likely value (fixed) thus resulting in a suboptimal channel interpolation and equalization process. In a wireless communication system, adaptive detection of channel delay-spread can be effectively used to operate the channel interpolation and equalization processes at near optimum [3]. This paper presents a new approach of adaptive estimation of the channel delay-spread using the cyclic-prefix of the OFDM symbols.

2. PROPOSED TECHNIQUE

The discrete complex-baseband OFDM signal $r(n)$ at the receiver for a single path channel can be given as [4]

$$r(n) = s(n)e^{j2\pi n\varepsilon/N} + g(n) \tag{1}$$

where $s(n)$ is the transmitted OFDM signal and $g(n)$ is additive white Gaussian noise (AWGN). ε and N denote the frequency offset error (normalized using the intercarrier spacing of Δf) and the total number of subcarriers, respectively. For a multipath channel $s(n)$ should be replaced by the multipath signal $p(n) = \sum_{i=0}^{I} \mu_i s(n-\tau_i)$, giving

$$r(n) = \left[\sum_{i=0}^{I} \mu_i s(n-\tau_i)\right] e^{j2\pi n\varepsilon/N} + g(n) \tag{2}$$

We define the multiple-argument correlation function $G_M^K(n)$ as

$$G_M^K(n) = \frac{1}{K}\sum_{k=0}^{K-1}\sum_{m=0}^{M-1} r(n+kN_t-m)r^*(n+kN_t-m-N) \tag{3}$$

where, $1 \le M \le N_g$, N_g is the length of the cyclic-prefix of OFDM symbols, and $N_t = N + N_g$ is the total length of an OFDM symbol. In (3), summation over index m indicates addition of conjugate products of samples N positions apart for consecutive samples. This is similar to the correlation function reported in [4] for the purpose of time and frequency synchronization, where the summation is performed over the length of cyclic-prefix (N_g). However in (3), this summation is performed for variable length of $1 \le M \le N_g$. Summation over the index k indicates the addition of conjugate products of samples N positions apart for consecutive OFDM symbols ($N_t = N + N_g$ samples). This is similar to the ensemble correlation function reported in [3] for OFDM timing recovery. Therefore, we have integrated both adjacent sample and adjacent symbol averaging concepts in the correlation function $G_M^K(n)$. Also, $G_M^K(n)$ can be written as

$$G_M^K(n) = \sum_{m=0}^{M-1} H^K(n-m) \tag{4}$$

where,

$$H^K(l) = \frac{1}{K}\sum_{k=0}^{K-1} r(l+kN_t)r^*(l+kN_t-N) \tag{5}$$

$$= \frac{1}{K}\sum_{k=0}^{K-1} J_k(l) \tag{6}$$

where, $J_k(l) = r(l+kN_t)r^*(l+kN_t-N)$ is the conjugate product of a sample pair N positions apart. Statistical properties of the term $J_k(l)$ is independent of k as it reflects OFDM symbol periodicity. Therefore, we consider a symbol independent term $J(l)$ defined as

$$J(l) = r(l)r^*(l-N) \tag{7}$$

In the next section we describe the proposed technique for a simple two-path channel. Generalization of the technique for a multipath channel is given in Section 2.2.

2.1. Two-path Channel

In this section we consider a wireless channel with only two paths to demonstrate the use of the correlation function $G_M^K(n)$ for delay-path estimation. For the two-path case $p(n) = s(n)+\mu s(n-\tau)$, where μ and τ are the *complex amplitude* and the *time delay* of the second path relative to the first path. Figure 1 shows the received two-path OFDM signal $p(n)$. Consider the first arrival path as the reference and

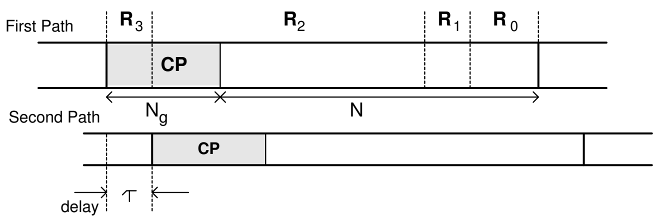

Fig. 1. Received two-path OFDM signal.

$n=1$ and $n=N+N_g$ as the time index values corresponding to the first and last samples of an OFDM symbol, respectively. Note that cyclic-prefix (CP) comes before the data portion of the OFDM symbol. Therefore, $\{n|1 \le n \le N_g\}$ is the CP and $\{n|N_g+1 \le n \le N+N_g\}$ is the data part of the OFDM symbol. In the context of a two-path channel with the second path having a relative delay of τ samples, we identify 4 distinct regions within an OFDM symbol. They are; $\mathcal{R}_0 = \{n|N+\tau+1 \le n \le N+N_g\}$, $\mathcal{R}_1 = \{n|N+1 \le n \le N+\tau\}$, $\mathcal{R}_2 = \{n|\tau+1 \le n \le N\}$, and $\mathcal{R}_3 = \{n|1 \le n \le \tau\}$. The expected value $E\{J(l)\}$ can be evaluated for $l \in \mathcal{R}_i$, where $0 \le i \le 3$, using the following statistical properties of the OFDM signal and AWGN noise.

$$E\{s(l_1)s^*(l_2)\} = \begin{cases} \sigma_s^2, & \text{if } l_1 = l_2 \\ 0, & \text{if } l_1 \ne l_2 \end{cases} \tag{8}$$

$$E\{s(l_1)g^*(l_2)\} = E\{g(l_1)s^*(l_2)\} = 0, \ \forall \ l_1, l_2 \tag{9}$$

$$E\{g(l_1)g^*(l_2)\} = \begin{cases} \sigma_n^2, & \text{if } l_1 = l_2 \\ 0, & \text{if } l_1 \ne l_2 \end{cases} \tag{10}$$

where, σ_s^2 and σ_n^2 are the variances of OFDM signal and noise samples, respectively. The property (8) is true since the OFDM signal is wideband [4]. Using above properties it can be shown that

$$E\{J(l)\} = \begin{cases} (1+|\mu|^2)\sigma_s^2 e^{j2\pi\varepsilon} & , \ l \in \mathcal{R}_0 \\ \sigma_s^2 e^{j2\pi\varepsilon} & , \ l \in \mathcal{R}_1 \\ 0 & , \ l \in \mathcal{R}_2 \\ |\mu|^2\sigma_s^2 e^{j2\pi\varepsilon} & , \ l \in \mathcal{R}_3 \end{cases} \tag{11}$$

As can be seen from (6) for sufficiently large K, $H^K(l)$ can be approximated by $E\{J(l)\}$.

$$H^K(l) \simeq E\{J(l)\} \tag{12}$$

As give in (4), $G_M^K(n)$ is the sum of $H^K(l)$ for M consecutive samples, $n-M+1$ to n. The magnitude of $G_M^K(n)$ for $M = N_g$ (i.e. $|G_{N_g}^K(n)|$) can be effectively used to determine symbol timing information,. According to (11) and (12), $|G_{N_g}^K(n)|$ maximizes at the last sample of each OFDM symbol (i.e. at $n = N+N_g$). This is true as the delayed path is normally has a smaller magnitude ($|\mu| < 1$). Therefore, the symbol timing estimation can be given as

$$\hat{T} = \arg\max_n |G_{N_g}^K(n)|. \tag{13}$$

Error of symbol timing estimation of (13) approaches zero for a time-invariant channel as $K \to \infty$. Any finite value of K causes a certain degree of estimation error due to the approximation in (12). The phase of $G_{N_g}^K(n)$ can be used to estimate frequency-offset (ε).

$$\hat{\varepsilon} = \frac{1}{2\pi}\angle G_{N_g}^K(n) \tag{14}$$

It should be noted that the symbol timing and frequency offset estimators given in (13) and (14) are similar to that reported in [4] for non-fading (i.e. AWGN only) channel. Our analysis shows that these estimators are asymptotically optimal for a frequency-selective channel as $K \to \infty$.

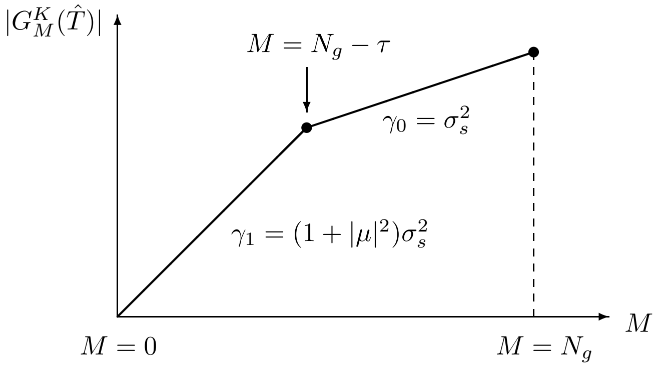

Fig. 2. Plot of $|G_M^K(\hat{T})|$ against M as $K \to \infty$, for the two-path channel scenario. γ_0 and γ_1 are the gradient of $|G_M^K(\hat{T})|$ for the segments $N_g - \tau \leq M \leq N_g$ and $1 \leq M \leq N_g - \tau$, respectively.

The relative delay (τ) and magnitude ($|\mu|$) of the second path in the two-path channel can be estimated observing $|G_M^K(\hat{T})|$ as a function of M, where $1 \leq M \leq N_g$. Figure 2 shows the plot of $|G_M^K(\hat{T})|$ as $K \to \infty$. As can be seen from Figure 2, the plot of $|G_M^K(\hat{T})|$ against M shows a *change of gradient* (i.e. a knee-point) at $M = N_g - \tau$. The relative magnitude of the second path ($|\mu|$) can be estimated as

$$|\hat{\mu}| = \sqrt{\frac{\gamma_1}{\gamma_0} - 1} \qquad (15)$$

where γ_0 and γ_1 are the gradient of $|G_M^K(\hat{T})|$ for the segments $N_g - \tau \leq M \leq N_g$ and $1 \leq M \leq N_g - \tau$, respectively. For finite K the line segments in Figure 2 are not perfectly straight, thus the knee-point ($M = N_g - \tau$) and the gradients γ_0 and γ_1 should be estimated using a best-fitting criterion.

2.2. Multipath Channel

In this section, we extend the basic delay-path estimation developed for a two-path channel in Section 2.1 to a multipath channel. Consider a multipath channel with the response $h(n) = \sum_{i=0}^{I} \mu_i \delta(n - \tau_i)$ consisting of $I + 1$ paths (first arrival path and I delay paths). Without loosing generality it is assumed that $\mu_0 = 1$ and $\tau_0 = 0$ (i.e. the first arrival path is the reference for delay and magnitude estimations), and $0 < \tau_1 < \tau_2 < \ldots < \tau_I < N_g$. For the multipath channel it can be shown that the plot of $|G_M^K(\hat{T})|$, for $1 \leq M \leq N_g$, consists of $I + 1$ number of straight line segments as $K \to \infty$. The ranges $\mathcal{M}_i$'s and the gradients γ_i's of the line segments are given by

$$\mathcal{M}_i = \{N_g - \tau_{i+1} \leq M \leq N_g - \tau_i\}, \quad 1 \leq i \leq I \qquad (16)$$

where, $\tau_{I+1} = N_g - 1$, and

$$\gamma_i = \sum_{j=0}^{i} |\mu_j|^2 \sigma_s^2, \quad 1 \leq i \leq I, \qquad \text{respectively.} \qquad (17)$$

For a finite K the plot of $|G_M^K(\hat{T})|$ will not consist of perfectly straight line segments. The knee points $N_g - \tau_i$, where $1 \leq i \leq I$, in the plot of $|G_M^K(\hat{T})|$ defining the boundaries of the of the regions $\mathcal{M}_i$'s can be determined using the following best fitting criterion.

$$\mathcal{T} = \arg \min_{\tau_1, \tau_2, \ldots, \tau_I} \sum_{i=o}^{I} \Psi_{\text{err}} \left(\left\{ |G_M^K(\hat{T})| \big| \mathcal{M}_i \right\} \right) \qquad (18)$$

where, $\mathcal{T} = \{\hat{\tau}_1, \hat{\tau}_2, \ldots, \hat{\tau}_I\}$, and

$$\hat{\gamma}_i = \Psi_{\text{grad}} \left(\left\{ |G_M^K(\hat{T})| \big| \hat{\mathcal{M}}_i \right\} \right), \quad 0 \leq i \leq I \qquad (19)$$

where, $\hat{\mathcal{M}}_i = \{N_g - \hat{\tau}_{i+1} \leq M \leq N_g - \hat{\tau}_i\}$, for $0 \leq i \leq I$. The function $\Psi_{\text{grad}}(\{.\})$ denotes the gradient of the best fitting (minimum mean-square-error) straight line segment to the data set given in the argument. The associated squared-error of fitting is given by $\Psi_{\text{err}}(\{.\})$. The magnitudes $|\mu_i|$'s of the delayed paths can be estimated using (17) and (19) as

$$|\hat{\mu}_i| = \sqrt{\frac{\hat{\gamma}_i - \hat{\gamma}_{i-1}}{\hat{\gamma}_0}}, \quad 1 \leq i \leq I. \qquad (20)$$

The computational complexity of this technique is mainly associated with the optimization task given in (18). However, as (18) involves a sequential decision making process with the cost function $\Psi_{\text{err}}(\{.\})$, it can be computationally efficiently solved using the Viterbi search algorithm. Since the technique is based on the knee-point detection in the plot of $|G_M^K(\hat{T})|$, the multipath components are expected to be sufficiently spaced in time with large enough magnitudes. The accuracy of the technique can be expected to degrade for closely spaced and weak (in magnitude) multipath components.

3. NUMERICAL RESULTS

An OFDM system with $N = 512$ subcarriers and cyclic-prefix length of $N_g = 64$ samples was used in simulations. A two-path channel with relative second path magnitude $\mu = 0.7$ and delay $\tau = 32$ samples was incorporated for evaluation. Frequency offset (normalized) was set to $\varepsilon = 0.3$. An average channel signal-to-noise (SNR) range of 0-25 dB at steps of 5 dB was selected. The second-path delay ($\hat{\tau}$) and the second-path magnitude ($|\hat{\mu}|$) were estimated using (18) and (20), respectively. Simulations were performed for $K = 16$, 32, and 64, each over a total of 2048 OFDM symbols for each SNR value. Figures 3 and 4 show the mean-square-error of $\hat{T}$ and $\hat{\varepsilon}$, respectively. Figure 5 shows the RMS error of the second path delay estimation ($\hat{\tau}$). The mean-square-error of second path magnitude estimation ($|\hat{\mu}|$) is shown in Figure 6.

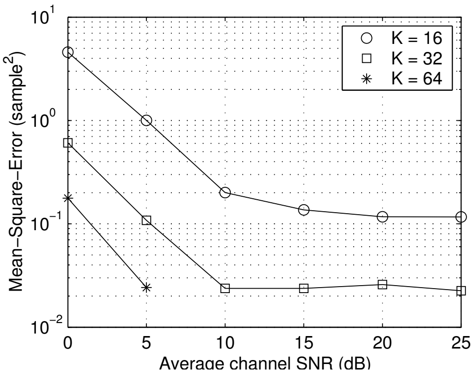

Fig. 3. Mean-square-error of the OFDM symbol timing estimation ($\hat{T}$), for $K = 16$, 32, and 64.

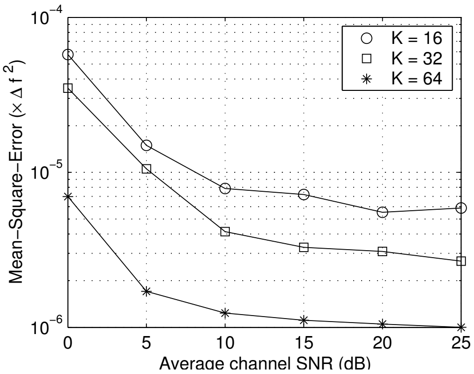

Fig. 4. Mean-square-error of the normalized frequency offset estimation ($\hat{\varepsilon}$), for $K = 16$, 32, and 64.

4. CONCLUSIONS

A new cyclic-prefix based delay-spread estimation technique for OFDM systems was presented. It was shown that the change of gradient (knee-point) of a correlation function is indicative of a delayed arrival path. Timing and magnitude information of delayed paths can be determined[1] by estimating the knee-point location and the gradients of the associated line segments of the correlation function. It was shown that the accuracy of the proposed technique increases as the number of adjacent OFDM symbols K, over which averaging is performed, is increased. Numerical results show that for $K = 64$ and SNR ≥ 10 dB, the RMS error of the delay estimation becomes less than 1 sample, and the mean-square-error of the magnitude estimation becomes less than 10^{-3}. Using the proposed technique the delay-spread information of the channel can be accurately and adaptively estimated, which can be effectively used to improve the accuracy of the channel interpolation process in OFDM systems. Slowly varying channel conditions allowing adequate adjacent symbol averaging (sufficiently large K) are the best situations where the proposed delay-spread estimating technique can be applied.

[1]However, this does not amount to a complete channel state detection because the *phase information* of the multipaths is not resolved.

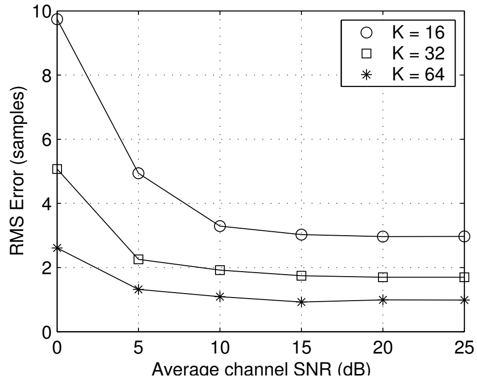

Fig. 5. Root-mean-square-error (RMS error) of the second-path delay estimation ($\hat{\tau}$), for $K = 16$, 32, and 64.

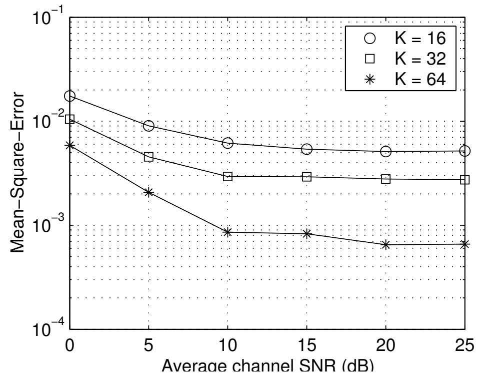

Fig. 6. Mean-square-error of the second-path magnitude estimation ($|\hat{\mu}|$), for $K = 16$, 32, and 64.

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