# MULTI-DIMENSIONAL HARMONIC ESTIMATION USING $K$-D RARE IN APPLICATION TO MIMO CHANNEL ESTIMATION


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## Abstract

In this paper, a new approach to the multi-dimensional harmonic retrieval problem is proposed. The novel method is based on a multi-dimensional extension of the RAnk Reduction Estimator (RARE), originally developed for DOA estimation in partly calibrated arrays. In the $K$-D RARE algorithm the frequency parameters in the various dimensions are sequentially estimates. The dimensionality of the estimation problem and therefore the computational load of the optimization procedure is successively reduced exploiting the rich multidimensional structure of the estimation problem. This important property yields various benefits like high estimation performance, weak identifiability conditions and automatically associated parameter estimates. The performance of the algorithm is illustrated at the example of MIMO communication channel estimation based on the double-directional channel model [1]. Numerical examples based on simulated and measured data recorded from the RUSK vector channel sounder at 2GHz are presented.


## 1. INTRODUCTION

Multi-dimensional harmonic retrieval problems arise in a large variety of important applications like estimation of a Multiple Input Multiple Output (MIMO) communication system where direction-of-departure (DOD), direction of arrival (DOA), time delays of arrival (TDOA) and Doppler are jointly estimated. Also certain signal separation problems in synthetic aperture radar, image motion estimation and chemistry applications can be solved under this framework.

Numerous parametric and nonparametric estimation methods have been proposed for the one-dimensional exponential retrieval problem. Only few of these techniques allow a simple extension to the multi-dimensional case at a reasonable computational load [2]. Simple application of 1D results separately in each dimension is only suboptimal in the sense that it does not exploit the benefits inherent in the multi-dimensional (mD) structure, leading to difficulties in mutually associating the parameter estimates obtained in the various dimensions and over-strict uniqueness conditions [3]. Contrariwise, many parametric high resolution methods specifically designed for the mD frequency estimation require mD, nonlinear, and non convex optimization so that the computational cost associated with the optimization procedure becomes prohibitively high.

In this paper a novel eigenspace-based estimation method for mD-exponential retrieval problems is proposed. The method can be viewed as a mD extension to the RAnk Reduction Estimator (RARE) [4], originally developed for DOA estimation in partly calibrated arrays. The method is computationally efficient due to its rooting-based implementation, makes explicit use of the rich $K$-D structure in the measurement data and therefore shows improved estimation performance compared to conventional search-free methods for mD frequency estimation.

## 2. SIGNAL MODEL

Consider a superposition of $L$ discrete time $K$-D exponentials corrupted by noise and let $\boldsymbol{\omega}_k = [\omega_{(k,1)}, \ldots, \omega_{(k,L)}]^T$ denote the frequency parameter vector of the $L$ discrete harmonics in the $k$th dimension. Furthermore let, the vector $\boldsymbol{h}_k(\omega_{(k,l)}) = \left[1, e^{j\omega_{(k,l)}}, \ldots, e^{j(M_k-1)\omega_{(k,l)}}\right]^T$ contain the contributions of the $l$th harmonic in the $k$th dimension and $M_k$ denote the corresponding sample size. The Khatri-Rao (column-wise Kronecker) product of matrix $\boldsymbol{A}$ and matrix $\boldsymbol{B}$ is defined as, $\boldsymbol{A} \circ \boldsymbol{B} = [\boldsymbol{a}_1 \otimes \boldsymbol{b}_1, \boldsymbol{a}_2 \otimes \boldsymbol{b}_2, \ldots])$, where $\boldsymbol{a}_k \otimes \boldsymbol{b}_k$ is the Kronecker matrix product of $\boldsymbol{a}_k$ and $\boldsymbol{b}_k$. Introducing the parameter vector $\boldsymbol{\Omega} = \left[\boldsymbol{\omega}_1^T, \ldots, \boldsymbol{\omega}_K^T\right]^T$, the $K$-D signal model associated with the harmonic retrieval problem can be formulated as

$$\boldsymbol{y}(i) = \boldsymbol{H}(\boldsymbol{\Omega})\boldsymbol{c}(i) + \boldsymbol{n}(i), \quad i = 1, \ldots, N \tag{1}$$

where the $K$-D exponential matrix

$$\boldsymbol{H}(\boldsymbol{\Omega}) = \boldsymbol{H}_1(\boldsymbol{\omega}_1) \circ \boldsymbol{H}_2(\boldsymbol{\omega}_2) \circ \ldots \circ \boldsymbol{H}_K(\boldsymbol{\omega}_K) \tag{2}$$

also referred to as the steering matrix, is composed of the individual 1D exponential matrices

$$\boldsymbol{H}_k(\boldsymbol{\omega}_k) = \left[\boldsymbol{h}_k(\omega_{(k,1)}), \ldots, \boldsymbol{h}_k(\omega_{(k,L)})\right] \in \mathbb{C}^{(M_k \times L)} \tag{3}$$

for $k = 1, \ldots, K$, $\boldsymbol{y}(i)$ denotes the measurement vector, $\boldsymbol{c}(i)$ stands for the complex envelope of the $L$ harmonics, $\boldsymbol{n}(i)$ is the vector of additive Gaussian noise and $N$ is the number of snapshots. Equation (1) describes the $K$-D harmonic retrieval problem which is efficiently solved by the conventional ESPRIT algorithm [5] and the mD-ESPRIT algorithm [2] for the 1D and the mD case, respectively. In the following we derive a new search-free eigenspace-based estimation method for the general case in (1) which yields highly accurate estimates of the parameters of interest.

Let the data covariance matrix be given by

$$\boldsymbol{R} = \mathrm{E}\left\{\boldsymbol{y}(i)\boldsymbol{y}^H(i)\right\} = \boldsymbol{E}_S\boldsymbol{\Lambda}_S\boldsymbol{E}_S^H + \boldsymbol{E}_N\boldsymbol{\Lambda}_N\boldsymbol{E}_N^H \tag{4}$$

where $(\cdot)^H$ denotes the Hermitian transpose, and $\mathrm{E}\{\cdot\}$ stands for statistical expectation. The diagonal matrices $\boldsymbol{\Lambda}_S \in \mathbb{R}^{(L\times L)}$ and $\boldsymbol{\Lambda}_N \in \mathbb{R}^{(M-L)\times(M-L)}$ contain the signal-subspace and the noise-subspace eigenvalues of $\boldsymbol{R}$, respectively. In turn, the columns of the matrices $\boldsymbol{E}_S \in \mathbb{C}^{(M\times L)}$ and $\boldsymbol{E}_N \in \mathbb{C}^{M\times(M-L)}$ denote the corresponding signal-subspace and noise-subspace eigenvectors for $M = \prod_{k=1}^{K} M_k$. The finite sample estimates are given by

$$\hat{\boldsymbol{R}} = \frac{1}{N}\sum_{i=1}^{N} \boldsymbol{y}(i)\boldsymbol{y}^H(i) = \hat{\boldsymbol{E}}_S\hat{\boldsymbol{\Lambda}}_S\hat{\boldsymbol{E}}_S^H + \hat{\boldsymbol{E}}_N\hat{\boldsymbol{\Lambda}}_N\hat{\boldsymbol{E}}_N^H \quad (5)$$

## 3. THE $K$-D RARE ALGORITHM

The $K$-D spectral MUSIC algorithm yields minimum function values for the true parameter estimates, that is

$$\begin{aligned} &P_{\mathrm{MUSIC}}(\omega_1,\ldots,\omega_K) \\ &\quad = \boldsymbol{h}^T(1/z_1,\ldots,1/z_K)\boldsymbol{E}_N\boldsymbol{E}_N^H\boldsymbol{h}(z_1,\ldots,z_K) = 0 \quad (6)\end{aligned}$$

where $z_k = e^{j\omega_k}$ denotes the $k$th exponential. Equation (6) represents a polynomial equation in $K$ variables which is generally hard to solve. The true source solutions can be obtained from the $k$-tuples $\left\{\left(z_{(1,i)}, z_{(2,i)}, \ldots, z_{(K,i)}\right)\right\}_{i=1}^{L}$ located on the unit circle that root the MUSIC criterion. The manifold vector in (6) can be represented as $\boldsymbol{h}(z_1, z_2, \ldots, z_K) = \boldsymbol{T}_1(z_1)\tilde{\boldsymbol{h}}_1(z_2, \ldots, z_K)$ with $\tilde{\boldsymbol{h}}_1(z_2, \ldots, z_K) = [\boldsymbol{h}_2(z_2)\otimes\ldots\otimes\boldsymbol{h}_K(z_K)] \in \mathbb{C}^{(m_1\times 1)}$. Instead of solving the minimization problem (6) on the original manifold $\boldsymbol{M} \triangleq \{\boldsymbol{h}(z_1,\ldots,z_K) : |z_1| = \ldots = |z_K| = 1\}$ we propose to relax the optimization problem searching for solutions on the so-called RARE manifold $\bar{\boldsymbol{M}} \triangleq \left\{\bar{\boldsymbol{h}}(z_1, \boldsymbol{c}_1) : |z_1| = 1, \right\}$ for an arbitrary non-zero vector $\boldsymbol{c}_1 \in \mathbb{C}^{(m_1\times 1)}$ and $\bar{\boldsymbol{h}}(z_1, \boldsymbol{c}_1) = \boldsymbol{T}_1(z_1)\boldsymbol{c}_1$ where $\boldsymbol{T}_1(z_1) = [\boldsymbol{h}_1(z_1)\otimes\boldsymbol{I}_{m_1}]$, $m_1 = \prod_{k=2}^{K} M_k$ and the $(m_1\times m_1)$ identity matrix $\boldsymbol{I}_{m_1}$. The relaxed MUSIC criteria can now be formulated as

$$\begin{aligned} P_{\mathrm{RARE}}(z_1, \boldsymbol{c}_1) &= \bar{\boldsymbol{h}}^T(1/z_1, \boldsymbol{c}_1)\boldsymbol{E}_N\boldsymbol{E}_N^H\bar{\boldsymbol{h}}(z_1, \boldsymbol{c}_1) \\ &= \boldsymbol{c}_1^H\boldsymbol{T}_1^T(1/z_1)\boldsymbol{E}_N\boldsymbol{E}_N^H\boldsymbol{T}_1(z_1)\boldsymbol{c}_1 \\ &= \boldsymbol{c}_1^H\bar{\boldsymbol{B}}_{m_1,1}(z_1)\boldsymbol{c}_1 = 0 \quad (7)\end{aligned}$$

where $\bar{\boldsymbol{B}}_{m_1,1}(z_1) \triangleq \boldsymbol{T}_1^T(1/z_1)\boldsymbol{E}_N\boldsymbol{E}_N^H\boldsymbol{T}_1(z_1) \in \mathbb{C}^{(m_1\times m_1)}$ denotes the RARE matrix polynomial. Interestingly, equation (7) has very simple solutions in the exponential $z_1$ which are given by the roots of the RARE polynomial

$$P_{\mathrm{RARE}}(z_1)|_{|z_1|=1} = \det\{\bar{\boldsymbol{B}}_{m_1,1}(z_1)\} = 0 \quad (8)$$

evaluated on the unit circle. It was proven in [4] that the solutions $\left\{z_{(1,l)}\right\}_{l=1}^{L}$ to (8) and the solutions of the original MUSIC criteria (6) are identical provided that the condition

$$L \leq m_1(M_1 - 1) = M - m_1 \quad (9)$$

is satisfied. In other words the true parameter vector $\boldsymbol{\omega}_1$ can uniquely be determined from the corresponding RARE polynomial without any knowledge of the remaining parameters $\boldsymbol{\omega}_2, \ldots, \boldsymbol{\omega}_K$.

The computational cost required for expanding the determinant of the RARE matrix polynomial becomes prohibitively high if the dimension of the polynomial matrix is large [6]. In the following we derive an equivalent formulation of (8) based on a RARE matrix polynomial of reduced dimension.

Using a well known property of block matrix determinants we can write (8) as

$$\begin{aligned} &P_{\mathrm{RARE}}(z_1)|_{|z_1|=1} = \det\{\bar{\boldsymbol{B}}_{m_1,1}(z_1)\} \\ &= \det\left\{\boldsymbol{T}_1^T(1/z_1)\boldsymbol{T}_1(z_1) - \boldsymbol{T}_1^T(1/z_1)\boldsymbol{E}_S\boldsymbol{E}_S^H\boldsymbol{T}_1(z_1)\right\} \\ &= 1/L\det\left\{\begin{bmatrix} \boldsymbol{I} & \boldsymbol{E}_S^H\boldsymbol{T}_1(z_1) \\ \boldsymbol{T}_1^T(1/z_1)\boldsymbol{E}_S & \boldsymbol{T}_1^T(1/z_1)\boldsymbol{T}_1(z_1)\end{bmatrix}\right\} \\ &= M/L\det\{\boldsymbol{B}_{L,1}(z_1)\} = 0 \quad (10)\end{aligned}$$

where $\boldsymbol{\Delta}_1 = \boldsymbol{T}_1^T(1/z_1)\boldsymbol{T}_1(z_1)$ is a constant diagonal matrix and $\boldsymbol{B}_{L,1}(z_1) \triangleq \boldsymbol{I} - \boldsymbol{E}_S^H\boldsymbol{T}_1(z_1)\boldsymbol{\Delta}_1^{-1}\boldsymbol{T}_1^T(1/z_1)\boldsymbol{E}_S \in \mathbb{C}^{(L\times L)}$ denotes the RARE matrix polynomial of dimension equal to the number $L$ of discrete exponentials.

The estimation of the remaining parameter vectors $\boldsymbol{\omega}_2$ to $\boldsymbol{\omega}_K$ is now straightforward. Due to the special structure of the steering matrix (2) the various dimensions of the parameter space can be interchanged. Corresponding RARE polynomial matrices in $z_2$ to $z_K$ can be formulated following similar considerations as above and using an appropriate permutation of the columns of the signal eigenvectors in (4). The parameter vectors $\boldsymbol{\omega}_2$ to $\boldsymbol{\omega}_K$ are uniquely determined from the roots of these matrix polynomials under similar conditions as in (9).

A mayor obstacle emerging in this approach is the difficulty of correctly associating the parameter estimates in the individual dimensions. The parameter vectors $\boldsymbol{\omega}_1, \ldots, \boldsymbol{\omega}_K$ are separately obtained. The computational cost due to the pairing of the solutions can be significant when the number $L$ of superposed signals is large and no efficient pairing procedure is available.

## 4. SUCCESSIVE DIMENSION REDUCTION

Assume that $n$ harmonics correspond to the same frequency component $\omega_1$. In this case the polynomial equation in (10) yields a signal root $\hat{z}_1$ of multiplicity $2n$ located on the unit circle[1]. Inserting $\hat{z}_1$ into (6) and using representation $\boldsymbol{h}(\hat{z}_1, z_2, \ldots, z_K) = \boldsymbol{T}_1(\hat{z}_1)\tilde{\boldsymbol{h}}_1(z_2, \ldots, z_K)$ we obtain the $(K-1)$-D root-MUSIC polynomial equation

$$\begin{aligned} &P_{\mathrm{MUSIC}}(\omega_2, \ldots, \omega_K|\hat{\omega}_1) \\ &\quad = \boldsymbol{h}^H(\hat{z}_1, z_2, \ldots, z_K)\boldsymbol{E}_N\boldsymbol{E}_N^H\boldsymbol{h}(\hat{z}_1, z_2, \ldots, z_K) \\ &\quad = \tilde{\boldsymbol{h}}_1^H(z_2, \ldots, z_K)\bar{\boldsymbol{B}}_{m_1,1}(\hat{z}_1)\tilde{\boldsymbol{h}}_1(z_2, \ldots, z_K) = 0 \quad (11)\end{aligned}$$

Comparing (6) and (11) it becomes apparent that the RARE concept enables us to decompose the $K$-D polynomial rooting problem into multiple $(K-1)$-D polynomial rooting problems which then can be solved sequentially. That is, the signal root $z_{(1,i)}$ inserted into the original $K$-D root-MUSIC function reduces the dimensionality of the rooting problem by one. Generalizing the procedure above and using the representation $[\boldsymbol{h}_p(z_p)\otimes\ldots\otimes\boldsymbol{h}_K(z_K)] = \boldsymbol{T}_p(z_p)\,[\boldsymbol{h}_{p+1}(z_{p+1})\otimes\ldots\otimes\boldsymbol{h}_K(z_K)]$ with $\boldsymbol{T}_p(z_p) =$

[1]Note, that the RARE polynomial is self-reciprocal in $z_1$, so that if $z_1$ is a root then $1/z_1^*$ is also a root of the RARE polynomial.

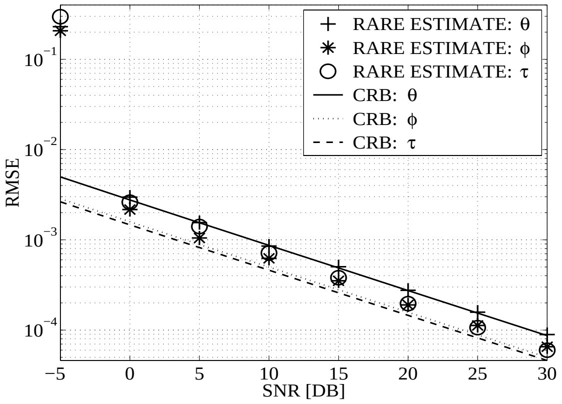


**Fig. 1**. RMSE's of estimates versus SNR.

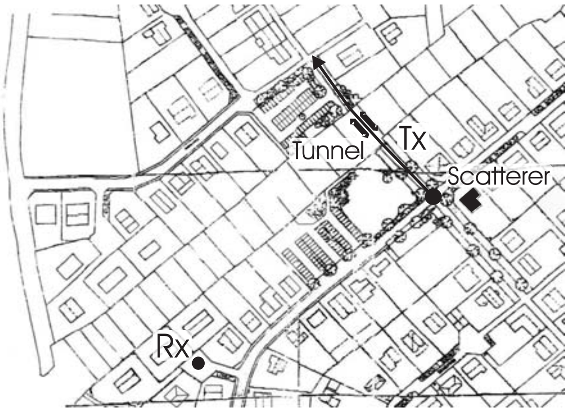


**Fig. 2**. Map of measurement route in Weikendorf.

$(\boldsymbol{h}_p(z_p) \otimes \boldsymbol{I}_{m_p})$ and $m_p = \prod_{k=(p+1)}^{K} M_k$, it is straightforward to decompose each $(K-p)$-D polynomial rooting problem into multiple $(K-p-1)$-D polynomial rooting problems in a recursive procedure. Given the parameter estimates $\hat{z}_1, \ldots \hat{z}_{(p-1)}$ the RARE matrix polynomial in $z_p$ becomes

$$P_{\text{RARE,p}}(z_p)|_{|z_p|=1} = \det\left\{\bar{\boldsymbol{B}}_{m_p,p}(z_p|\hat{z}_1, \ldots \hat{z}_{(p-1)})\right\} \quad (12)$$

were the matrix polynomial

$$\begin{aligned}&\bar{\boldsymbol{B}}_{m_p,p}(z_p|\hat{z}_1, \ldots \hat{z}_{\tilde{p}}) \\ &\quad \triangleq \ \boldsymbol{T}_p^T(1/z_p)\bar{\boldsymbol{B}}_{m_{\tilde{p}},\tilde{p}}(\hat{z}_{\tilde{p}}|\hat{z}_1, \ldots \hat{z}_{(p-2)})\boldsymbol{T}_p(z_p) \quad (13)\end{aligned}$$

$\in \mathbb{C}^{(m_p \times m_p)}$ for $\tilde{p} = p-1$ is recursively computed. According to (10) we have

$$\begin{aligned}P_{\text{RARE,p}}(z_p)|_{|z_p|=1} &= \det\left\{\bar{\boldsymbol{B}}_{m_p,p}(z_p|\hat{z}_1, \ldots \hat{z}_{(p-1)})\right\} \\ &= 1/L \det\left\{\begin{bmatrix} \boldsymbol{I} & \boldsymbol{E}_{S,p}^H \boldsymbol{T}_p(z_p) \\ \boldsymbol{T}_p^T(1/z_p)\boldsymbol{E}_{S,p} & \boldsymbol{\Delta}_p \end{bmatrix}\right\} \\ &= M/L \det\left\{\boldsymbol{I} - \boldsymbol{E}_{S,p}^H \boldsymbol{T}_p(z_p)\boldsymbol{\Delta}_p^{-1}\boldsymbol{T}_p^T(1/z_p)\boldsymbol{E}_{S,p}\right\} \\ &= M/L \det\{\boldsymbol{B}_{L,p}(z_p|\hat{z}_1, \ldots \hat{z}_{(p-1)})\} = 0 \quad (14)\end{aligned}$$

where $\boldsymbol{\Delta}_p = \boldsymbol{T}^T(1/z_p)\boldsymbol{\Delta}_{(p-1)}\boldsymbol{T}(z_p)$, $\boldsymbol{E}_{S,p} = \left[\boldsymbol{h}_1(\hat{z}_1) \otimes \ldots \otimes \boldsymbol{h}_{p-1}(\hat{z}_{(p-1)}) \otimes \boldsymbol{I}_{m_p}\right]^H \boldsymbol{E}_S \in \mathbb{C}^{(m_p \times L)}$, and the matrix polynomial $\boldsymbol{B}_{L,p}(z_p|\hat{z}_1, \ldots \hat{z}_{(p-1)}) \triangleq \boldsymbol{I} - \boldsymbol{E}_{S,p}^H \boldsymbol{T}_p(z_p)\boldsymbol{\Delta}_p^{-1}\boldsymbol{T}_p^T(1/z_p)\boldsymbol{E}_{S,p} \in \mathbb{C}^{(L \times L)}$.

Due to noise effects in the realistic case the signal roots $\boldsymbol{z}_1$ to $\boldsymbol{z}_K$ evaluated from (10) and (14) for $k = 1, \ldots, K$, are displaced from its ideal positions on the unit circle. In the following we consider only the roots inside the unit circle.

### 4.1. Implementation

*Initialization:* Compute the eigendecomposition of $\hat{\boldsymbol{R}}$ to obtain the matrices $\hat{\boldsymbol{E}}_S$ and $\hat{\boldsymbol{E}}_N$. Root the polynomial in (10) and select the $L$ largest roots $\hat{z}_{(1,1)}, \ldots, \hat{z}_{(1,L)}$. Compute $\boldsymbol{E}_{S,2}$ for $\left\{\hat{z}_{(1,l)}\right\}_{l=1}^{L}$. Set $p = 2$.

*Recursion:* Given the $(p-1)$-tuples $\left\{(\hat{z}_{(1,i)}, \ldots, \hat{z}_{(p-1,i)})\right\}_{i=1}^{I}$ of multiplicity $n_{(p-1,i)}$ , determine the $n_{(p-1,i)}$ largest roots of (13), for $\forall i$. Set $p = p+1$.

*Post processing:* From all $K$-tuples obtained in the recursion select the $L$ signal root $K$-tuples $(\hat{z}_{(1,l)}, \ldots, \hat{z}_{(K,l)})$ for $l = 1, \ldots, L$ with maximum $r_l = 1/K \sum_{k=1}^{K} |\hat{z}_{(k,l)}|$. Alternatively, select the $L$ signal $K$-tuples that minimize the $K$-D MUSIC function in (6). Compute estimates of the parameter vector $\hat{\boldsymbol{\Omega}}$ from the $L$ signal root $K$-tuples.

*Remark:* In each recursion step, the multiplicity of the roots can easily be determined from clustering the mD frequency estimates according to their estimated mD separation angle obtained in the previous recursions. Without loss of generality and independently from the true frequency parameters the multiplicity of each root $p$-tuple $(\hat{z}_{(1,i)}, \ldots, \hat{z}_{(p,i)})$ can also be chosen as $n_i = L$. However, overestimating the multiplicity of the roots increases the number of polynomials that need to be rooted in each recursion and therefore augments the computational complexity of the algorithm significantly.

## 5. SIMULATION RESULTS

### 5.1. Synthetic Data

In this section simulation results using synthetic data are presented. Computer simulations are performed for the 3D case with sample sizes $M_1 = M_2 = M_3 = 5$. The $(5^3 \times 5^3)$ data covariance matrix is computed from $N = 200$ snapshots and a number of $L = 3$ equi-powered exponentials is assumed with the frequency parameters $\boldsymbol{\omega}_1 = (0.55\pi, 0.719\pi, 0.906\pi)$, $\boldsymbol{\omega}_2 = (0.41\pi, 0.777\pi, 0.276\pi)$ and $\boldsymbol{\omega}_3 = (0.34\pi, 0.906\pi, 0.358\pi)$. The RMSE of the parameter estimates $\hat{\boldsymbol{\omega}}_1$, $\hat{\boldsymbol{\omega}}_2$ and $\hat{\boldsymbol{\omega}}_3$ obtained by the $K$-D RARE algorithm averaged over 100 simulation runs are plotted versus the SNR in Figure 1. A comparison to the individual CRBs reveals that the new method yields estimation performance close to the optimal bound.

### 5.2. Measurement Data

The measurement data used for this paper was recorded during a measurement campaign in Weikendorf, a suburban area in a small town north of Vienna in autumn 2001. Measurements were performed by a vector channel sounder RUSK ATM, manufactured by MEDAV [1]. The sounder operated at a center frequency of 2

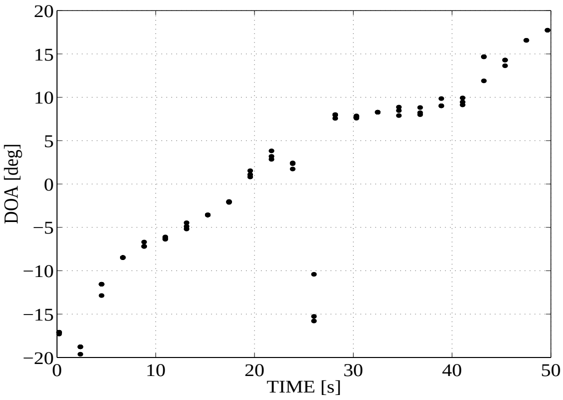


**Fig. 3**. 3D-RARE DOA estimates versus time.

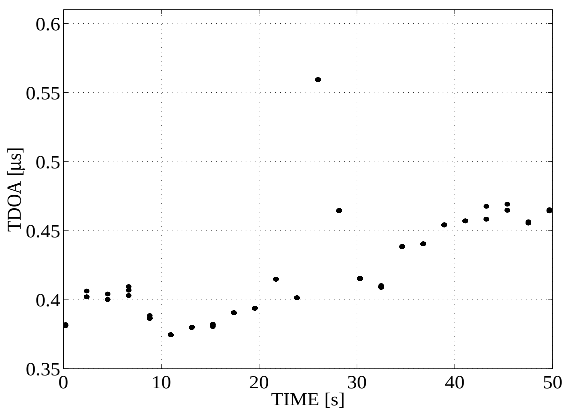


**Fig. 4**. 3D-RARE TDOA estimates versus time.

GHz with an output power of 2 Watt and a transmitted signal bandwidth of 120 MHz. At the mobile station a uniform circular array composed of 15 monopoles arranged at an inter-element spacing of 0.43$\lambda$ was mounted on top of a small trolley at a height of about 1.5m. At the receiver site a uniform linear array[2] composed of 8 elements with half wavelength distance between adjacent sensors was mounted on a lift in about 20m height. With above arrangement, consecutive sets of the $(15 \times 8)$ individual transfer functions, cross-multiplexed in time, were measured from correlation analysis of the received and the transmitted signal. The acquisition period of one snapshot was limited to $3.2\mu s$ corresponding to a maximum path length of about 1km. During the measurements the receiver was moved at speeds of about 5km/h on the sidewalk. Rx-position and Tx-position, as well as the motion of the transmitter are marked in the site map in Figure 2. The transmitter was passing through a pedestrian tunnel approximately between times $t = 25$ s and $t = 30$ s of the measurement run. TDOA and DOA estimates obtained with 3D RARE are displayed in Figure 3 and Figure 4 relative to the orientation of the array.

The results show that during the first 25 seconds the propagation scenario is dominated by a strong line-of-sight component surrounded by local scattering pathes from trees and buildings. The trace of the DOA estimates in Figure 3 and also the corresponding TDOA estimates match exactly the motion of the transmitter depicted in Figure 2 for the direct path. At time 25 the trolley reaches the pedestrian tunnel and a second path resulting from scattering at the building (see Figure 2) appears at a DOA of approximately $-15°$. This path corresponds to a significantly larger access delay of about 0.55 to 0.58 $\mu s$. By the time the Tx moves out of the tunnel the dominant LOS component with local scattering is newly tracked by the 3D-RARE algorithm.

## 6. SUMMARY AND CONCLUSIONS

A novel method for $K$-D harmonic exponential estimation has been derived as a multi-dimensional extension of the conventional RARE algorithm. High resolution frequency parameter estimates are obtained from the proposed method in a search-free procedure at low computational complexity. The parameters in the various dimensions are independently estimated exploiting the rich structure of the $K$-D measurement model and the estimates of the parameters of interest are automatically associated. Simulation results based on synthetic and measured data of a MIMO communication channel underline the strong performance of the new approach.

[2]provided by T-NOVA, Germany

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