Time-Varying and Frequency-Selective Channel Estimation with Unequally Spaced Pilot Symbols

Jingxian Wu, Chengshan Xiao
Department of Electrical & Computer Engineering
University of Missouri, Columbia, MO 65211, USA
Jan C. Olivier
Nokia Research Center
6000 Connection drive, Irving, Texas, 75039, USA

***Abstract*— In this paper, an accurate and computationally efficient algorithm is proposed for estimating time-varying and frequency-selective fading channel with unequally spaced pilot symbols. By employing the time-varying coefficient polynomial interpolation method, it is proved that the time-varying channel impulse response can be estimated by the product of a constant interpolation matrix and the fading information at pilot symbol positions. Furthermore, a least square off-line training algorithm is presented to optimally calculate the constant matrix, taking into consideration of the statistics of channel fading and noise. Simulation results indicate that the bit error rate performance of our new estimation algorithm is close to that of the perfect channel estimation.**

I. Introduction

With the increasing demands for high quality multimedia mobile communication, new mobile standards are developed to adopt higher packet data throughput rate. Frequency-selective fading channel with intersymbol interference (ISI) is the primary impediment for reliable high date rate mobile communication. Channel estimation and equalization techniques can be jointly used to combat the adverse impairments caused by the frequency-selective fading. Equalization algorithms, such as Maximum Likelihood Sequence Estimation (MLSE) [1], can optimally eliminate ISI provided the impulse response of the fading channel is known. The channel impulse response can be estimated by pilot symbol assisted modulation (PSAM), which has been studied extensively in the recent literature [2]-[4]. However, most of the previous work on PSAM are focused on flat fading channel and equally spaced pilot symbols, and these methods can not always successfully be extended to the frequency-selective channel case and unequally spaced pilot symbols as required in the new emerging standards.

A new algorithm is proposed for estimating time-varying and frequency-selective (or dispersive) fading channel with unequally spaced pilot symbol assisted modulations. Based on polynomial interpolation and off-line training, this algorithm successfully balances the estimation accuracy and computational complexity of the system. Along with channel equalization algorithms, the impairments caused by time-varying and frequency-selective fading can be effectively mitigated.

The rest of this paper is organized as follows. The system model is given in Section II. Section III presents the channel new channel estimation algorithm with an off-line training scheme. Simulations are carried out to evaluate the performance of the combined channel estimation and equalization algorithms and the results are presented in Section IV. Section V concludes the paper.

II. System Model

The baseband equivalent system model is shown in Figure 1. When the modulated signal along with the inserted pilot symbols passes through the wireless channel, it will be corrupted by frequency-selective fading and additive noise. The signal at the output of the matched filter can be written as

$$y(t) = \sum_{n} s(n)h(t, t - nT_s) + z(t), \tag{1}$$

where $s(n)$ denotes both the data symbols and pilot symbols transmitted in the system, and $z(t)$ is the additive noise component of the output signal. $h(t,\tau) = u(t) \otimes g(t,\tau) \otimes u^*(-t)$ is the combined impulse response (CIR) of the channel, where $u(t)$ and $u^*(-t)$ are the pulse shaping filter and matched filter of the system, $g(t,\tau)$ is the physical impulse response of the time-varying frequency-selective fading channel, and the symbol $\otimes$ denotes convolution. $g(t,\tau)$ often take the form of $\sum_i g(t,\tau_i)\delta(\tau - \tau_i)$.

The output of the matched filter is sampled at the times $t = nT_s$, yielding

$$y(n) = \sum_{k=0}^{K-1} s(n-k)h_n(k) + z(n), \tag{2}$$

where $y(n) = y(nT_s)$, $z(n) = z(nT_s)$, and $K = \lceil \tau_{max}/T_s \rceil$ is the memory length of the discrete-time CIR $h_n(k) = h(nT_s, kT_s)$.

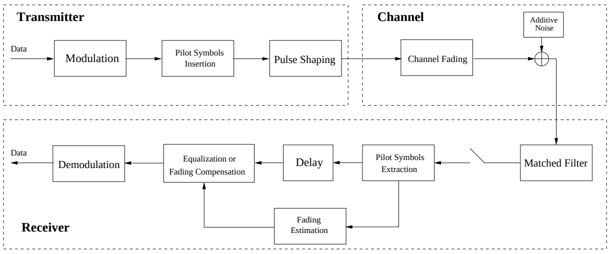

Fig. 1. System Model

The transmitted symbols are divided into slots, with the slot structure depicted in Figure 2. Without losing generality, we assume that each slot consists of N symbols with L pilot symbols at locations of $(n_1, n_2, \cdots, n_L)$.

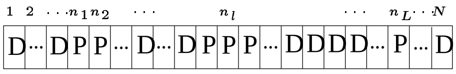

Fig. 2. Transmitted Slot Structure

For a group of $(2K-1)$ consecutive pilot symbols $[s(j), s(j+1), \cdots, s(j+2K-2)]$, we can write (2) as the following explicit equation

$$\mathbf{y}_j = \mathbf{S}_j \mathbf{h}_j + \mathbf{z}_j, \tag{3}$$

where $\mathbf{y}_j = [y(j+K-1), \cdots, y(j+2K-2)]^t$, $\mathbf{h}_j = [h_j(0), \cdots, h_j(K-1)]^t$, and $\mathbf{z}_j = [z(j+K-1), \cdots, z(j+2K-2)]^t$ are the received signal vector, the CIR vector and the noise vector, respectively, and $\mathbf{S}_j$ is the pilot matrix defined as

$$\begin{bmatrix} s(j+K-1) & \cdots & s(j+1) & s(j) \\ s(j+K) & \cdots & s(j+2) & s(j+1) \\ \vdots & \ddots & \ddots & \vdots \\ s(j+2K-2) & \cdots & s(j+K) & s(j+K-1) \end{bmatrix}. \tag{4}$$

Here we assume that the CIR is constant over the time duration of $2K-1$ symbols and the length of consecutive pilot symbols are no less than $2K-1$ symbols. These assumptions are true for all the current mobile standards (eg. TDMA IS136+ [5], GSM and EDGE [6], and W-CDMA [7]).

Based on equations (3), The least square error estimation of $\mathbf{h_j}$ is given by

$$\hat{\mathbf{h}}_j = \mathbf{S_j^\dagger} \mathbf{y_j}, \tag{5}$$

where $\mathbf{S_j^\dagger}$ is the inverse of $\mathbf{S_j}$ when it has full rank or pseudo-inverse of $\mathbf{S_j}$ when it is a singular matrix. Once the CIR at pilot locations $\hat{\mathbf{h}}_j$ are estimated from (5), it can be used to estimate the CIR of the entire slot.

III. A New Algorithm for Channel Estimation

In this section, we first present a polynomial interpolation method to estimate the channel impulse response, then a least square off-line training method is proposed to improve the accuracy and efficiency of the algorithm.

A. Polynomial Interpolation

Let $p_m(t)$ be a pre-selected mth-order polynomial, $c_m(k)$ be an unknown (or to-be-determined) complex-valued coefficient, and $\tilde{h}_n(k)$ can be expressed by Mth-order polynomial as follows

$$\tilde{h}_n(k) = \sum_{m=0}^{M} c_m(k) p_m(nT_s). \tag{6}$$

Writing (6) into matrix format, we can get

$$\widetilde{\mathbf{H}} = \mathbf{P}\mathbf{C}^h \tag{7}$$

where $(\cdot)^h$ is the Hermitian operation, the element on the ith row and jth column of $\widetilde{\mathbf{H}}$ is $(\widetilde{\mathbf{H}})_{i,j} = \tilde{h}_i(j-1)$, and $(\mathbf{P})_{(i,j)} = p_{j-1}(iT_s)$, $(\mathbf{C})_{(i,j)} = c_{j-1}(i-1)$.

In order to solve the coefficient matrix $\mathbf{C}$ with minimizing the estimation error, we consider the CIR at the pilot symbol locations, and define a cost function as follows

$$J_p = tr\left[(\widehat{\mathbf{H}}_T - \mathbf{T}\mathbf{C}^h)(\widehat{\mathbf{H}}_T - \mathbf{T}\mathbf{C}^h)^h\right], \tag{8}$$

where $tr(\cdot)$ denotes the trace of a matrix, $\widehat{\mathbf{H}}_T \in \mathbb{C}^{L\times K}$ and $\mathbf{T} \in \mathbb{C}^{L\times(M+1)}$ are sub-matrix of $\widehat{\mathbf{H}}$ and $\mathbf{P}$, respectively, and their elements $(\widehat{\mathbf{H}}_T)_{(i,j)} = \hat{h}_{n_i}(j-1)$, $(\mathbf{T})_{(i,j)} = p_{j-1}(n_i T_s)$ are corresponding to pilot symbol locations of one slot.

Since J_p is a quadratic function of $\mathbf{C}$, the coefficient matrix $\mathbf{C}$ that minimizes J_p can be obtained from the equation $\frac{\partial J_p}{\partial \mathbf{C}^*} = 0$, where $(\cdot)^*$ denotes complex conjugate, and the solution is

$$\mathbf{C} = \widehat{\mathbf{H}}_T^h \mathbf{T}(\mathbf{T}^h\mathbf{T})^\dagger. \tag{9}$$

Theoretically speaking, once the interpolation polynomials $p_m(t)$ and M are selected, the coefficient matrix $\mathbf{C}$ can be obtained from (9). This approach implies that $\mathbf{C}$ has to be calculated for every slot, thus the computational load can be heavy for real time applications. However, if we move one step forward, we will get a very interesting result. From (7) and (9), we immediately have

$$\widetilde{\mathbf{H}} = \mathbf{P}(\mathbf{T}^h\mathbf{T})^\dagger \mathbf{T}^h \widehat{\mathbf{H}}_T. \tag{10}$$

If we define $\mathbf{U} = \mathbf{P}(\mathbf{T}^h\mathbf{T})^\dagger\mathbf{T}^h$, then

$$\widetilde{\mathbf{H}} = \mathbf{U}\widehat{\mathbf{H}}_T. \tag{11}$$

It is clear now that once the interpolation polynomials $p_m(t)$ and M are chosen, $\mathbf{U}$ is a constant matrix. We are now in a position to conclude a lemma as follows:

Lemma 1: For frequency-selective fading channel, the time-varying channel impulse response matrix over an entire slot can be estimated by the product of a constant matrix and the time-varying fading matrix at pilot symbol locations.

The polynomial interpolation method provides good estimation accuracy when the signal-to-noise ratio (SNR) is high and the Doppler frequency of the fading is low. To obtain good channel estimation accuracy in typical SNR and Doppler frequency ranges of an "on-line" mobile channel, we need to take into consideration of the statistical characteristics of the channel fading and noise, therefore, we present an off-line training algorithm to optimize the constant matrix $\mathbf{U}$ in the rest of this section.

B. Off-line Training

For the off-line training, let $\mathcal{H}$ be Q slots fading matrix with no noise, $\hat{\mathcal{H}}$ be noise-corrupted fading matrix at pilot symbol locations. Both $\mathcal{H}$ and $\hat{\mathcal{H}}$ are partitioned as follows

$$\begin{aligned} \mathcal{H} &= [\,\mathbf{H}_1 \;\vdots\; \mathbf{H}_2 \;\vdots\; \cdots \;\vdots\; \mathbf{H}_Q\,] \\ \hat{\mathcal{H}} &= [\,\widehat{\mathbf{H}}_{T_1} \;\vdots\; \widehat{\mathbf{H}}_{T_2} \;\vdots\; \cdots \;\vdots\; \widehat{\mathbf{H}}_{T_Q}\,], \end{aligned}$$

where $\mathbf{H}_q \in \mathbb{C}^{N\times K}$ is the qth training slot's fading matrix with $(\mathbf{H}_q)_{(i,j)} = h_i(j-1)$, and $\widehat{\mathbf{H}}_{T_q} \in \mathbb{C}^{L\times K}$ is the noise corrupted fading matrix corresponding to pilot symbol locations. According to Lemma 1, $\mathbf{H}_q$ and $\widehat{\mathbf{H}}_{T_q}$ have the following relation

$$\widetilde{\mathbf{H}}_q = \mathbf{U}\widehat{\mathbf{H}}_{T_q}, \qquad (12)$$

where $\mathbf{U}$ is the constant interpolation matrix to be optimized. To minimize the estimation error, the cost function is defined by

$$J = tr\left[(\mathcal{H} - \mathbf{U}\hat{\mathcal{H}})^h(\mathcal{H} - \mathbf{U}\hat{\mathcal{H}})\right] \qquad (13)$$

With this cost function, the optimal solution for the constant matrix $\mathbf{U}$ is given by the following theorem.

Theorem 1: The necessary and sufficient condition for minimizing J given by (13) is

$$\mathbf{U} = \begin{cases} (\mathcal{H}\hat{\mathcal{H}}^h)_r\left[(\hat{\mathcal{H}}\hat{\mathcal{H}}^h)_r\right]^\dagger, & \text{real-valued} \\ \mathcal{H}\hat{\mathcal{H}}^h(\hat{\mathcal{H}}\hat{\mathcal{H}}^h)^\dagger, & \text{complex-valued} \end{cases} \qquad (14)$$

where $(X)_r$ stands for the real part of X.

Proof: The proof is similar to the procedure of obtaining the matrix $\mathbf{C}$, details are omitted here for brevity.

Before concluding this section, we have the following remarks.

Remark 1: When the channel fading of each branch is Rayleigh fading and the noise is white Gaussian noise, the real part of the complex-valued matrix $\mathbf{U}$ is almost identical to the real-valued matrix $\mathbf{U}$, and the imaginary part of the complex-valued matrix $\mathbf{U}$ is negligible compared to its real part.

Remark 2: After having estimated the time-varying and frequency- selective channel fading matrix $\widetilde{\mathbf{H}}$, the transmitted symbol sequence $s(n)$ can be detected by means of known equalization algorithms such as maximum-likelihood sequence estimation (MLSE) [1], delayed decision feedback sequence estimation (DDFSE) [8], or reduced state sequence estimation (RSSE) [9].

IV. Simulation Results

In this section, we are going to show by simulations the accuracy of our new channel estimation algorithm for both time-varying frequency-selective fading channels and time-varying flat fading channels.

The up-link of 8-PSK IS-136+ radio system [5] is used as an example in the simulations. The frequency-selective fading channel is modeled as a uncorrelated equal power two ray Rayleigh fading channel [10] [11] with the time spacing of T_s between the main and the second rays. $Q = 100,000$ slots are used in the off-line training process. These Q slots consist of a spread of SNR values between 15 and 25 dB inclusive, and the Doppler frequencies of 10, 80 and 180 Hz inclusive.

The normalized mean square error of the estimation is defined as

$$\bar{e^2} = \frac{1}{N}\cdot\frac{tr\{E[(\mathbf{H}-\widetilde{\mathbf{H}})(\mathbf{H}-\widetilde{\mathbf{H}})^{\mathbf{h}}]\}}{tr\{E[\mathbf{H}\mathbf{H}^{\mathbf{h}}]\}}, \qquad (15)$$

where N is the number of symbols in one slot, $\widetilde{\mathbf{H}}$ is the estimated CIR matrix, and $\mathbf{H}$ is the original CIR matrix of the frequency-selective fading channel. The normalized MSE $\bar{e^2}$ at different values of E_b/N_0 are shown in Figure 3. From the figure we can see that $\bar{e^2}$ decreases rapidly with the increase of E_b/N_0. When $E_b/N_0 = 15dB$, $\bar{e^2}$ is in the order of 10^{-5}, which means that our channel estimation algorithm has a high accuracy.

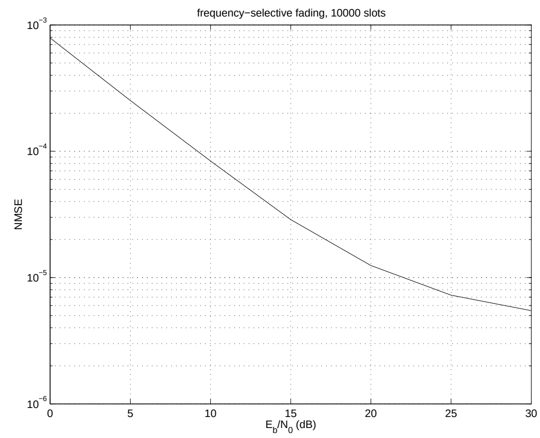

Fig. 3. The normalized MSE vs. E_b/N_0 (max $F_d = 180$) Hz

The waveform of the estimated and original CIR's amplitude and phase within one slot duration are shown in Figure 4. As can be seen from the fugures, the estimated CIR has very good agreement with that of the actual fading channel in terms of both amplitude and phase, and the result is consistent with the low normalized MSE in Figure 3.

The estimated CIRs are used for channel compensation in flat fading channel and used for channel equalization for frequency-selective fading channel. The BER performances of the compensator and the MLSE equalizer with the estimated CIRs are

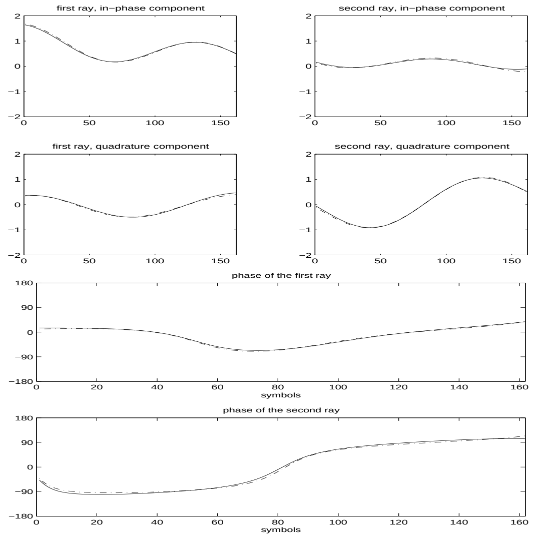

Fig. 4. Estimated and original CIR of one slot(SNR = 20dB, max F_d = 180 Hz). Solid lines stand for the estimated amplitude, and the dash and dot lines stand for the actual amplitude.

shown in Figure 5 and 6, respectively. The figures indicate that the actual estimation BER performance of the compensator is within 1 dB apart from the perfect estimation situation at typical SNR range in flat fading channel, and the estimation BER performance of the equalizer in frequency-selective fading channel is about 2 dB worse than that of the ideal estimation case. An interesting result is that the receiver gives better BER performance when the channel is frequency-selective, because it is unlikely that both of the two paths will deeply fade together.

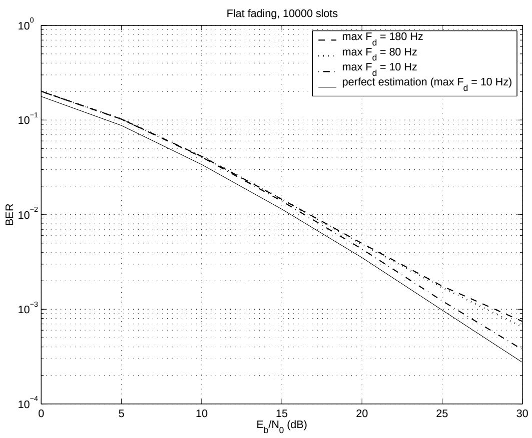

Fig. 5. BER performance of the compensator

V. Conclusions

In this paper, an accurate and computationally efficient channel impulse response estimation algorithm with unequally spaced pilot symbols is proposed, and a least square off-line training method is introduced to optimize the estimation accuracy as well as to reduce the real time computational load. Simulation results show that the channel estimation algorithm is accurate in terms of both mean square error and bit error rate. The normalized MSE of the estimation is in the order of 10^{-5} at typical SNRs, and the BER performance of the estimation algorithm is close to the results of perfect estimation.

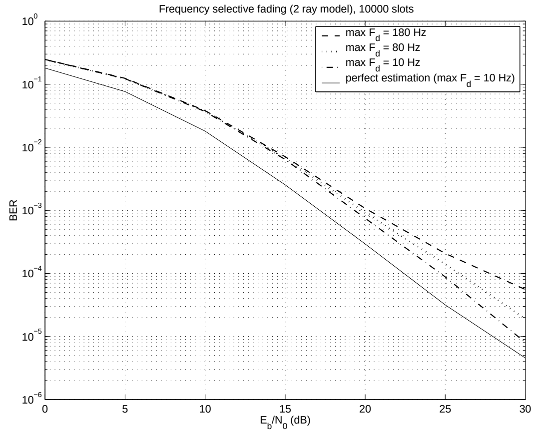

Fig. 6. BER performance of the equalizer

References

[1] J.G. Proakis, *Digital Communication*,, 4th ed. Boston, MA:McGraw-Hill, 2001.

[2] A. Aghamohammadi, H. Meyr, and G. Asheid, “A new method for phase synchronization and automatic gain control of linearly modulated signals on frequency-flat fading channels,” *IEEE Trans. Commun.*, vol.39, pp.25-29, 1991.

[3] J.K. Cavers, “An analysis of pilot symbol assisted modulation for Rayleigh fading channels,” *IEEE trans. Veh. Technol.*, vol.40, pp. 686-693, Nov. 1991.

[4] C. Xiao and J.C. Oliver, “Nonselective fading channel estimation with non-uniformly spaced pilot symbols,” *Int. J. Wireless Inform. Networks*, vol.7, pp.177-185, 2000.

[5] TIA Technical Sub-Committee TR-45.3, “TIA/EIA-136-131-A Draft Text: Digital Traffic Channel Layer 1,” Nov. 1998.

[6] TESI. Tdoc SMG2 95/97, “Enhanced Data rate for GSM Evolution (EDGE) Feasibility Study, Work Item 184; Improved Data Rates through Optimized Modulation,” version 0.3, Dec. 1997.

[7] UMTS, “Selection procedure for the choice of radio transmission technologies of UMTS,” *UMTS 30.03 version 3.2.0 ETSI*, Apr. 1998.

[8] A. Duel-Hallen and C. Heegards, “Delayed decision-feedback sequence estimation,” *IEEE Trans. Commun.*, vol.37, pp.428-436, May 1989.

[9] V. M. Eyuboglu and S.U. Quereshi, ”Reduced-state sequence estimation with set partitioning and decision feedback,” *IEEE Trans. Commun.*, vol. 36, pp. 13-20, Jan 1988.

[10] S. Chennakeshu and G.J. Saulnier, “Differential Detection of $\pi/4$-shifted-DQPSK for digital cellular radio,” *IEEE Trans. Veh. Technol.*, vol.42, pp.46-57, Feb. 1993.

[11] V. Fung, T.S. Rappaport and B. Thoma, “Bit error simulation for $\pi/4$ DQPSK mobile radio communications using two-ray and measurement-based impulse response models,” *IEEE J. Select. Areas Commun.*, vol.11, pp.393-405, Apr. 1993.

[12] A. Stamoulis, S.N. Diggavi, and N. Al-Dhahir, “Intercarrier interference in MIMO OFDM,” *IEEE Trans. Signal Processing*, vol.50, pp.2451-2464, Oct. 2002.