EM ALGORITHM-BASED TIMING SYNCHRONIZATION IN TURBO RECEIVERS

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ABSTRACT

The current paper addresses the issue of estimating the sampling instant in turbo receivers. The proposed synchronizer is based on the expectation-maximization (EM) algorithm and takes benefit from the soft information delivered by the turbo system. Performance of the proposed synchronizer is illustrated by simulation results. In particular, the mean and the variance of the estimator as well as the bit error rate reached by the synchronized system are reported.

1. INTRODUCTION

Since the discovery of turbo codes by Berrou [1], the so-called turbo principle has been extended to a number of operations to be performed by a receiver notably joint decoding and demodulation, joint decoding and equalization.

In addition to detection and decoding, a receiver has also to estimate a number of synchronization parameters in order to work properly. A consequence of the low-SNR operating point in turbo receivers is that classical estimators fail to provide good estimates of the synchronization parameters. However, such iterative receivers are able to deliver soft information on bits or symbols. It would therefore be relevant to try using this soft information in order to help the synchronizer. The idea of using soft information to estimate parameters has already been applied in a number of contributions. Reference [2], for instance, proposes to combine soft decision-directed carrier phase estimation with turbo decoding. In [3] the carrier phase synchronizer is embedded in a maximum a posteriori decoder and exploits the extrinsic information generated at each turbo iteration. Paper [4] presents a unifying framework for ML synchronization in turbo systems by means of the expectation-maximization (EM) algorithm [5].

In this paper, we will focus on the particular issue of timing synchronization in the turbo receivers. The proposed synchronizer will be derived from the EM algorithm. The sequel of this paper will then be organized as follows. In section 2, the considered transceiver will be presented. EM algorithm principles will be introduced in section 3 and be applied to timing synchronization in section 4. The implementation of the proposed EM-based synchronizer in a turbo receiver will then be discussed in section 5. Finally, in section 6, the performance of the synchronizer will be illustrated through simulation results.

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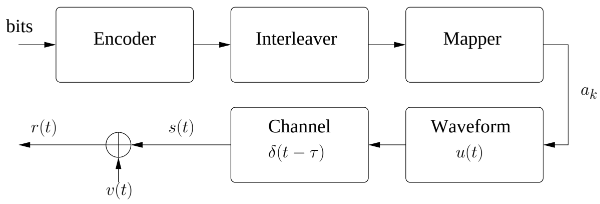

Fig. 1. Transmitter.

2. SYSTEM MODEL

In this paper we will focus on a bit-interleaved coded modulation (BICM) scheme. The transmitter (Fig. 1) is then made up of a binary convolutional encoder and a constellation mapper separated by a bit interleaver. In the baseband formalism, the signal at the transmitter output may then be written as

$$s(t) = \sum_k a_k \, u(t - kT), \tag{1}$$

where a_k's are complex symbols belonging to constellation alphabet $\mathcal{A}$, T is the symbol period and $u(t)$ is a unit energy square-root raised-cosine pulse with roll-off α. Assume that $s(t)$ is sent over an AWGN channel introducing a time delay τ, the received signal is

$$r(t) = \sum_k a_k \, u(t - kT - \tau) + v(t), \tag{2}$$

where $v(t)$ is the complex envelope of an additive white gaussian noise with power spectral density $N_0/2$. At the

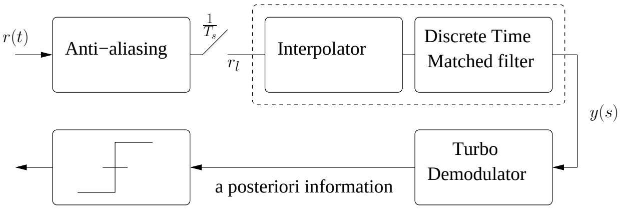

Fig. 2. Receiver.

receiver (Fig. 2), after anti-aliasing filtering, $r(t)$ is sampled at a rate of $1/T_s$ (with $T_s < T/(1+\alpha)$) leading to samples

$$r_l \triangleq r(lT_s) = \sum_k a_k\, u(lT_s - kT - \tau) + v_l, \qquad (3)$$

where v_l is a white gaussian noise with variance $2N_0/T_s$. The resulting samples r_l are passed through a discrete-time matched filter i.e.

$$y(s) = \sum_l r_l\, u^*(lT_s - s),$$

where s denotes the time at which the matched filter output is computed. Finally, we assume that statistics $y(s)$ are processed in a turbo demodulator. Such a device introduced in [6] performs iterative joint demodulation and decoding through the exchange of extrinsic information between a soft-input soft-ouput (SISO) demodulator and a SISO decoder.

Not computing the matched filter output $y(s)$ at the proper instant (i.e. $s = kT + \tau$) usually leads to a strong degradation of the system performance. The problem addressed in the sequel will therefore be the estimation of the unknown parameter τ. Our approach will be based on the EM algorithm.

3. ML ESTIMATION AND EM ALGORITHM

Let $\mathbf{r}$ denote a random vector and let $\mathbf{b}$ indicate a deterministic vector of parameters to be estimated from the observation of the received vector $\mathbf{r}$. Assume that $\mathbf{r}$ also depends on a random nuisance vector $\mathbf{a}$ independent of $\mathbf{b}$ and with a priori probability density function $p(\mathbf{a})$. The maximum likelihood estimate of $\mathbf{b}$ is then the value $\hat{\mathbf{b}}$ which maximizes the probability of observing vector $\mathbf{r}$ i.e.

$$\hat{\mathbf{b}} = \arg\max_{\tilde{\mathbf{b}}}\{\ln p(\mathbf{r}|\tilde{\mathbf{b}})\}, \qquad (4)$$

where

$$p(\mathbf{r}|\tilde{\mathbf{b}}) = \int_{\mathbf{a}} p(\mathbf{r}|\mathbf{a},\tilde{\mathbf{b}})\, p(\mathbf{a})\, d\mathbf{a}, \qquad (5)$$

and $\tilde{\mathbf{b}}$ is a trial value of $\mathbf{b}$. The EM algorithm is a method which allows to resolve iteratively the maximization problem defined in (4). Formally, if we set $\mathbf{z} \triangleq [\mathbf{r}^T, \mathbf{a}^T]^T$, the EM algorithm states that the sequence $\hat{\mathbf{b}}^{(n)}$ defined by

$$\mathcal{Q}(\tilde{\mathbf{b}}, \hat{\mathbf{b}}^{(n-1)}) = \int_{\mathbf{z}} p(\mathbf{z}|\mathbf{r}, \hat{\mathbf{b}}^{(\mathbf{n-1})})\ \ln p(\mathbf{z}|\tilde{\mathbf{b}})\ d\mathbf{z}, \qquad (6)$$

$$\hat{\mathbf{b}}^{(n)} = \arg\max_{\tilde{\mathbf{b}}}\{\mathcal{Q}(\tilde{\mathbf{b}}, \hat{\mathbf{b}}^{(n-1)})\}, \qquad (7)$$

converges under fairly general conditions towards the ML estimate (4). In the particular case of parameter vector $\mathbf{b}$ independent of vector $\mathbf{a}$, it can be shown [4] that the $\mathcal{Q}$-function defined in (6) reduces to

$$\mathcal{Q}(\tilde{\mathbf{b}}, \hat{\mathbf{b}}^{(n-1)}) = \int_{\mathbf{a}} p(\mathbf{a}|\mathbf{r}, \hat{\mathbf{b}}^{(n-1)})\ \ln p(\mathbf{r}|\mathbf{a}, \tilde{\mathbf{b}})\ d\mathbf{a}. \qquad (8)$$

The solution of (4) can then be found iteratively by only using posterior probabilities $p(\mathbf{a}|\mathbf{r}, \hat{\mathbf{b}}^{(n)})$ and the log-likelihood function $\ln p(\mathbf{r}|\mathbf{a}, \tilde{\mathbf{b}})$.

4. SYNCHRONIZATION AND EM ALGORITHM

In this section we will apply the EM framework described in the previous section to the particular case of timing synchronization for a BICM transmission . In this context, the vector $\mathbf{a}$ contains the values of the K unknown transmitted data symbols $(a_0, a_1, ..., a_{K-1}) \in \mathcal{A}^K$, the parameter vector $\mathbf{b}$ to be estimated only contains the symbol timing τ whereas the observation vector $\mathbf{r}$ contains the values of all the samples r_l.

Using the expression of the received samples r_l and neglecting terms independent of $\tilde{\mathbf{b}}$, the log-likelihood function $\ln p(\mathbf{r}|\mathbf{a}, \tilde{\mathbf{b}})$ present in (8) can then be written as

$$\ln p(\mathbf{r}|\mathbf{a}, \tilde{\mathbf{b}}) = \mathrm{Re}\Big\{\sum_{k=0}^{K-1} a_k^*\ y(kT + \tilde{\tau})\Big\}, \qquad (9)$$

where $y(kT + \tilde{\tau})$ corresponds to the matched filter output evaluated at $kT + \tilde{\tau}$. Let us define for each transmitted symbol a_k

$$\eta_k(\mathbf{r}, \hat{\mathbf{b}}^{(n-1)}) \triangleq \int_{\mathbf{a}\in\mathcal{A}^K} a_k\ p(\mathbf{a}|\mathbf{r}, \hat{\mathbf{b}}^{(n-1)})\, d\mathbf{a}$$
$$= \sum_{a\in\mathcal{A}} a\ p(a_k = a|\mathbf{r}, \hat{\mathbf{b}}^{(n-1)}). \qquad (10)$$

Using this definition and replacing $\ln p(\mathbf{r}|\mathbf{a}, \tilde{\mathbf{b}})$ by (9) in (8), we get

$$\mathcal{Q}(\tilde{\mathbf{b}}, \hat{\mathbf{b}}^{(n-1)}) = \mathrm{Re}\Big\{\sum_{k=0}^{K-1} \eta_k^*(\mathbf{r}, \hat{\mathbf{b}}^{(n-1)})\, y(kT + \tilde{\tau})\Big\}. \qquad (11)$$

Notice the similarity between (9) and (11) : the latter is formally obtained from the former by simply replacing the actual symbol a_k by their respective a posteriori expected values $\eta_k(\mathbf{r}, \hat{\mathbf{b}}^{(n-1)})$. These mean values are not constellation points but rather weighted averages over all constellation points according to the posterior probabilities $p(a_k|\mathbf{r}, \hat{\mathbf{b}}^{(n-1)})$. The proposed EM-based synchronization method may therefore be seen as a "soft decision" directed method.

5. EM ALGORITHM IMPLEMENTATION IN TURBO RECEIVERS

5.1. A Posteriori Probability Computation

Thanks to the particular structure of the log-likelihood function (9), we see from (10) that only the marginal posterior probabilities $p(a_k|\mathbf{r}, \hat{\mathbf{b}}^{(n-1)})$ are required to compute $\mathcal{Q}(\tilde{\mathbf{b}}, \hat{\mathbf{b}}^{(n-1)})$. The latter probabilities are however not directly available since the turbo demodulation scheme computes a posteriori probabilities on bits rather than on symbols. However, if we assume that, thanks to the presence of the interleaver, the bits transmitted in one symbol are independent, symbol posterior probabilities $p(a_k|\mathbf{r}, \hat{\mathbf{b}}^{(n-1)})$ may be approximated as

$$p(a_k|\mathbf{r}, \hat{\mathbf{b}}^{(n-1)}) \simeq \prod_{m=1}^{M} p(c_k^m|\mathbf{r}, \hat{\mathbf{b}}^{(n-1)}), \qquad (12)$$

where M is the number of bits contained in symbol a_k, c_k^m is the m$^{\text{th}}$ bit of a_k and $p(c_k^m|\mathbf{r}, \hat{\mathbf{b}}^{(n-1)})$ is the bit a posteriori probabilities delivered by the SISO decoder at turbo iteration n. Using approximation (12), we may perform a new EM-step at each turbo iteration and therefore merge the synchronization iterations (EM algorithm) into those of detection (turbo demodulation).

5.2. $\mathcal{Q}$-function Maximization

At each iteration the EM algorithm requires to find the estimate $\hat{\tau}^{(n)}$ which maximizes (11). Such a maximization problem has however no analytical solution and we will resort to a Newton-Raphson method in order to solve it i.e.

$$\hat{\tau}^{(n)} = \hat{\tau}^{(n-1)} - \left(\frac{\partial \mathcal{Q}}{\partial \tilde{\tau}}\right)_{|\tilde{\tau}=\hat{\tau}^{(n-1)}} \left(\frac{\partial^2 \mathcal{Q}}{\partial \tilde{\tau}^2}\right)^{-1}_{|\tilde{\tau}=\hat{\tau}^{(n-1)}}, \quad (13)$$

where $\hat{\tau}^{(n)}$ denotes the timing estimate at the turbo iteration n. Note from (11) that the first and second derivative of $\mathcal{Q}(\tilde{\mathbf{b}}, \hat{\mathbf{b}}^{(n-1)})$ require to compute the first and second derivative of the matched filter output. In this paper, we will approximate them by second-order centered finite differences.

6. SIMULATION RESULTS

In this section the performance of the proposed synchronization method will be studied through simulation results.

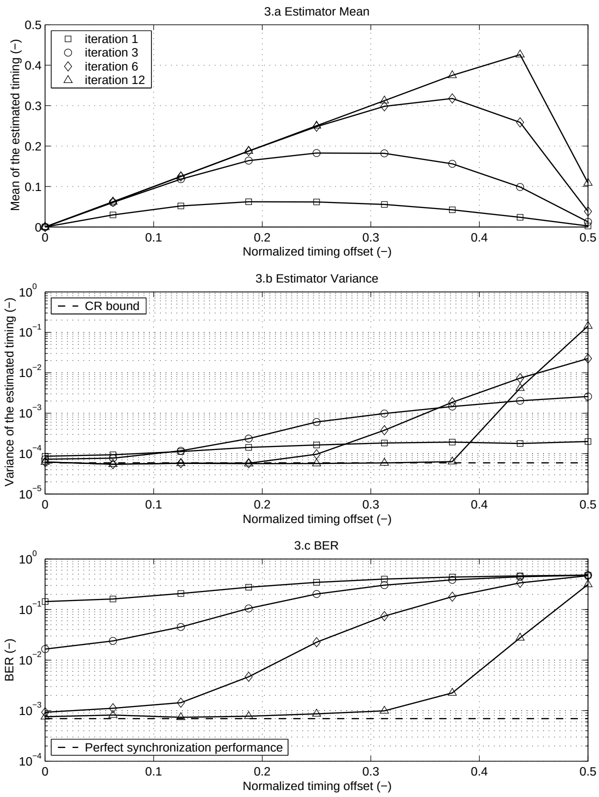

Fig. 3. Estimator mean, estimator variance and BER versus normalized timing offset for $E_b/N_0 = 4dB$.

At the transmitter, we consider a rate-$\frac{1}{2}$ non-systematic convolutional encoder with polynomial generators $(g_1, g_2) = (5, 7)_8$ and use 16-QAM modulation. A mapping proposed by ten Brink in [6] and referred to as medium unconditioned bit-wise mutual information mapping is used. The pulse waveform is a square-root raised cosine with roll-off 0.2. The interleaver is totally random and a different permutation is used at each frame. The considered interpolator [7] is designed in order to minimize, on the bandwidth of the useful signal $s(t)$, the quadratic error between the ideal interpolator frequency response and the interpolator frequency response. The number of taps of the interpolator is set to 20. The simulations have been run for frames of 500 16-QAM symbols and 12 turbo iterations have been performed. For each new frame the initial timing estimate is initialize to 0.

Fig. 3 represents the synchronizer performance (mean and variance) and the bit error rate (BER) for $E_b/N_0 = 4$dB and for a normalized timing offset τ/T ranging from 0 to 0.5. The dashed curves represent the data-aided (DA) Cramer-Rao bound in Fig. 3.b and the BER reached by a perfectly synchronized system in Fig. 3.c.

We notice that the timing estimation is unbiased and reaches the Cramer-Rao bound for values of τ/T up to about 0.4. In almost all this range, the estimation error is therefore small enough for the system to reach the BER of a perfectly

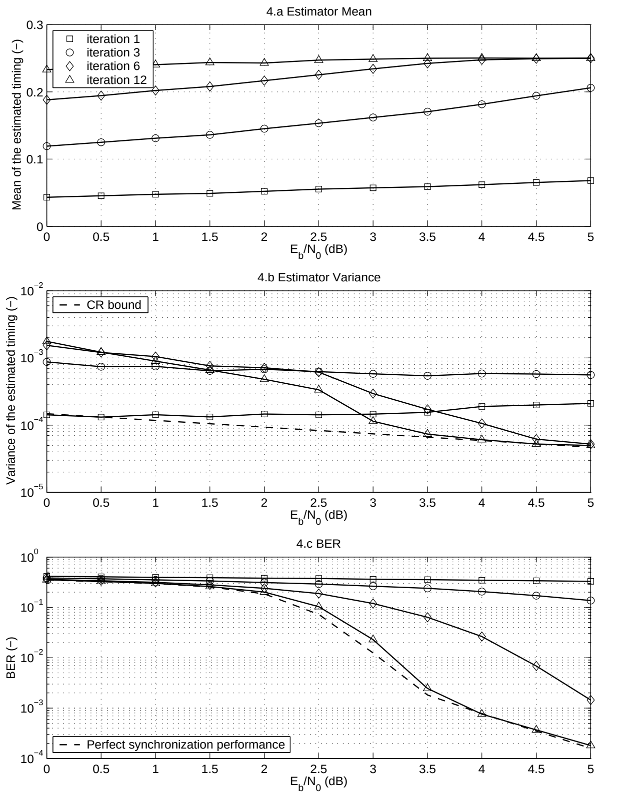

Fig. 4. Estimator mean, estimator variance and BER versus E_b/N_0 for normalized timing offset $\tau/T = 0.25$.

synchronized system. For values of τ/T greater than 0.4, the estimator is biased and its variance moves away from the Cramer-Rao bound. This may be explained by observing that successive estimates $\hat{\tau}^{(n)}$ sometimes converge towards $T - \tau$ instead of to the actual timing τ. The estimate mean remains however positive, which expresses that the system converges more often to the actual timing offset than to $T - \tau$. In particular, the mean curve does not cancel out for $\tau/T = 0.5T$ unlike the non data-aided (NDA) estimator mean curve. This may be explained by the presence of the encoder in the transmission scheme. Indeed, it limits the number of possible transmitted sequences and therefore enables the proposed synchronizer to benefit from an a priori information about the transmitted sequence. Our timing estimator can then be regarded as a "code-aided" synchronizer i.e. is an intermediate case between a non data-aided and a data-aided estimator.

Fig. 4.a, 4.b and 4.c represent the mean, the variance and the BER obtained for a normalized timing offset $\tau/T = 0.25$ and for E_b/N_0-ratios ranging from 0dB to 5dB. The parameters are the same as for the previous simulations. We see that the synchronizer and the turbo demodulator are actually complementary. Indeed, on the one hand, the synchronizer requires a low BER in order to deliver good-quality estimate (i.e unbiased and with small variance). On the other hand, the turbo-demodulator needs a good timing estimate in order to decrease the BER. This is the reason why the estimate variance starts converging to the Cramer-Rao bound for an E_b/N_0-ratio located in the so-called "waterfall" region. In the same way, it explains why the turbo demodulator reaches the same BER as the perfectly synchronized system only when the synchronizer has converged to the Cramer-Rao bound. Note however that one may observe that the convergence to such a BER requires more iterations than for a perfectly synchronized system since in the considered system the receiver has to perform both the detection and the synchronization.

7. CONCLUSION

In this paper we derive a timing synchronizer from the EM algorithm. It is shown that this synchronizer is actually well-suited to the soft iterative structure of the turbo systems: it can be embedded in the receiver without significant increase of the complexity and may take benefit from the available soft information. Simulation results show that the proposed timing estimator is unbiased and reaches the Cramer-Rao bound over a wide range of timing offsets. Over this range, the bit error rate reached by the synchronized system does not suffer from any degradation with respect to a perfectly synchronized system.

8. REFERENCES

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