ESTIMATION OF MULTIPLE DELAYS IN A SLOWLY FADING CHANNEL

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ABSTRACT

The time-of-arrival estimation of several short delayed replicas of a known signal in a single antenna receiver is an ill-conditioned problem, which improves if the amplitudes are not completely static. We present a signal model that includes slow amplitude variations using a Karhunen-Loève expansion, and the corresponding Maximum Likelihood estimator. The results show that a clear performance improvement in the delay estimation can be achieved by including these variations in the model.

1. INTRODUCTION

In a single antenna receiver, the estimation of the delays of several static replicas of a known signal becomes an ill-conditioned problem as the separation among delays decreases [1]. Nevertheless, this scenario is quite usual in satellite navigation and in wireless communications systems; see [1] and [2]. One factor that improves this situation is the slow fading which varies the amplitudes of the replicas, as long as the signal can be observed for a long enough period or in several time slots. In this paper, we assume that the slow fading does not change the delays of the impinging replicas during the observation period, but that it changes their amplitudes due to the rotation of the carrier phase and to the varying characteristics of the channel. This approximation is valid if the carrier frequency is much greater than the signal bandwidth.

In the next section we present a signal model of this scenario, based on two Karhunen-Loéve expansion, one for the transmitted signal, and one for the fading amplitudes. This general model is particularized in section 3 for a DS-CDMA signal with long spreading code, in which the data modulation has been eliminated using a Decision-Directed scheme. The estimation is then performed at the output of a bank of correlators in order to reduce the problem size. Section 4 presents the Conditional Maximum Likelihood estimator. Finally, section 5 contains simulaton results.

Notation. '$\equiv$' is used to perform definitions. The notations $[\mathbf{b}]_n$, $[\mathbf{B}]_{p,q}$ refer to a column vector $\mathbf{b}$ or a matrix $\mathbf{B}$ by specifying a generic element with index n or indexes p, q. A centred dot '$\cdot$' refers to all possible values of the index. The range (interval) of values x such that $a \leq x \leq b$ is denoted by $[a, b]$. '$\odot$' is the Hadamard (element-by-element) product. $\mathbf{I}$ and $\mathbf{1}$ denote an identity matrix and a column vector of ones of proper size, respectively. $\mathbf{1}_{N_1 \times N_2}$ is a $N_1 \times N_2$ matrix of ones. $(\cdot)^{\mathrm{H}}$ performs the Hermitian (transpose-conjugate) of a matrix. '$\diamond$' is the Katri-Rao product, (see equation (19)). '$\times$' is the product defined in (10). Cardinals are written with a letter N and a mnemonic sub-script: N_{s} is the number of samples, N_{g} is the length of a vector $\mathbf{g}$, and so on.

2. SIGNAL MODEL

Let us consider an scenario in which several replicas of a known signal arrive with different delays at a receiver equipped with a single antenna. The receiver samples the incoming signal at $1/T_{\mathrm{s}}$ rate in the interval $[0, (N_{\mathrm{s}} - 1)T_{\mathrm{s}}]$. We assume that the delays remain constant during this interval, but that the amplitudes vary slowly in it, which is the typical behaviour of a slowly fading channel. The low-pass equivalent of the incoming signal is

$$\mathrm{y}(t) = \sum_{k=1}^{N_{\mathrm{m}}} \mathrm{s}(t - \tau_k)\mathrm{a}_k(t) + \mathrm{n}(t), \tag{1}$$

where $\mathrm{s}(t)$ is the transmitted signal, $\mathrm{a}_k(t)$ is the complex amplitude of the k-th replica, and $\mathrm{n}(t)$ is a white complex Gaussian process with variance σ^2. After sampling, we obtain the model

$$\mathbf{y} = \sum_{k=1}^{N_{\mathrm{m}}} \mathbf{s}(\tau_k) \odot \mathbf{a}_k + \mathbf{n}, \tag{2}$$

where we have defined a vector for each signal in (1), ($n = 1, \ldots, N_{\mathrm{s}}, k = 1, \ldots, N_{\mathrm{m}}$):

$$\begin{aligned} &[\mathbf{y}]_n \equiv \mathrm{y}((n-1)T_{\mathrm{s}}),\ [\mathbf{s}(\tau)]_n \equiv \mathrm{s}((n-1)T_{\mathrm{s}} - \tau), \\ &[\mathbf{a}_k]_n \equiv \mathrm{a}_k((n-1)T_{\mathrm{s}}), \quad \text{and} \quad [\mathbf{n}]_n \equiv \mathrm{n}((n-1)T_{\mathrm{s}}). \end{aligned} \tag{3}$$

Next, we assume that there are two truncated Karhunen-Lòeve expansions that approximate $\mathbf{s}(\tau)$ and $\mathbf{a}_k$ respectively with negligible error, i.e. we may write

$$\mathbf{s}(\tau) = \mathbf{C}\mathbf{g}(\tau), \quad \text{and} \quad \mathbf{a}_k = \mathbf{K}\boldsymbol{\eta}_k. \tag{4}$$

As we will see in the next section, $\mathbf{s}(\tau)$ can be the result of a linear modulation, with $\mathbf{C}$ being a convolution matrix associated to the symbols, and $\mathbf{g}(\tau)$ the pulse shape employed. The expansion of $\mathbf{a}_k$ can be obtained by sampling the spectrum of $\mathrm{a}_k(t)$. Define first the Vandermonde vector

$$[\boldsymbol{\phi}(f)]_n \equiv \mathrm{e}^{j2\pi f(n-1)T_\mathrm{s}}, \; n = 1, \ldots, N_\mathrm{s}. \tag{5}$$

Assuming that $\mathrm{a}_k(t)$ has bandwidth B, it can be approximated by sampling its spectrum $\mathrm{a}_k(f)$ at frequencies f_1, f_2, ..., f_{N_k}:

$$\begin{aligned}[\mathbf{a}_k]_n = \textstyle\int_B \mathrm{a}(f)\mathrm{e}^{j2\pi f(n-1)T_\mathrm{s}}df \approx \\ \textstyle\sum_{p=1}^{N_\mathrm{k}} \mathrm{a}(f_p)\mathrm{e}^{j2\pi f_p(n-1)T_\mathrm{s}}(f_{p+1} - f_p) = \\ \textstyle\sum_{p=1}^{N_\mathrm{k}} \mathrm{a}(f_p)[\boldsymbol{\phi}(f_p)]_n(f_{p+1} - f_p),\end{aligned} \tag{6}$$

For all n and a sufficient N_k, we have

$$\mathbf{a}_k = \sum_{p=1}^{N_\mathrm{k}} \mathrm{a}(f_p)(f_{p+1} - f_p)\,\boldsymbol{\phi}(f_p). \tag{7}$$

Thus, $\mathbf{a}_k$ belongs to the span of $\boldsymbol{\phi}(f_1), \ldots, \boldsymbol{\phi}(f_{N_\mathrm{k}})$. This allows us to model the amplitudes as

$$\mathbf{a}_k = \mathbf{K}\boldsymbol{\eta}_k \;\text{ with }\; \mathbf{K} \equiv [\boldsymbol{\phi}(f_1), \ldots, \boldsymbol{\phi}(f_{N_\mathrm{k}})], \tag{8}$$

$\boldsymbol{\eta}_k$ being an unknown vector parameter. We proceed to substitute the expansions (4) and (7) in (2):

$$\begin{aligned}\mathbf{y} = \sum_{k=1}^{N_\mathrm{m}} (\mathbf{C}\mathbf{g}(\tau_k)) \odot (\mathbf{K}\boldsymbol{\eta}_k)) + \mathbf{n} = \\ \sum_{k=1}^{N_\mathrm{m}} ((\sum_{q=1}^{N_\mathrm{g}} [\mathbf{C}]_{\cdot,q}[\mathbf{g}(\tau_k)]_q) \odot (\sum_{r=1}^{N_\mathrm{k}} [\mathbf{K}]_{\cdot,r}[\boldsymbol{\eta}_k]_r)) + \mathbf{n} = \\ \sum_{r=1}^{N_\mathrm{k}} \sum_{q=1}^{N_\mathrm{g}} ([\mathbf{C}]_{\cdot,q} \odot [\mathbf{K}]_{\cdot,r}) \sum_{k=1}^{N_\mathrm{m}} [\mathbf{g}(\tau_k)]_q[\boldsymbol{\eta}_k]_r + \mathbf{n}.\end{aligned} \tag{9}$$

The signal in this equation is a linear combination of vectors $[\mathbf{C}]_{\cdot,q} \odot [\mathbf{K}]_{\cdot,r}$. Thus, we may write (9) in terms of the following product, ($q = 1, \ldots, N_\mathrm{g}, r = 1, \ldots, N_\mathrm{k}$):

$$[\mathbf{C} \times \mathbf{K}]_{\cdot,N_\mathrm{g}(r-1)+q} \equiv [\mathbf{C}]_{\cdot,q} \odot [\mathbf{K}]_{\cdot,r}. \tag{10}$$

In the third sum in (9), if we vary q and r as in the definition of '$\times$', we obtain the column vector

$$\sum_{k=1}^{N_\mathrm{m}} \mathrm{diag}(\mathbf{g}(\tau_k), \overset{N_\mathrm{k})}{\ldots}, \mathbf{g}(\tau_k))\boldsymbol{\eta}_k. \tag{11}$$

Equations (10) and (11) allow us to achieve a compact model. Define

$$\begin{aligned}\mathbf{G}_\mathrm{o}(\tau_k) \equiv \mathrm{diag}(\mathbf{g}(\tau_k), \overset{N_\mathrm{k})}{\ldots}, \mathbf{g}(\tau_k)), \\ \mathbf{G}(\boldsymbol{\tau}) \equiv [\mathbf{G}_\mathrm{o}(\tau_1), \mathbf{G}_\mathrm{o}(\tau_2), \ldots, \mathbf{G}_\mathrm{o}(\tau_{N_\mathrm{m}})], \\ \boldsymbol{\eta} \equiv [\boldsymbol{\eta}_1^\mathrm{T}, \boldsymbol{\eta}_2^\mathrm{T}, \ldots, \boldsymbol{\eta}_{N_\mathrm{m}}^\mathrm{T}]^\mathrm{T}.\end{aligned} \tag{12}$$

Then, (9) can be written as

$$\mathbf{y} = (\mathbf{C} \times \mathbf{K})\mathbf{G}(\boldsymbol{\tau})\boldsymbol{\eta} + \mathbf{n}. \tag{13}$$

We have obtained a signal model with a familiar form in array processing [3], but in which several consecutive columns of $\mathbf{G}(\boldsymbol{\tau})$ depend on the same element of $\boldsymbol{\tau}$. The model without fading would result if $\mathbf{K} = \mathbf{1}$.

3. APPLICATION TO A DS-CDMA SIGNAL WITH LONG SPREADING CODE

We proceed to apply the model (13) to a DS-CDMA signal with long spreading code as the ones employed in navigation systems [4], by specifying the factorisation $\mathbf{s}(\tau) = \mathbf{C}\mathbf{g}(\tau)$ from the modulation in $\mathrm{s}(t)$. Then, the estimation problem will be posed at the output of a bank of correlators in order to reduce the problem size.

Assume that the transmitted signal is the convolution of a delta train $\mathrm{c}(t)$, that contains the spreading code and the data modulation, and a pulse shape of (approximately) finite duration $\mathrm{g}(t)$,

$$\mathrm{s}(t) = \mathrm{c}(t) * \mathrm{g}(t). \tag{14}$$

If $1/T_\mathrm{s}$ is greater than the Nyquist sampling frequency of $\mathrm{s}(t)$, we may represent $\mathrm{c}(t)$ as a train of sincs,

$$\mathrm{c}(t) = \sum_{r=-\infty}^{+\infty} c_r \mathrm{sinc}\left(\frac{t - rT_\mathrm{s}}{T_\mathrm{s}}\right). \tag{15}$$

For simplicity, assume that there is one codeword per symbol, and that one codeword is formed by N_c samples at the $1/T_\mathrm{s}$ sampling rate. Then, if a_p is the p-th symbol, and w_q the q-th sample of the codeword, the sequence in (15) is $c_{N_\mathrm{c}(p-1)+q} = a_p w_q$.

Given that the Sampling Theorem holds, the convolution in (14) can be performed in the discrete domain:

$$\mathrm{s}(nT_\mathrm{s} - \tau) = \sum_{r=-\infty}^{\infty} \mathrm{c}(nT_\mathrm{s} - rT_\mathrm{s})\mathrm{g}(rT_\mathrm{s} - \tau). \tag{16}$$

Since the pulse is approximately time-limited, $\mathrm{g}(t - \tau)$ can be regarded as zero outside an interval $[0, (N_\mathrm{g} - 1)T_\mathrm{s}]$. This makes the sum in (16) finite, allowing us to obtain a factorisation like the one in (4), in which $\mathbf{C}$ is a convolution matrix, and $\mathbf{g}(\tau)$ contains the samples of $\mathrm{g}(t - \tau)$ inside

$[0,(N_g - 1)T_s]$. Specifically, the terms in (4) are, ($n = 1,\ldots,N_s$, $q = 1,\ldots,N_g$),

$$[\mathbf{C}]_{n,q} \equiv c_{n-q}, \;\; [\mathbf{g}(\tau)]_q \equiv \mathrm{g}(qT_s - \tau). \qquad (17)$$

Given that the spreading codes are long, the number of rows of $\mathbf{C}$ is usually enormous. In order to reduce this number, we assume that the data modulation has been removed from $\mathbf{C}$ and that the estimation is performed at the output of a bank of correlators. Besides, in order to simplify the implementation further, we assume that the receiver correlates with $\mathbf{C} \times \widetilde{\mathbf{K}}$, $\widetilde{\mathbf{K}}$ being a version of $\mathbf{K}$ that is constant inside each codeword. To correlate using $\widetilde{\mathbf{K}}$ produces almost no performance loss, given that the variation of the amplitudes is negligible in these short periods (slow fading). A convenient implementation would consist of detecting the data and then eliminating it, (Decision-Directed scheme), at the output of a bank of correlators matched to a single codeword. This implementation would be acceptable if the signal replicas do not block the data detection. Thus, with these assumptions, the correlators' output $\mathbf{y}_c$ is

$$\mathbf{y}_c = (\mathbf{C} \times \widetilde{\mathbf{K}})^{\mathrm{H}}(\mathbf{C} \times \mathbf{K})\mathbf{G}(\boldsymbol{\tau})\boldsymbol{\eta} + (\mathbf{C} \times \widetilde{\mathbf{K}})^{\mathrm{H}}\mathbf{n}. \qquad (18)$$

To put this model in operation requires to calculate beforehand the constant matrix in the signal term and the noise covariance matrix: $\mathbf{R}_S \equiv (\mathbf{C} \times \widetilde{\mathbf{K}})^{\mathrm{H}}(\mathbf{C} \times \mathbf{K})$ and $\mathbf{R}_N \equiv (\mathbf{C} \times \widetilde{\mathbf{K}})^{\mathrm{H}}(\mathbf{C} \times \widetilde{\mathbf{K}})$. Since the data modulation has been removed, $\mathbf{C}$ is the convolution matrix of a periodic code. This allows us to calculate $\mathbf{R}_S$ and $\mathbf{R}_N$ in a number of operations independent of N_s. First, define the Katri-Rao product, which is a column-wise Kronecker product. If $\mathbf{A}$ and $\mathbf{B}$ have both N columns and N_A and N_B rows respectively, we have, ($p = 1,\ldots,N_A$, $q = 1,\ldots,N_B$, $r = 1,\ldots,N$),

$$[\mathbf{A} \diamond \mathbf{B}]_{N_B(p-1)+q,r} \equiv [\mathbf{A}]_{p,r}[\mathbf{B}]_{q,r}. \qquad (19)$$

Then, $\mathbf{C}$ has the structure

$$\mathbf{C} = [\mathbf{C}_o^{\mathrm{T}},\ldots,\mathbf{C}_o^{\mathrm{T}}]^{\mathrm{T}} = \mathbf{1}_{N_b \times N_g} \diamond \mathbf{C}_o, \qquad (20)$$

where $\mathbf{C}_o$ is the convolution matrix of a single codeword, and it is possible to extract a compatible structure to this from $\mathbf{K}$:

$$\begin{aligned}&[\mathbf{K}]_{N_{sc}(p-1)+q,r} = \mathrm{e}^{2\pi f_r T_s (N_{sc}(p-1)+q)} = \\ &[\mathbf{K}_1]_{p,r}[\mathbf{K}_2]_{q,r} = [\mathbf{K}_1 \diamond \mathbf{K}_2]_{N_{sc}(p-1)+q,r}.\end{aligned} \qquad (21)$$

In this equation, we have defined $[\mathbf{K}_1]_{p,r} \equiv \mathrm{e}^{2\pi f_r T_s N_{sc}(p-1)}$ and $[\mathbf{K}_2]_{q,r} \equiv \mathrm{e}^{2\pi f_r T_s q}$. $\mathbf{K}_2$ describes the variation of $\mathbf{K}$ inside one codeword. Thus, we obtain $\widetilde{\mathbf{K}}$ by substituting this matrix by an all ones matrix:

$$\widetilde{\mathbf{K}} \equiv \mathbf{K}_1 \diamond \mathbf{1}_{N_c \times N_k}. \qquad (22)$$

Using (20), (22) and the join properties of the '$\times$' and '$\diamond$' operators, it can be shown that $\mathbf{R}_S$ and $\mathbf{R}_N$ follow the formulas

$$\begin{aligned}\mathbf{R}_S = &\left[(\mathbf{1}_{N_b \times N_g} \times \mathbf{K}_1)^{\mathrm{H}}(\mathbf{1}_{N_b \times N_g} \times \mathbf{K}_1)\right] \odot \\ &\left[(\mathbf{C}_o \times \mathbf{1}_{N_{sc} \times N_k})^{\mathrm{H}}(\mathbf{C}_o \times \mathbf{K}_2)\right], \\ \mathbf{R}_N = &\left[(\mathbf{1}_{N_b \times N_g} \times \mathbf{K}_1)^{\mathrm{H}}(\mathbf{1}_{N_b \times N_g} \times \mathbf{K}_1)\right] \odot \\ &\left[(\mathbf{C}_o \times \mathbf{1}_{N_{sc} \times N_k})^{\mathrm{H}}(\mathbf{C}_o \times \mathbf{1}_{N_{sc} \times N_k})\right].\end{aligned} \qquad (23)$$

With these matrices, we achieve the model

$$\mathbf{y}_w = \mathbf{L}\mathbf{R}_S\mathbf{G}(\boldsymbol{\tau})\boldsymbol{\eta} + \mathbf{n}_w, \qquad (24)$$

in which $\mathbf{y}_c$ is the whitened correlation bank output , $\mathbf{L}$ the Cholesky factor of $\mathbf{R}_N^{-1}$, ($\mathbf{L}^{\mathrm{H}}\mathbf{L} = \mathbf{R}_N^{-1}$), and $\mathbf{n}_c$ has covariance $\mathbf{I}\sigma^2$.

4. THE CONDITIONAL MAXIMUM LIKELIHOOD ESTIMATOR

The Maximum Likelihood estimator of $\boldsymbol{\eta}$ and $\boldsymbol{\tau}$ in (24) is equivalent to the least squares estimator given that the noise is white [5]. Defining $\mathbf{S}_w(\boldsymbol{\tau}) \equiv \mathbf{L}\mathbf{R}_S\mathbf{G}(\boldsymbol{\tau})$ and suppressing the τ dependency, the ML estimation in (24) is

$$[\hat{\mathbf{a}}, \hat{\boldsymbol{\tau}}] = \arg\min_{\boldsymbol{\eta},\boldsymbol{\tau}} \|\mathbf{y}_w - \mathbf{S}_w\boldsymbol{\eta}\|^2. \qquad (25)$$

For a fixed $\boldsymbol{\tau}$, the global minimum is obtained at $\boldsymbol{\eta} = \mathbf{S}_w^{\dagger}\mathbf{y}_w$. Thus, it is only necessary to minimise the cost function

$$L = \|\mathbf{y}_w - \mathbf{S}_w\mathbf{S}_w^{\dagger}\mathbf{y}_w\|^2. \qquad (26)$$

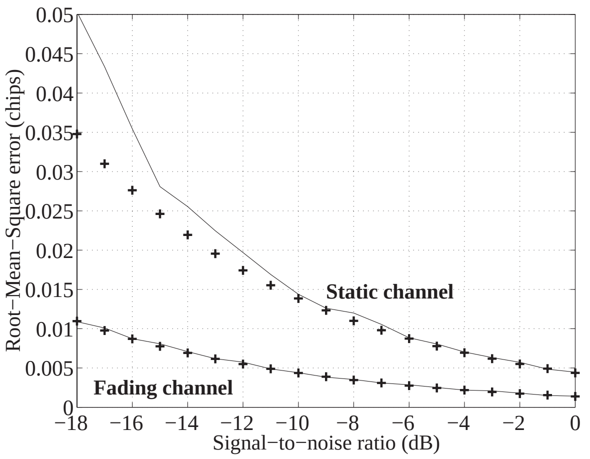
The minimisation can be performed using a variant of Newton's method, the Modified Variable Projection (MVP) Method in [6]. Assuming first that each column of $\mathbf{S}_w$ depends on a different element of $\boldsymbol{\tau}$, the MVP method updates the p-th iteration, $\boldsymbol{\tau}^{(p)}$, using

$$\boldsymbol{\tau}^{(p+1)} = \boldsymbol{\tau}^{(p)} - \mu_p\mathbf{M}^{-1}\nabla\mathrm{L}, \qquad (27)$$

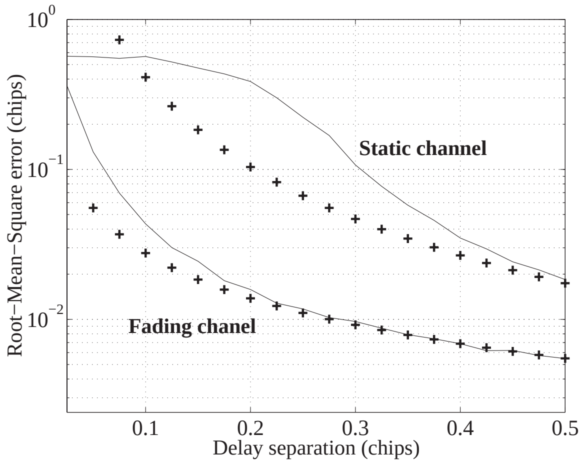
where μ_p adjusts the step length to assure a descendant direction,

$$\begin{aligned}&\mathbf{M} \equiv 2\,\mathrm{Re}\{[\mathbf{D}^{\mathrm{H}}(\mathbf{I}-\mathbf{P})\mathbf{D}] \odot [\mathbf{S}_w^{\dagger}\mathbf{y}_w\mathbf{y}_w^{\mathrm{H}}(\mathbf{S}_w^{\dagger})^{\mathrm{H}}]\}, \quad \text{and} \\ &\nabla\mathrm{L} \equiv -2\,\mathrm{Re}\{\mathrm{diag}\{\mathbf{S}_w^{\dagger}\mathbf{y}_w\mathbf{y}_w^{\mathrm{H}}(\mathbf{I}-\mathbf{P})\mathbf{D}\}\}.\end{aligned} \qquad (28)$$

In these formulas, $\mathbf{P} \equiv \mathbf{S}_w\mathbf{S}_w^{\dagger}$ and $[\mathbf{D}]_{\cdot,k} \equiv \partial\,[\mathbf{S}_w]_{\cdot,k}/\partial\tau_k$, $k = 1,\ldots,N_m$. Since several consecutive columns of $\mathbf{S}_w$ depend on the same element of $\boldsymbol{\tau}$, it is necessary to apply the Chain Rule to $\nabla\mathrm{L}$ and $\mathbf{M}$ before using them in (27). This amounts to adding up into a single element the elements in $\nabla\mathrm{L}$ that correspond to the same τ_k and, in the same way, to adding up into a single element the elements in $\mathbf{M}$ that correspond to the same (τ_k, τ_r) pair, $k, r = 1,\ldots,N_m$. The resulting vector and matrix can be employed in (27).



(a) RMS error versus SNR.



(b) RMS error versus delay separation.

Fig. 1. RMS error and Cramer-Rao Bound versus (a) SNR, and (b) delay separation. Crosses mark the Cramer-Rao Bounds.

5. SIMULATION RESULTS

The signal model has been simulated with a GPS C/A signal but modulated with Root-Raised cosine pulses, roll-off $\beta = 0.2$. The chip rate was 1023 MChips/sec, one codeword having 1023 chips. The sampling rate was 2 samples/chip. There were two signal replicas with delays $\tau_1 = -0.1$ chips and $\tau_2 = 0.4$ chips respectively, and the amplitude of the second replica was 10 dB below the amplitude of the first replica. The phases were equal. The first replica was almost static: there was a frequency component at 1 Hz and at -1 Hz 20 dB lower than the component at 0 Hz, the amplitude being real. The amplitude of the second replica had a 3 Hz modulation, being also real. The frequencies in (7) employed by the model were from -4 to 4 Hz with 1 Hz spacing, and the pulse was truncated to $N_{\mathrm{g}} = 13$ samples to which 8 zero padding samples were added. Figure 1 (a) shows the performance of the ML estimator in Root-Mean-Square (RMS) error in a static and fading channels respectively. The inclusion of the fading in the model improves the performance clearly. Without fading and for low Signal-to-Noise (SNR) ratios, the ML estimator is not effecient due to the ill-condition of the problem. In Figure 1 (b) the RMS error for a SNR ratio of 12 dB has been plotted versus the delay separation between the signal replicas. In both cases the problem becomes ill-conditioned as the delay separation decreases but the RMS error is clearly lower in the fading case.

6. CONCLUSIONS

We have introduced a model of the slow fading in the problem of estimating the delays of several replicas of a know signal in a single-antenna receiver. The results show that the performance improves with the presence of the slow fading, if this effect is exploited by the estimator. The ability to separate closely delayed replicas is improved.

7. REFERENCES

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