OPTIMIZATION OF SYMBOL TIMING RECOVERY FOR MULTI-USER DS-CDMA RECEIVERS

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ABSTRACT

In this paper, we consider a direct-sequence code-division multiple-access system operating in downlink. We propose to improve the performances of a conventional delay tracking algorithm, i.e. the early-late algorithm, by means of prefiltering of the received signal. This concept has been studied by D'Andrea and Luise in [1] for the clock recovery scheme proposed by Gardner [2]. Our main contribution is to extend this concept in a multi-user CDMA context. In this extension, we have to treat multiple-access interference and to deal with the two "timing scales", i.e. symbol and chip duration, due to spread spectrum. The analysis shows improved tracking performances in comparison to the standard early-late algorithm.

1. INTRODUCTION

The code tracking performance of the Early-late synchronizer [3] [4] is affected by the presence of intersymbol interference (ISI) and multiple-access interference (MAI) in the received signal.

Guenach and Vandendorpe [5] proposed an interference cancellation receiver. They derived the likelihood function for delay estimation in a multi-user context and mentioned an early-late implementation with the interference mitigation term. Interference mitigation requires the knowledge of all user's symbols.

We resort to a different approach based on the standard early-late algorithm. A new early-late implementation is proposed in which the ISI and MAI terms are taking into account by using the concept of prefiltering, which needs only the knowledge of one user symbols (we choose user one by convention). The concept of prefiltering has been developed for the first time for the analog receivers and then adapted to digital receivers. It consists in inserting a filter in the loop and computing the optimal filter's coefficients which minimizes the timing variance (due to noise and ISI). We will generalize this approach to the early-late receiver in the context of multi-user DS-CDMA signal.

This paper is organized as follows : in section 2, a multi-user transmission model is proposed. The standard early-late algorithm is described in section 3 and its improved version with the prefilter is described in section 4. Numerical results are presented in section 5, and conclusions are given in section 6.

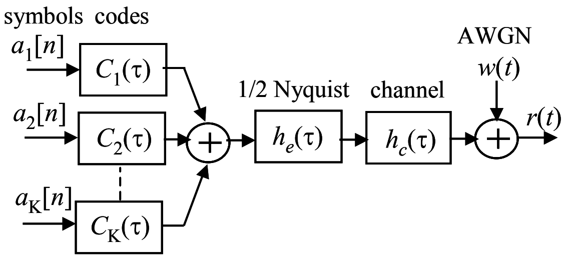

Fig. 1. Multi-user system.

2. MULTI-USER TRANSMISSION MODEL

The continuous-time baseband representation (complex envelope) of the received signal is modeled as (see Fig. 1.):

$$r(t) = Ts\sum_{k=1}^{K}\sum_{n} a_k[n] s_k(t - nTs - \tau_0) + w(t), \qquad (1)$$

where $a_k[n]$ are the BPSK symbols transmitted by the k-th source at the "symbol times" nTs. K is the number of users. The transmitted symbols are stationary, with zero-mean and power A^2, considered to be uncorrelated temporally, from one user to another, and also uncorrelated with the additive noise.
$s_k(\tau) = (c_k * h_e)(\tau)$ is the signature of the k-th user, which results from the convolution between the k-th spreading code and the half Nyquist h_e (square root raised cosine filter). The codes of different users are made from Q binary entities named chips. The impulse response of one code for the k-th user is defined by

$$c_k(\tau) = \sum_{q=0}^{Q-1} c_k[q]\delta(\tau - qTc), \qquad (2)$$

where $Tc = Ts/Q$ is the chip duration. $w(t)$ is a baseband additive white complex gaussian noise, with two-sided power spectral density $N_0/2$.

We consider the following context of downlink multi-user communication [6]:

- The number of users K is less than or equal to the spreading factor Q.
- The K active codes (taken among a set of Q known codes) are assumed to be known at the receiver.
- All K users share the same propagation channel $h_c(\tau)$.
- The baseband channel is supposed to be a single path (AWGN) with a delay factor τ_0 : $h_c(\tau) = \delta(\tau - \tau_0)$

3. THE EARLY-LATE RECEIVER

The early-late receiver is a closed-loop clock synchronizer. Its purpose is to estimate the channel delay τ_0 in order to provide an estimation of the optimum time sampling $mTs+\tau_0$. The estimate of τ_0 is updated at a symbol rate by an error signal $e[m]$ filtered by the loop filter $g[m]$. The recursive equation of the early-late loop is thus defined as :

$$\hat{\tau}_0[m+1] = \hat{\tau}_0[m] + (g * e)[m], \tag{3}$$

where $\hat{\tau}_0[m]$ is the estimate of τ_0 at instant mTs and $g[m]$ is the impulse response of the loop filter. For calculation simplicity and without loss of generality, we choose $g[m] = \mu\,\delta[m]$, with δ the Kronecker function, to obtain a first order loop recursive equation.

The loop error signal $e[m]$ is computed as follows :

$$e[m] = \mathrm{Re}\left\{ \hat{a}_1^*[m] \left[y\left(mTs + \hat{\tau}_0[m] + \frac{Tc}{2} \right) - y\left(mTs + \hat{\tau}_0[m] - \frac{Tc}{2} \right) \right] \right\}$$

where : (4)

- $y(t)$ is the output of the matched filter $s_1^H(\tau)$ when $r(t)$ is applied, i.e. : $y(t) = (r * s_1^H)(t)$. By using (1), it can be written as :

$$y(t) = Ts \sum_{k=1}^{K} \sum_{n} a_k[n] \Gamma_{k1}(t - nTs - \tau_0) + n(t), \tag{5}$$

  where $\Gamma_{k1}(\tau) = (s_k * s_1^H)(\tau)$ is the global cross-correlation function between user 1 and k, and $n(t)$ is the filtered version of $w(t)$. By convention, the exponent $(.)^H$ represents hermitian transform i.e. $f^H(\tau) = f^*(-\tau)$ for a given function f.

- $\hat{a}_1[m]$ is the estimation of $a_1[m]$. For Decision-Directed operating mode, estimated symbols are supposed available and possible estimation errors are also neglected.

Here we consider a fully digital implementation of the early-late receiver with non synchronous sampling since it is quite feasible given the today's technology. Effects of interpolation errors can be neglected by choosing an optimal parabolic interpolation [7].

The signal $r(t)$ is passed through an anti-aliasing filter (AAF) before being sampled at a rate of two samples per chip. This sampling rate allows to have no loss of information in the case of signals having an excess bandwidth less than 100%. The sampling clock is fixed and independent from the transmitter clock. The interpolator is controlled by the loop output to provide the samples at the desired interpolation instants. the early late samples $y\left(mTs + \hat{\tau}_0[m] \pm Tc/2\right)$ are then computed by applying the cross-correlation between the interpolated signal and the code of user #1 shifted by $\pm\,Tc/2$. They are used by the Timing Error Detector (TED) to generate the loop error signal $e[m]$ as shown in Fig. 2.

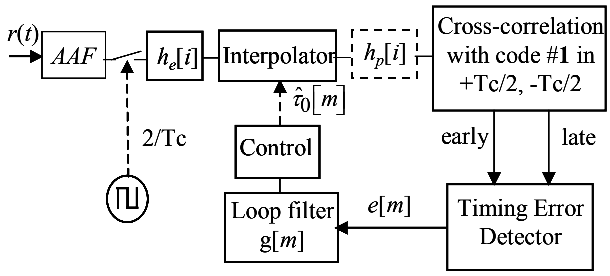

Fig. 2. Standard early-late receiver (without the prefilter $h_p[i]$) or improved early-late receiver (with $h_p[i]$).

The loop error signal can be decomposed to the sum of the conditional expectation $E\left\{e[m] \middle| \hat{\tau}_0[m]\right\}$ and a random term $N[m]$:

$$e[m] = E\left\{e[m] \middle| \hat{\tau}_0[m]\right\} + N[m]. \tag{6}$$

The first term, called the S-curve, is a function of the timing error:

$$E\left\{e[m] \middle| \hat{\tau}_0[m]\right\} = S\left(\tau_0 - \hat{\tau}_0[m]\right), \tag{7}$$

and the second term, called the loop noise, is defined as:

$$N[m] = e[m] - S\left(\tau_0 - \hat{\tau}_0[m]\right). \tag{8}$$

In the context of small fluctuations of the timing error, it is possible to linearize the S-curve arround its stable equilibrium point [8]:

$$S\left(\tau_0 - \hat{\tau}_0[m]\right) = D\left(\tau_0 - \hat{\tau}_0[m]\right), \tag{9}$$

where D is the slope of the S-curve at the stable equilibrium point.

The computation of the timing variance shows that [9] it depends on the autocorrelation of the loop noise $\Gamma_N[l]$:

$$\sigma^2 = \frac{2B_L Ts}{D^2} \sum_{l=-\infty}^{+\infty} \Gamma_N[l] (1 - \mu D)^{|l|}, \tag{10}$$

where $B_L Ts$ is the normalized equivalent noise bandwidth of the loop. $B_L Ts$ is given by [9]:

$$B_L Ts = \frac{\mu D}{2(2 - \mu D)}.$$

The computation of the loop noise autocorrelation will constitute a first step in the calculation of the timing variance.

4. IMPROVEMENT OF THE EARLY-LATE LOOP

We insert a prefilter of finite impulse response in the early-late loop between the interpolator and the cross-correlator (see Fig. 2.). It works at a rate of two samples per chip. For the same reasons as those developed in [1], we choose a symmetric prefilter noted $h_p[i]$, $i=-N_p,\ldots 0,\ldots,N_p$.

This section is organized as follows. The first step is the computation of the timing variance, which depends on the prefilter coefficients. Then the timing variance is minimized with respect to the prefilter coefficients. Next we propose a more compact form of the timing variance by introducing a matrix formulation. This will enable us to find an analytic solution to the minimization problem.

The following notations will be used in this section:

$$f^{\Delta}(t)=f\left(t+\frac{Tc}{2}\right)-f\left(t-\frac{Tc}{2}\right),$$

$$f^{(u)}(t)=f\left(t+u\frac{Tc}{2}\right)+f\left(t-u\frac{Tc}{2}\right),$$

$$f^{(u,p)}(t)=f^{(u)}\left(t+p\frac{Tc}{2}\right)+f^{(u)}\left(t-p\frac{Tc}{2}\right),$$

where f represents any desired function.

The new error signal can be expressed as:

$$e[m]=\mathrm{Re}\left\{\hat{a}_1^*[m]\left[y'\left(mTs+\hat{\tau}_0[m]+\frac{Tc}{2}\right)-y'\left(mTs+\hat{\tau}_0[m]-\frac{Tc}{2}\right)\right]\right\} \quad (11)$$

where

$$y'(t)=\sum_{p=-N_p}^{N_p}h_p[p]y\left(t-p\frac{Tc}{2}\right). \quad (12)$$

Inserting (5) in (12) yields:

$$y'(t)=Ts\sum_{k=1}^{K}\sum_{n}a_k[n]\Gamma'_{k1}(t-nTs-\tau_0)+n'(t), \quad (13)$$

where

$$\Gamma'_{k1}(t)=\sum_{p=-N_p}^{N_p}h_p[p]\Gamma_{k1}\left(t-p\frac{Tc}{2}\right), \quad (14)$$

$$n'(t)=\sum_{p=-N_p}^{N_p}h_p[p]n\left(t-p\frac{Tc}{2}\right). \quad (15)$$

Now, remembering that if $\hat{\tau}_0[m]=\tau_0$, then $e[m]=N[m]$; the loop noise autocorrelation takes the form:

$$\Gamma_N[l]=E\left\{e[m]\,e^*[m-l]\right\}. \quad (16)$$

Substituting (11) in the above expression yields:

$$\Gamma_N[l]=E\Big\{\mathrm{Re}\left\{\hat{a}_1^*[m]y'^{\Delta}(mTs+\tau_0)\right\} .\mathrm{Re}\left\{\hat{a}_1^*[m-l]y'^{\Delta}((m-l)Ts+\tau_0)\right\}\Big\} \quad (17)$$

We develop the product of the two real parts by using the equality of Re{z} Re{z'} = Re{$zz'+zz'^*$}/2, where z and z' are two complex numbers and z^* is the complex conjugate of z. By substituting (13) in (17) and after some calculations we obtain the expression of the loop noise autocorrelation :

$$\Gamma_N[0]=Ts^2A^4\sum_{n\neq 0}\left(\Gamma'^{\Delta}_{11}(nTs)\right)^2+Ts^2A^4\sum_{k\neq 1}\sum_{n=-\infty}^{+\infty}\left(\Gamma'^{\Delta}_{k1}(nTs)\right)^2 + A^2\left(\Gamma_{n'}[0]-\Gamma_{n'}[2]\right), \quad (18)$$

and $\Gamma_N[l]=Ts^2A^4\Gamma'^{\Delta}_{11}(-lTs)\Gamma'^{\Delta}_{11}(lTs)$ for $l\neq 0$,

where

$\Gamma_{n'}[i]=2N_0\sum_{u=-N_p}^{N_p}\sum_{p=-N_p}^{N_p}\Gamma_{11}\left((u-p+i)\frac{Tc}{2}\right)h_p[p]h_p^*[u]$ is the expression of the autocorrelation of $n'(t)$ sampled at a rate of two samples per chip.

The loop noise autocorellation in l=0 consists of three terms. The first term corresponds to the ISI, the second term is due to the MAI and the third one represents thermal noise. In a multi-user context, the terms where $l\neq 0$ can be neglected. Under these conditions, considering $N(m)$ as a white noise is a good approximation. The variance is thus evaluated as:

$$\sigma^2=\frac{2B_LTs}{D^2}\Gamma_N[0]. \quad (19)$$

Then (18) can be written in matrix notation as a quadratic form :

$$\Gamma_N[0]=\mathbf{h}^t\mathbf{\Gamma}\mathbf{h}, \quad (20)$$

where $\mathbf{h}=\left[h_p[0]\ldots h_p[N_p]\right]^t$ is the vector of prefilter coefficients and $\mathbf{\Gamma}=Ts^2A^4\mathbf{\Gamma_{ISI}}+Ts^2A^4\mathbf{\Gamma_{MAI}}+2A^2N_0\mathbf{\Gamma_{TN}}$ the matrix containing the three interference terms.

$\mathbf{\Gamma_{ISI}}$ and $\mathbf{\Gamma_{MAI}}$ are defined as :

$$\mathbf{\Gamma_{ISI}}=\sum_{n\neq 0}\mathbf{v}_{1,n}\mathbf{v}_{1,n}^t, \quad (21)$$

$$\mathbf{\Gamma_{MAI}}=\sum_{k\neq 1}\sum_{n}\mathbf{v}_{k,n}\mathbf{v}_{k,n}^t, \quad (22)$$

where $\mathbf{v}_{k,n}=\left[\Gamma^{\Delta}_{k1}(nTs)\ \ \Gamma^{\Delta,(1)}_{k1}(nTs)\ldots\Gamma^{\Delta,(N_p)}_{k1}(nTs)\right]^t$.

Now let us consider the following matrix :

$$\mathbf{B}_i=\begin{bmatrix}\Gamma_{11}\left(i\frac{Tc}{2}\right) & \Gamma_{11}^{(1)}\left(i\frac{Tc}{2}\right) & \cdots & \Gamma_{11}^{(N_p)}\left(i\frac{Tc}{2}\right)\\ \Gamma_{11}^{(1)}\left(i\frac{Tc}{2}\right) & \Gamma_{11}^{(1,1)}\left(i\frac{Tc}{2}\right) & \cdots & \Gamma_{11}^{(1,N_p)}\left(i\frac{Tc}{2}\right)\\ \vdots & \vdots & \ddots & \vdots\\ \Gamma_{11}^{(N_p)}\left(i\frac{Tc}{2}\right) & \Gamma_{11}^{(1,N_p)}\left(i\frac{Tc}{2}\right) & \cdots & \Gamma_{11}^{(N_p,N_p)}\left(i\frac{Tc}{2}\right)\end{bmatrix}, \quad (23)$$

$\mathbf{\Gamma_{TN}}$ is given by : $\mathbf{\Gamma_{TN}}=\mathbf{B}_0-\mathbf{B}_2$. (24)

In the following we will minimize the timing variance with respect to $\mathbf{h}$. We have to introduce a constraint to avoid the trivial solution where each coefficient equals zero. We choose the constraint to obtain the same slope of the S-curve at the stable equilibrium point as in the absence of the prefilter. The constraint can be written as $D(\mathbf{h})=D$. Taking the expectation of the error signal gives the expression of the S-curve. Then we compute the derivative of the S-curve and we obtain the constraint, expressed in matrix notation:

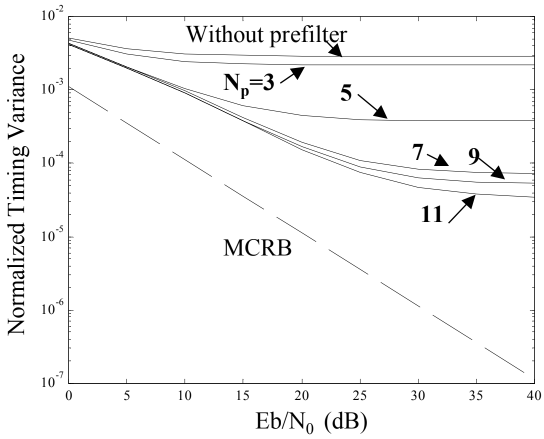

Fig. 3: Normalized timing variance for different values of N_p

$$\mathbf{x}^t\mathbf{h} = \dot{\Gamma}_{11}^{\Delta}(0), \qquad (25)$$

where $\mathbf{x}^t = \left[\dot{\Gamma}_{11}^{\Delta}(0) \quad \dot{\Gamma}_{11}^{\Delta,(1)}(0) \quad \cdots \quad \dot{\Gamma}_{11}^{\Delta,(N_p)}(0) \right]$.

The dot is used for the time derivative.

To minimize the timing variance with the constraint defined above, we introduce a Lagrange multiplier λ and we define the function F:

$$F(\mathbf{h},\lambda) = \sigma^2(\mathbf{h}) + \lambda\left(\mathbf{x}^t\mathbf{h} - \dot{\Gamma}_{11}^{\Delta}(0)\right). \qquad (26)$$

We minimize F with respect to both variables $\mathbf{h}$ and λ and we find the expression for the optimal coefficients :

$$\mathbf{h}_{\mathbf{opt}} = \frac{\dot{\Gamma}_{11}^{\Delta}(0)}{\mathbf{x}^t\mathbf{\Gamma}^{-1}\mathbf{x}}\mathbf{\Gamma}^{-1}\mathbf{x}. \qquad (27)$$

5. NUMERICAL RESULTS

In this section, we present a numerical analysis of the results obtained in the previous section. We use Hadamard codes of length Q=16. The chip shaping filter is a square-root raised cosine with the roll-off factor of 0.22. The loop bandwidth is such that B_LTs=5*10^{-3}. We consider a downlink communication with K=9 users. Fig. 3. shows the normalized timing variance as a function of Eb/N$_0$ for 3, 5, 7, 9 and 11 prefilter coefficients. We also calculate the timing variance of the standard early-late (without prefiltering). The Modified Cramer-Rao Bound (MCRB) [10] is used as a lower limit to the timing variance. It is seen that the standard early-late has a floor timing jitter due to the ISI and MAI terms. The prefilter contributes to mitigate the interference terms and as a result, the performance curve of the improved early-late approaches to the MCRB. When the number of coefficients increases, the timing variance becomes much closer to the MCRB.

There is another point to be mentioned about the number of coefficients. If we increase the number of the coefficients beyond N_p=7, we obtain a neligible improvement in the performance. Hence, a small number of coefficients would represent the best compromise between cost and performance. Regarding this compromise, the best choice for the prefilter length is to be less than 1/4 of the code length (the prefilter works with samples taken at time intervals of Tc/2).

6. CONCLUSION

In this paper, the symbol timing recovery is studied for multi-user DS-CDMA. A novel improved early-late structure with prefiltering is proposed, together with an analytical solution for calculating the prefilter coefficients. The proposed structure is tested by simulations, considering 9 users with Hadamard codes of length 16. In order to evaluate the resulting synchronization performances, we compared them with the performances of the classical early-late algorithm for different signal-to-noise ratios. The results show considerable improvements for a small number of coefficients, which represents a prefilter length inferior to 1/4 of the code length.

7. REFERENCES

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