BLIND CODE TIMING AND CARRIER OFFSET ESTIMATION FOR DS-CDMA SYSTEMS

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ABSTRACT

In this paper, we consider the problem of joint carrier offset and code timing estimation for CDMA (code division multiple access) systems. In contrast to most existing schemes which require multi-dimensional search over the parameter space, we propose a blind estimator that solves the joint estimation problem algebraically. By exploiting the noise subspace of the covariance matrix of the received data, the multiuser estimation is decoupled into parallel estimations of individual users, which makes computations efficient. The proposed estimator is non-iterative and near-far resistant. It can deal with frequency-selective and time-varying channels. The performance of the proposed scheme is illustrated by some computer simulations.

1. INTRODUCTION

Initial spreading code and carrier frequency synchronization that precedes symbol detection is a challenging problem in direct-sequence (DS) code-division multiple-access (CDMA) systems [1]. A conventional technique is to search serially through all potential code phases and frequencies for the desired user, meanwhile treating the multi-access interference (MAI) as noise [1, ch. 5]. This approach, although easy to implement, suffers the MAI, particularly in a near-far environment [2], [3]. A number of MAI-resistant synchronization schemes have been introduced recently (e.g., [2]–[4] and references therein). Most of these schemes, however, consider only code synchronization, assuming that carrier synchronization has been achieved at a prior stage. Joint treatment of carrier and code synchronizations acheives better performance than that of each estimator used separately, since the optimum maximum likelihood (ML) estimator is a joint processor which yields estimates for all parameters simultaneously. The ML estimator is mainly of theoretical interest due to its exponential complexity. For practical applications, suboptimal joint synchronizer/estimators with reasonable complexity are highly desired. However, very limited studies on joint synchronization are available. A notable exception is a joint carrier offset and code timing estimator proposed in [5], where frequency-nonselective and time-invariant channels were considered. This estimator involves a multi-dimensional (MD) search over the parameter space, which is computationally involved and requires accurate initial parameter estimates that are often difficult to obtain.

In this paper, we consider the problem of joint carrier offset and code timing estimation for CDMA (code division multiple access) systems. We propose an algebraic, blind estimator which jointly estimates the carrier offset and code timing. The proposed estimator is computationally attractive since it decouples multiuser estimation into parallel estimations of individual users; furthermore, unlike most existing schemes which require multi-dimensional search over the parameter space, the proposed estimator is non-iterative. It is a near-far resistant and can deal with frequency-selective and time-varying channels.

Notation: Vectors (matrices) are denoted by boldface lower (upper) case letters; all vectors are column vectors; $\mathrm{diag}\{\mathbf{g}\}$ is a diagonal matrix with the elements of the vector $\mathbf{g}$ placed on the diagonal; $\mathbf{I}_M$ is the $M \times M$ identity matrix; $\mathbf{0}$ is a vector or matrix with all zero elements; $\|\cdot\|$ denotes matrix/vector Frobenius norm; superscripts $(\cdot)^T$, $(\cdot)^H$ and $(\cdot)^\dagger$ denote transpose, conjugate transpose, and pseudo-inverse respectively; and finally $E\{\cdot\}$ denotes statistical expectation.

2. DATA MODEL

Consider a baseband asynchronous K-user DS-CDMA system. The transmitted signal for user k is given by $s_k(t) = \sum_{m=0}^{M-1} d_k(m)$ $\psi_k(t - mT_s)$, where M is the number of symbols considered for synchronization, $d_k(m)$ and $\psi_k(t)$ denote the mth data symbol and spreading waveform, respectively, for user k, and $T_s = NT_c$ denotes the symbol interval, with T_c and N being the chip interval and spreading gain, respectively. Signal $s_k(t)$ passes through a baseband time-varying frequency-selective channel. The received signal is given by

$$y(t) = \sum_{k=1}^{K} \sum_{l=1}^{L_k} \alpha_{k,l}(t) s_k(t - \tau_{k,l}) \mathrm{e}^{j\Omega_k (t-\tau_{k,l})} + n(t), \quad (1)$$

where L_k denotes the number of paths of user k, Ω_k denotes the carrier frequency offset, $\alpha_{k,l}(t)$ and $\tau_{k,l}$ denote the time-varying fading coefficient and code timing, respectively, associated with path l of user k, and $n(t)$ denotes the noise. We assume that the delays of a particular user are distinct and remain (approximately) unchanged during acquisition. We also assume that the delay spread is within one symbol interval, i.e., $\tau_{k,l} < T_s$. This could be the case when the cell size is small relative to the transmission rate, or due to a prior coarse synchronization which pulls the timing uncertainty to within a symbol interval [3]. The receiver front-end is a chip-matched filter (CMF) which outputs samples $y(l) = y(t)\big|_{t=lT_i}$, where $T_i = T_c/Q$ is the sampling interval, with $Q \geq 1$ denoting the *oversampling factor* (an integer). It is convenient to write $\tau_{k,l}$ as $\tau_{k,l} = (p_{k,l} + \mu_{k,l})T_i$, where $p_{k,l}$ denotes an integer between 0 and $NQ - 1$ and $\mu_{k,l} \in [0, 1)$ denotes the fractional delay.

Let $\mathbf{y}(m) \triangleq [y(mNQ), \cdots, y(mNQ + NQ - 1)]^T$, $\mathbf{c}_k \triangleq [c_k(0), \cdots, c_k(NQ-1)]^T$, where $c_k(n) = \frac{1}{T_i} \int_{(n-1)T_i}^{nT_i} \psi_k(t)\mathrm{d}t$. Due to asynchronism, two adjacent symbols in each path con-

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tribute to $\mathbf{y}(m)$ [2]– [5]

$$\mathbf{y}(m) = \sum_{k=1}^{K} \mathbf{\Sigma}_k(\omega_k)\mathbf{A}_k(\boldsymbol{\tau}_k)\boldsymbol{\beta}_k(m) + \mathbf{n}(m), \tag{2}$$

$m = 0, 1, \cdots, M-1$, where $\omega_k \triangleq \Omega_k T_i$ denotes the *normalized carrier offset*, $\boldsymbol{\tau}_k \triangleq [\tau_{k,1}, \cdots, \tau_{k,L_k}]^T$,

$$\begin{aligned}
\mathbf{\Sigma}_k(\omega_k) &\triangleq \mathrm{diag}\left\{1, \mathrm{e}^{j\omega_k}, \cdots, \mathrm{e}^{j\omega_k(NQ-1)}\right\}, \\
\mathbf{A}_k(\boldsymbol{\tau}_k) &\triangleq [\mathbf{a}_k(\tau_{k,1}), \bar{\mathbf{a}}_k(\tau_{k,1}), \cdots, \mathbf{a}_k(\tau_{k,L_k}), \bar{\mathbf{a}}_k(\tau_{k,L_k})], \\
\boldsymbol{\beta}_k(m) &\triangleq [\beta_{k,1}(m), \bar{\beta}_{k,1}(m), \cdots, \beta_{k,L_k}(m), \bar{\beta}_{k,L_k}(m)]^T, \\
\beta_{k,l}(m) &\triangleq \alpha_{k,l}(m) d_k(m-1)\mathrm{e}^{j\omega_k m NQ}, \\
\bar{\beta}_{k,l}(m) &\triangleq \alpha_{k,l}(m) d_k(m)\mathrm{e}^{j\omega_k m NQ},
\end{aligned}$$

and $\mathbf{n}(m)$ denotes the $NQ \times 1$ noise vector. Furthermore, we have [4]:

$$\mathbf{a}_k(\tau_{k,l}) = \mathbf{F}_k(p_{k,l})\boldsymbol{\mu}_{k,l}, \quad \bar{\mathbf{a}}_k(\tau_{k,l}) = \bar{\mathbf{F}}_k(p_{k,l})\boldsymbol{\mu}_{k,l}, \tag{3}$$

where $\boldsymbol{\mu}_{k,l} \triangleq [1-\mu_{k,l}, \mu_{k,l}]^T$, and $\mathbf{F}_k(p_{k,l})$ [respectively, $\bar{\mathbf{F}}_k(p_{k,l})$] are $NQ \times 2$ matrices with the first and second column consisting of the *acyclic left shift* (resp., *acyclic right shift*) of $\mathbf{c}_k$ by $p_{k,l}$ and $p_{k,l}+1$ samples, respectively; see [4, Eqns. (38) and (39)] for exact expressions of $\mathbf{F}_k(p_{k,l})$ and $\bar{\mathbf{F}}_k(p_{k,l})$.

The problem of interest is to estimate the code timing $\{\boldsymbol{\tau}_k\}_{k=1}^K$, and the carrier offset $\{\omega_k\}_{k=1}^K$ from the received data $\{\mathbf{y}(m)\}_{m=0}^{M-1}$, without any knowledge of the transmitted information symbols. The optimum approach to solving this problem can be formulated by means of the maximum likelihood (ML) estimation (e.g., [6]). However, such optimum schemes require a multi-dimensional search that has a complexity growing exponentially with the size of the estimation problem. In what follows, we present a scheme that is computationally efficient with close-to-optimum performance.

3. JOINT CARRIER AND CODE ESTIMATION

Let $\mathbf{R}_y \triangleq E\{\mathbf{y}(m)\mathbf{y}(m)^H\}$ denote the data covariance matrix, and $L \triangleq \sum_{k=1}^K L_k$ the total number of paths of all users. Assuming $NQ > 2L$, the eigendecomposition of $\mathbf{R}_y$ can be expressed as: $\mathbf{R}_y = \mathbf{E}_s\mathbf{\Lambda}_s\mathbf{E}_s^H + \sigma_n^2\mathbf{E}_n\mathbf{E}_n^H$, where $\mathbf{\Lambda}_s$ is a diagonal matrix made from the $2L$ largest eigenvalues associated with the eigenvectors that form $\mathbf{E}_s \in \mathbb{C}^{NQ\times 2L}$, and $\mathbf{E}_n \in \mathbb{C}^{NQ\times(NQ-2L)}$ contains the eigenvectors corresponding to the smallest eigenvalue σ_n^2 with multiplicity $NQ-2L$, with σ_n^2 denoting the variance of the channel noise. Since $\mathbf{E}_n$ spans the orthogonal complement of the signal subspace, we have

$$\mathbf{E}_n^H\mathbf{\Sigma}(\omega_k)\mathbf{A}_k(\boldsymbol{\tau}_k) = \mathbf{0}, \quad k = 1, \cdots, K. \tag{4}$$

Estimates ω_k and $\boldsymbol{\tau}_k$ can be obtained by solving the above nonlinear equation through a search over an (L_k+1) - dimensional parameter space, which is computationally involved and suffers local convergence. Note that the scheme in [5] may be considered a special case of the above approach when the channel is frequency-flat and time-invariant.

To seek an alternative solution, we invoke the theory of polynomial matrices (e.g., [7]). For the lth path of user k, (4) is equivalent to (hereafter, we drop the subscripts k and l for notational simplicity):

$$\mathbf{E}_n^H\mathbf{\Sigma}(\omega)\mathbf{a}(\tau) = \mathbf{0}, \quad \mathbf{E}_n^H\mathbf{\Sigma}(\omega)\bar{\mathbf{a}}(\tau) = \mathbf{0}.$$

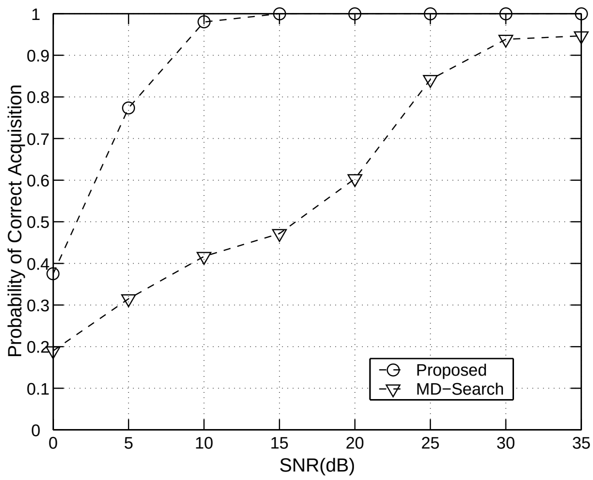


Fig. 1. Probability of correct acquisition versus SNR in time-varying two-path fading channel when $Q = 2$, $K = 5$, $M = 200$, $N = 15$ and NFR = 10 dB.

Using the notation $z \triangleq \mathrm{e}^{j\omega}$ and, hence, $\mathbf{\Sigma}(z) = \mathrm{diag}\{1, z, \cdots, z^{NQ-1}\}$, we have [cf. (3)]

$$\mathbf{\Psi}(z)\boldsymbol{\mu} = \mathbf{0}, \quad \bar{\mathbf{\Psi}}(z)\boldsymbol{\mu} = \mathbf{0}, \tag{5}$$

where $\mathbf{\Psi}(z) \triangleq \mathbf{E}_n^H\mathbf{\Sigma}(z)\mathbf{F}(p)$ $\bar{\mathbf{\Psi}}(z) \triangleq \mathbf{E}_n^H\mathbf{\Sigma}(z)\bar{\mathbf{F}}(p)$ are both $(NQ-2L)\times 2$ polynomial matrices in z of order $NQ-1$. Eqn. (5) indicates that $\boldsymbol{\mu}$, which is always non-trivial by definition [cf. (3)], is a null vector of both $\mathbf{\Psi}(z)$ and $\bar{\mathbf{\Psi}}(z)$. Therefore, the nullity of $\mathbf{\Psi}(z)$ and $\bar{\mathbf{\Psi}}(z)$ is at least one. In effect, since $\mathbf{\Psi}(z)$ and $\bar{\mathbf{\Psi}}(z)$ have two columns, we can see that the nullity is exactly one unless the range spaces of these matrices are trivial, which is possible only for all-zero spreading sequences. Let

$$\mathbf{E}_n^H \triangleq [\mathbf{e}_{n,1}, \quad \mathbf{e}_{n,2}, \quad \cdots, \quad \mathbf{e}_{n,NQ}],$$

$$\mathbf{F}(p) = \begin{bmatrix} \mathbf{f}_1^H \\ \mathbf{f}_2^H \\ \vdots \\ \mathbf{f}_{NQ}^H \end{bmatrix}, \quad \bar{\mathbf{F}}(p) = \begin{bmatrix} \bar{\mathbf{f}}_1^H \\ \bar{\mathbf{f}}_2^H \\ \vdots \\ \bar{\mathbf{f}}_{NQ}^H \end{bmatrix},$$

where $\mathbf{e}_{n,i}$ is the ith column of $\mathbf{E}_n^H$, and $\mathbf{f}_i^H(p)$, $\bar{\mathbf{f}}_i^H(p)$ are the ith rows of $\mathbf{F}(p)$ and $\bar{\mathbf{F}}(p)$ respectively. The polynomial matrices $\mathbf{\Psi}(z)$ and $\bar{\mathbf{\Psi}}(z)$ can then be expressed as:

$$\mathbf{\Psi}(z) = \sum_{i=1}^{NQ} \mathbf{e}_{n,i}\mathbf{f}_i^H(p)z^{i-1}, \quad \bar{\mathbf{\Psi}}(z) = \sum_{i=1}^{NQ} \mathbf{e}_{n,i}\bar{\mathbf{f}}_i^H(p)z^{i-1}.$$

Let

$$\mathbf{\Psi}(z) \triangleq [\boldsymbol{\psi}_1(z), \ \boldsymbol{\psi}_2(z)], \quad \bar{\mathbf{\Psi}}(z) \triangleq [\bar{\boldsymbol{\psi}}_1(z), \ \bar{\boldsymbol{\psi}}_2(z)],$$

where $\boldsymbol{\psi}_1(z)$, $\boldsymbol{\psi}_2(z)$ correspond to the first and second columns of $\mathbf{\Psi}(z)$, and $\bar{\boldsymbol{\psi}}_1(z)$, $\bar{\boldsymbol{\psi}}_2(z)$ correspond to the first and second columns of $\bar{\mathbf{\Psi}}(z)$. The previous analysis also indicates that $\boldsymbol{\psi}_1(z)$ and $\boldsymbol{\psi}_2(z)$ [resp., $\bar{\boldsymbol{\psi}}_1(z)$ and $\bar{\boldsymbol{\psi}}_2(z)$] are linearly dependent. Hence, we can construct projection matrices $\mathbf{P}_{\boldsymbol{\psi}_2}^{\perp}(z)$ and $\bar{\mathbf{P}}_{\bar{\boldsymbol{\psi}}_2}^{\perp}(z)$ that project to the orthogonal complement of vectors $\boldsymbol{\psi}_2$ and $\bar{\boldsymbol{\psi}}_2$, respectively. That is,

$$\mathbf{P}_{\boldsymbol{\psi}_2}^{\perp}(z)\boldsymbol{\psi}_1(z) = \mathbf{0}, \quad \bar{\mathbf{P}}_{\bar{\boldsymbol{\psi}}_2}^{\perp}(z)\bar{\boldsymbol{\psi}}_1(z) = \mathbf{0}. \tag{6}$$

To construct $\mathbf{P}^{\perp}_{\boldsymbol{\psi}_2}(z)$ and $\bar{\mathbf{P}}^{\perp}_{\bar{\boldsymbol{\psi}}_2}(z)$, we note that by the Bezout identity [7, p. 379] (also see [8] for another application), there exist $1 \times (NQ - 2L)$ polynomial vectors $\mathbf{g}^H(z)$ and $\bar{\mathbf{g}}^H(z)$ such that

$$\mathbf{g}^H(z)\boldsymbol{\psi}_2(z) = z^{-n_o}, \quad \bar{\mathbf{g}}^H(z)\bar{\boldsymbol{\psi}}_2(z) = z^{-\bar{n}_o}, \tag{7}$$

for some appropriate delays n_o and $\bar{n}_o$. To construct the polynomial vectors $\mathbf{g}^H(z)$ and $\bar{\mathbf{g}}^H(z)$, rewrite $\boldsymbol{\psi}_2(z)$, $\bar{\boldsymbol{\psi}}_2(z)$, $\mathbf{g}^H(z)$, and $\bar{\mathbf{g}}^H(z)$ as:

$$\boldsymbol{\psi}_2(z) = \sum_{i=0}^{NQ-1} \boldsymbol{\psi}_{2,i} z^i, \quad \bar{\boldsymbol{\psi}}_2(z) = \sum_{i=0}^{NQ-1} \bar{\boldsymbol{\psi}}_{2,i} z^i,$$
$$\mathbf{g}^H(z) = \sum_{i=0}^{B-1} \mathbf{g}_i^H z^i, \quad \bar{\mathbf{g}}^H(z) = \sum_{i=0}^{B-1} \bar{\mathbf{g}}_i^H z^i.$$

Let

$$\mathbf{g}_{\mathbf{g}}^H = \left[\mathbf{g}_{B-1}^H, \quad \mathbf{g}_{B-2}^H, \quad \cdots, \quad \mathbf{g}_0^H\right]_{1\times B(NQ-2L)},$$
$$\bar{\mathbf{g}}_{\bar{\mathbf{g}}}^H = \left[\bar{\mathbf{g}}_{B-1}^H, \quad \bar{\mathbf{g}}_{B-2}^H, \quad \cdots, \quad \bar{\mathbf{g}}_0^H\right]_{1\times B(NQ-2L)},$$

where $\mathbf{g}_{\mathbf{g}}^H$, $\bar{\mathbf{g}}_{\bar{\mathbf{g}}}^H$ are block vectors of the polynomial vectors $\mathbf{g}^H(z)$ and $\bar{\mathbf{g}}^H(z)$ respectively, and $B > (NQ-1)/(NQ-2L-1)$ is so chosen to guarantee that the block matrices $\boldsymbol{\Upsilon}$ and $\bar{\boldsymbol{\Upsilon}}$ (shown below) are both tall matrices. Then the time domain convolution of (7) is given by

$$\mathbf{g}_{\mathbf{g}}^H \underbrace{\begin{bmatrix} \boldsymbol{\psi}_{2,0} & \boldsymbol{\psi}_{2,1} & \cdots & \boldsymbol{\psi}_{2,NQ-1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\psi}_{2,0} & \boldsymbol{\psi}_{2,1} & \cdots & \boldsymbol{\psi}_{2,NQ-1} & \cdots & \mathbf{0} \\ \vdots & & \ddots & & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \boldsymbol{\psi}_{2,0} & \cdots & \cdots & \boldsymbol{\psi}_{2,NQ-1} \end{bmatrix}}_{\triangleq \boldsymbol{\Upsilon}_{B(NQ-2L)\times(NQ+B-1)}}$$
$$= \underbrace{[0, \quad \cdots, \quad 1, \quad \cdots, \quad 0]}_{\triangleq \boldsymbol{\zeta}_{1\times(NQ+B-1)}},$$
$$\bar{\mathbf{g}}_{\bar{\mathbf{g}}}^H \underbrace{\begin{bmatrix} \bar{\boldsymbol{\psi}}_{2,0} & \bar{\boldsymbol{\psi}}_{2,1} & \cdots & \bar{\boldsymbol{\psi}}_{2,NQ-1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \bar{\boldsymbol{\psi}}_{2,0} & \bar{\boldsymbol{\psi}}_{2,1} & \cdots & \bar{\boldsymbol{\psi}}_{2,NQ-1} & \cdots & \mathbf{0} \\ \vdots & & \ddots & & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \bar{\boldsymbol{\psi}}_{2,0} & \cdots & \cdots & \bar{\boldsymbol{\psi}}_{2,NQ-1} \end{bmatrix}}_{\triangleq \bar{\boldsymbol{\Upsilon}}_{B(NQ-2L)\times(NQ+B-1)}}$$
$$= \underbrace{[0, \quad \cdots, \quad 1, \quad \cdots, \quad 0]}_{\triangleq \bar{\boldsymbol{\zeta}}_{1\times(NQ+B-1)}}. \tag{8}$$

Notice that $\boldsymbol{\zeta}$ and $\bar{\boldsymbol{\zeta}}$ each contains a unit element at the n_0th and the $\bar{n}_0$th location, respectively. The delays n_0 and $\bar{n}_0$ can be found as the numbers at which the only non-zero elements of the n_0th, $\bar{n}_0$th columns and n_0th, $\bar{n}_0$th rows of $\boldsymbol{\Upsilon}^\dagger\boldsymbol{\Upsilon}$ and $\bar{\boldsymbol{\Upsilon}}^\dagger\bar{\boldsymbol{\Upsilon}}$ respectively are equal to one. With the knowledge of n_0 and $\bar{n}_0$, $\mathbf{g}_{\mathbf{g}}^H$ and $\bar{\mathbf{g}}_{\bar{\mathbf{g}}}^H$ can be calculated as

$$\mathbf{g}_{\mathbf{g}}^H = \boldsymbol{\zeta}\boldsymbol{\Upsilon}^\dagger, \quad \bar{\mathbf{g}}_{\bar{\mathbf{g}}}^H = \bar{\boldsymbol{\zeta}}\bar{\boldsymbol{\Upsilon}}^\dagger,$$

from which we construct $\mathbf{g}^H(z)$ and $\bar{\mathbf{g}}^H(z)$. It follows that

$$\begin{aligned} \mathbf{P}^{\perp}_{\boldsymbol{\psi}_2}(z) &= z^{-n_0}\mathbf{I}_{NQ-2L} - \boldsymbol{\psi}_2(z)\mathbf{g}^H(z), \\ \bar{\mathbf{P}}^{\perp}_{\bar{\boldsymbol{\psi}}_2}(z) &= z^{-\bar{n}_0}\mathbf{I}_{NQ-2L} - \bar{\boldsymbol{\psi}}_2(z)\bar{\mathbf{g}}^H(z). \end{aligned} \tag{9}$$

Using (6), along with the projection matrices constructed in (9), we can obtain an estimate of the carrier offset as follows:

$$\hat{\omega} = \arg\min_{\omega} \left\{ \|\mathbf{P}^{\perp}_{\boldsymbol{\psi}_2}(z)\boldsymbol{\psi}_1(z)\|^2 + \|\bar{\mathbf{P}}^{\perp}_{\bar{\boldsymbol{\psi}}_2}(z)\bar{\boldsymbol{\psi}}_1(z)\|^2 \right\}, \tag{10}$$

which need be minimized for all possible values of p. This can be done by using polynomial rooting, similar to the root-MUSIC algorithm [9]. Once an estimate of ω is known, $\boldsymbol{\Psi}(z)$ and $\bar{\boldsymbol{\Psi}}(z)$ are parameterized by the integer delay p. Hence, we may write them as $\boldsymbol{\Psi}(p)$ and $\bar{\boldsymbol{\Psi}}(p)$. It follows from (5) that we can use the following criterion to estimate the integer and fractional delay:

$$\{\hat{p}, \hat{\mu}\} = \arg\min_{p,\mu} \left\{ \|\boldsymbol{\Psi}(p)\boldsymbol{\mu}\|^2 + \|\bar{\boldsymbol{\Psi}}(p)\boldsymbol{\mu}\|^2 \right\}. \tag{11}$$

In the multipath case, there are L_k solutions corresponding to the L_k paths of user k, all achieving identically the same minimum of the cost function, which is zero if $\mathbf{E}_n$ is known exactly (see discussions next). We remark that the above criterion is equivalent to the one employed in [2, Eqn. (23)] for code acquisition assuming no carrier offset. As shown there, it can be efficiently minimized by a sequence of polynomial rooting.

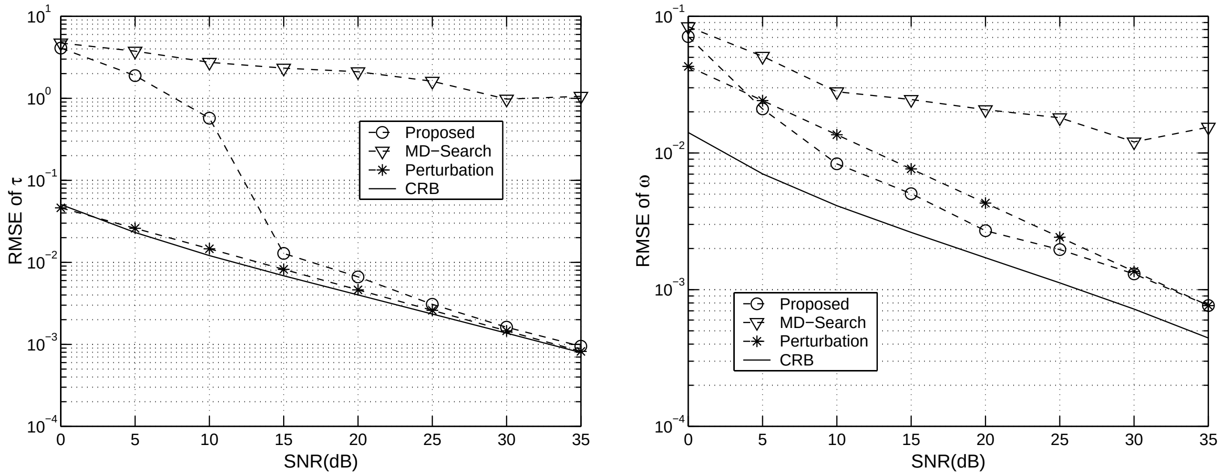
Remark 1: The ideal noise eigenvectors $\mathbf{E}_n$ have to be estimated from the observed data. We can use the sample eigenvector estimates obtained from the eigendecomposition of the sample covariance matrix $\hat{\mathbf{R}}_y \triangleq \frac{1}{M}\sum_{m=0}^{M-1}\mathbf{y}(m)\mathbf{y}^H(m)$. Alternatively, they may be computed adaptively via subspace tracking algorithms (e.g., [10]). Due to finite-sample errors in the noise eigenvector estimates, (6) holds only approximately in this case. Hence, we seek code timing estimates as those achieving the smallest minima of (11).

Remark 2: For the delays and carrier offsets to be uniquely identifiable, it is necessary for the matrices $\mathbf{P}^{\perp}_{\boldsymbol{\psi}_2}(z)$ and $\bar{\mathbf{P}}^{\perp}_{\bar{\boldsymbol{\psi}}_2}(z)$ in (6) as well as $\boldsymbol{\Psi}(z)$ and $\bar{\boldsymbol{\Psi}}(z)$ in (5) to have nullity exactly equal to one. The exact identification conditions and their implications are being investigated and will be reported elsewhere.

Remark 3: A necessary condition for this method to work is to have $NQ > 2L$ so that $\mathbf{E}_n$ is non-trivial. For large L (e.g., in overloaded systems and/or with large delay spread), the oversampling factor Q would have to be increased, which, in turn, would require excess bandwidth to avoid ill-conditioning problems. An interesting future subject is to compare both training-based and blind estimators with comparable bandwidth expansion, in order to determine which of these two different types of methods utilize bandwidth more efficiently.

4. NUMERICAL RESULTS

We consider an asynchronous DS-CDMA system that uses $N = 15$ large Kasami codes and BPSK (binary phase shift keying) constellation. We consider a near-fare environment whereby the transmitted power P_1 for the desired user is scaled so that $P_1 = 1$, whereas the power for the $K-1$ interfering users in *all* simulations follows a log normal distribution: $P_k/P_1 = 10^{0.1P}, P \sim N(10, 100)$, for $k = 2, \cdots, K$. Note that all the interfering users transmit at a mean power level 10 dB higher than that of the desired user, i.e., the *near-far ratio* (NFR) is 10 dB for all examples. The simulated channel is frequency-selective, and time-varying, generated by the Jakes' model [11] with a normalized Doppler rate $f_D T_s$ of 0.0067 (i.e., carrier frequency = 900 MHz, symbol rate = 10 kHz, and mobile speed = 50 mi/hr) with f_D denoting the maximum doppler rate. The *normalized* carrier offset is set to $\omega_k = 0.1$ for the desired user. The additive noise $n(t)$ is white Gaussian with zero-mean and power spectral density of $N_0/2$. We compare the proposed estimator and the multi-dimensional search (MD-Search) based scheme (4). The latter is initialized by estimates obtained by the method in [2], assuming zero initial carrier offset,



(a) (b)

Fig. 2. Performance versus SNR in time-varying two-path fading channel when $Q = 2, K = 5, M = 200, N = 15$ and NFR $= 10$ dB. (a) RMSE of τ. (b) RMSE of ω.

and then iterates using the Matlab nonlinear optimization routine `fminsearch` till convergence. We show the performance of the proposed and the MD-Search methods as the SNR varies from 0 to 35 dB, in time-varying, two path Rayleigh fading channel. The number of users $K = 5, M = 200, Q = 2$, and NFR $= 10$ dB. The information symbols are generated randomly, and then fixed for all trials. One performance measure is the *probability of correct acquisition* (PCA), defined as the probability of the event that the code-timing estimation error is less than $T_c/2$. Another performance measure, at a finer scale, is the root mean squared error (RMSE) of the code timing and carrier offset estimates. Figure 1 shows the PCA for the two method. The MD-Search scheme is seen to suffer local convergence caused by poor initialization, yielding a lower PCA, whereas the proposed method shows a high performance with a small SNR threshold. Figures 2(a) and 2(b) depict the RMSE of the code timing and carrier offset estimates, along with the perturbation analysis and the Cramér-Rao bound (CRB) derived in [12]. The MD-Search yields higher RMSE than the proposed scheme, due to the initialization problem. Figure 2(b) also indicates that the (carrier offset) RMSE for the MD-Search saturates at high SNR, meanwhile the proposed scheme does not have this problem. As the SNR increases, it is seen that the simulated RMSE, of both carrier offset and code timing, and perturbation results are very close to each other and approach the CRB.

5. CONCLUSION

In this paper, we have considered the problem of joint carrier offset and code timing estimation for DS-CDMA systems. The proposed algorithm decouples the multiuser estimation into single user estimations, which makes it computationally attractive; furthermore, the carrier offset and code timing are estimated algebraically to avoid the existing non-linear optimization techniques. The proposed scheme is non-iterative and near-far resistant. It can deal with frequency selective and time-varying channels.

6. REFERENCES

[1] R. L. Peterson, R. E. Ziemer, and D. E. Borth, *Introduction to Spread Spectrum Communications*, Prentice Hall, Englewood Cliffs, NJ, 1995.

[2] E. G. Ström, S. Parkvall, S. L. Miller, and B. E. Ottersten, "Propagation delay estimation in asynchronous direct-sequence code-division multiple access systems," *IEEE Transactions on Communications*, vol. 44, no. 1, pp. 84–93, January 1996.

[3] R. F. Smith and S. L. Miller, "Acquisition performance of an adaptive receiver for DS-CDMA," *IEEE Transactions on Communications*, vol. 47, no. 9, pp. 1416–1424, September 1999.

[4] H. Li, J. Li, and S. L. Miller, "Decoupled multiuser code-timing estimation for code-division multiple-access communication systems," *IEEE Transactions on Communications*, vol. 49, no. 8, pp. 1425–1436, August 2001.

[5] M. Erić and M. Obradović, "Subspace-based joint time-delay and frequency-shift estimation in asynchronous systems," *Electronics Letters*, vol. 33, no. 14, pp. 1193–1195, July 1997.

[6] P. Stoica and A. Nehorai, "Performance study of conditional and unconditional direction of arrival estimation," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 38, no. 10, pp. 1783–1795, October 1990.

[7] T. Kailath, *Linear Systesm*, Prentice Hall, Englewood Cliffs, NJ, 1980.

[8] K. Li and H. Liu, "Joint channel and carrier offset estimation in CDMA communications," *IEEE Transactions on Signal Processing*, vol. 47, no. 7, pp. 1811–1822, July 1999.

[9] A. J. Barabell, "Improving the resolution performance of eigenstructure-based direction-finding algorithms," in *Proceedings of the International Conference on Acoustics, Speech, and Signal Processing*, Boston, MA, April 1983, pp. 336–339.

[10] X. Li and H. Fan, "Blind channel identification: Subspace tracking method without rank estimation," *IEEE Transactions on Signal Processing*, vol. 49, no. 10, pp. 2372–2382, October 2001.

[11] W. C. Jakes, Jr., *Microwave Mobile Communications*, Wiley-Interscience, 1974.

[12] K. Amleh and H. Li, "Joint carrier offset and code timing estimation for DS-CDMA systems," submitted to *IEEE Transactions on Signal Processing*, 2002.