BER PERFORMANCE OF SOFTWARE RADIO MULTIRATE RECEIVERS WITH NONSYNCHRONIZED IF-SAMPLING AND DIGITAL TIMING CORRECTION

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ABSTRACT

This contribution deals with the design of an all digital multirate receiver with nonsynchronized IF sampling and digital timing correction, that can be used in software defined radios. By performing timing correction prior to matched filtering, the complexity of the matched filter is reduced, but at the expense of additional aliasing. Design parameters, yielding low degradations with respect to synchronized baseband sampling, are provided.

1. INTRODUCTION

1.1. Software Radio

A Software Defined Radio (SDR) can be seen a transceiver whose functions are realized as software running on a suitable hardware platform. Radio functionalities are as much as possible replaced by digital signal processing (DSP).

Some of the critical tasks of a software radio have been identified as: analog to digital conversion (ADC), digital mixing, sample rate conversion and filtering. In this contribution we mainly focus on the last three aspects. ADC is assumed to be performed with infinite digitization bandwidth and precision.

1.2. Digital Mixing and Sample Rate Conversion

In a traditional super-heterodyne receiver four stages can be discerned: the RF, IF, BB and DSP stages. In the radio frequency (RF) stage the received signals at the output of the antenna are filtered, amplified and downconverted to the intermediate frequency (IF). In the IF stage the signal is bandlimited. Then the IF signal is downconverted to baseband (BB), filtered (by a matched filter, to maximize the signal to noise ratio (SNR)), synchronously sampled and converted to the digital domain by the ADC. Finally, general DSP is performed in order to detect the digital data conveyed by the transmitted signal. None of the analog components are reconfigurable: for each symbol rate different filters need to be used. For the remainder of this paper we shall refer to this receiver as the *reference structure*.

In this contribution, we sample the IF signal reducing the number of analog components as compared to baseband sampling. Sampling is performed at a fixed rate, $1/T_s$. This implies different clock rates (corresponding to variable data rates) need to be supplied virtually, by means of digital sample rate conversion (SRC). This is closely related to the well-known problem of digital timing correction, which is the focus of this contribution. As will be shown, aliasing will be a main concern. SRC can be applied in combination with matched filtering [1]. A significant reduction in receiver complexity can be accomplished by performing timing correction prior to matched filtering: this configuration, given in Fig. 1, makes the matched filter independent of the symbol rate. This structure has been investigated in [2] assuming nonsynchronized baseband sampling.

2. SYSTEM DESCRIPTION

2.1. Receiver Operation

We consider multirate burst transmission, where the symbol rate during a burst is constant, but can change from one burst to the next. We denote the signal at the input of the ADC by $r(t) = s(t) + n(t)$, where $n(t)$ is real additive white gaussian noise (AWGN) with power spectral density equal to $N_0/2$ and $s(t)$ is the IF signal:

$$s(t) = \sqrt{2E_s}\Re\left[\sum_k a_k p(t - kT - \tau) e^{j2\pi f_{IF}t} e^{j\theta}\right] \quad (1)$$

where f_{IF} is the IF, $\{a_k\}$ are the uncorrelated data symbols with $E\left[|a_k|^2\right] = 1$, E_s is the energy per symbol, $p(t)$ is a square root cosine rolloff unit energy transmit pulse with rolloff α_p and a one-sided bandwidth $B = (1+\alpha_p)/(2T)$, θ is the carrier phase and τ is the propagation delay of the complex envelope. Note that τ is constant for a given burst, but can vary from burst to burst. Two cases are distinguished: the symbol interval, T, can take on any values from an interval (T_{min}, T_{max}). Alternatively, the symbol interval takes on values from a discrete set: $\{T_{min}, \dots, T_{max}\}$. Consequently, the bandwidth B of the transmit pulse is in a corresponding interval (B_{min}, B_{max}).

As shown in Fig. 1 the received signal is applied to an anti-aliasing (AA) bandpass filter, sampled by a free-running clock at a rate $1/T_s$, and frequency-translated from IF to baseband by performing a constant-speed rotation of $-2\pi f_{IF} T_s$ rad/sample. The AA filter has a bandwidth $B_{AA} = B_{max}(1+e)$ for some $e \geq 0$ and has a flat frequency response for $|f|$ in the interval $(f_{IF} - B_{max}, f_{IF} + B_{max})$. Subsequently, synchronized samples of the downconverted AA filter output signal, taken at a multiple (N) of the symbol rate (i.e., at instants $mT/N + \tau$), are obtained by means of an *interpolator*. The resulting synchronized samples are applied to a discrete-time filter that is matched to the transmit pulse $p(t)$. The matched filter output is decimated by a factor N, yielding samples at the decision instants $kT + \tau$. The matched filter also suppresses (part of) the double-frequency terms (at $-2f_{IF}$ + multiples of $1/T_s$) that result from the downconversion of the sampled output of the anti-aliasing filter. The matched filter output samples are fed to the decision device, that detects the transmitted symbols $\{a_k\}$. Denoting $rem(x)$ as the fractional part of x, i.e., $rem(x) = x - floor(x)$, we define $r = rem(2f_{IF}T_s)$. Consequently, the shifted HF components are centered at $(k-r)/T_s$ for $k \in \mathbb{Z}$.

2.2. Interpolation

Polynomial interpolation [3] is used to reconstruct, from samples $x(nT_s)$, a signal $x(t)$ at instants t_k, according to

$$x_I(t_k) = \sum_n x(nT_s)\, p_I(t_k - nT_s) \tag{2}$$

where $p_I(t)$ is the interpolating pulse. Denoting the Fourier transform (FT) of $x(t)$ by $X(f)$, interpolation is perfect (i.e., $x_I(t_k) = x(t_k)$) provided $X(f) = 0$ for $|f| > 1/T_s$ and $p_I(t) = sinc(t/T)$, with $sinc(x) = \sin(\pi x)/(\pi x)$. Otherwise $x_I(t_k) \neq x(t_k)$. Generally, polynomial interpolators have a frequency response which is fairly flat around $f = 0$ and has nulls around non-zero multiples of k/T_s, $k \in \mathbb{Z}$. The finite number of filter taps inevitably gives rise to non-ideal interpolation (i.e., $x_I(t_k) \neq x(t_k)$).

In our application we select $t_k = kT/N + \tau$: the interpolator output rate is an integer multiple (N) of the symbol rate $1/T$. It is assumed that τ is known at the receiver. In practice, this quantity needs to be estimated by a timing recovery circuit. For the remainder of this paper, we use linear interpolation. Generalizations to higher order polynomial interpolators is straightforward.

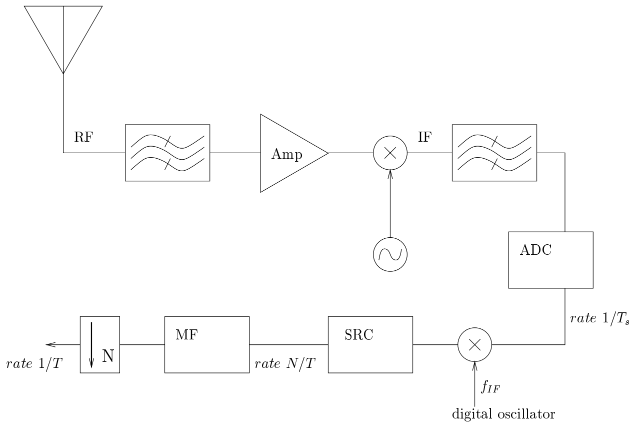

Fig. 1. Proposed receiver with IF sampling

3. SYSTEM ANALYSIS

3.1. Receiver Complexity

The design parameters N, T_s and r have a different impact on receiver complexity: the length of the matched filter is proportional to N, the complexity of the ADC after the AA filter is related to T_s and r has essentially no impact on receiver complexity. For practical reasons it is preferred to keep f_{IF} as low as possible.

3.2. Aliasing

A mathematically equivalent representation of the interpolator output, based on continuous-time filters can be used to examine the receiver: we apply the downconverted samples at the output of the AA filter to a reconstruction filter with impulse response $p_I(t)$ and (virtually) resample the resulting continuous-time signal at a rate N/T. Signal and noise components around $f = k/T_s$, for $k \neq 0$, are severely attenuated by the interpolator, while components centered around $f = (k-r)/T_s$ have a magnitude proportional to $sinc^2(k-r)$. For $k = 0$ and $k = 1$, this magnitude can be significant. If these components after virtual resampling end up within the bandwidth of the matched filter, severe degradations will occur. We denote the noise power at the output of the matched filter by σ^2_{tot}, with $\sigma^2_{tot} = \sigma^2_{LP} + \sum_k \sigma^2_{HF,k}$, where σ^2_{LP} is total noise power due to LP components, and $\sigma^2_{HF,k}$ is the noise power due to the HF component centered around $f = (k-r)/T_s$ at the output of the matched filter. A similar decomposition of signal power P_{tot} at the output of the matched filter can be made. To analyze the impact of each contribution to the overall performance it is important to note that any frequency f_1 at the interpolator input ends up at frequency $f = rem(f_1 T/N)\, N/T$ in the folded spectrum, $[0, N/T[$ at the interpolator output.

3.3. Performance measure

To determine the performance of the receiver in Fig. 1, we calculate the bit error rate (BER) degradation as compared to the reference structure. This degradation is defined by:

$$BER\,deg\ (dB) = 10\log\left(\frac{E_b/N_0}{(E_b/N_0)_{ref}}\right) \qquad (3)$$

where E_b/N_0 is the SNR needed to achieve the same BER as in the reference system that operates at a SNR of $(E_b/N_0)_{ref}$. It turns out that the BER degradation, with respect to the reference structure can be approximated by a simple function of E_b/N_0, σ^2_{tot} and P_{tot} [4]. This degradation depends on r through σ^2_{HF} and P_{HF}.

3.4. Reducing Aliasing

By properly selecting the design parameters r, N and T_s, we try to eliminate the aliasing at the interpolator output that is caused by $HF_0(f)$ and $HF_1(f)$, which are the largest HF components at the input of the interpolator. The results presented here can easily be extended to eliminate aliasing from $HF_k(f)$ with k different from 0 or 1. We look for T_s, N and r, such that $\sigma^2_{HF,0}$ and $\sigma^2_{HF,1}$ (and a fortiori $P_{HF,0}$ and $P_{HF,1}$) are both zero for all considered symbol rates.

3.4.1. Case 1: discrete symbol rates

Here, $T \in \{T_{min} = T_1, T_2 \ldots, T_N = T_{max}\}$. For each T_l, we determine a range for r, R_l, for which $\sigma^2_{HF,0}$ and $\sigma^2_{HF,1}$ are both zero. The intersection of these ranges R_l defines the desired range. It is easily verified that $\sigma^2_{HF,k} = 0$ for $T = T_l$ when the following set of inequalities holds:

$$B < \frac{N}{T_l} rem\left(\frac{k-r}{T_s N}T_l\right) - B_{AA} \qquad (4)$$

$$\frac{N}{T_l} - B > \frac{N}{T_l} rem\left(\frac{k-r}{T_s N}T_l\right) + B_{AA} \qquad (5)$$

Considering (4) and (5) for $k = 0$ and introducing the following quantities: $\alpha(T) = \frac{NT_s}{T}$, $\beta(T) = T_s(B + B_{AA})$, $A^s_M(T) = M\alpha(T) + \beta(T)$ and $A^e_M(T) = (M+1)\alpha(T) - \beta(T)$ it follows that $\sigma^2_{HF,0} = 0$ when $r \in R^0_l \bigcap [0,1[$, where

$$R^0_l = \bigcup_{M=M_{min}}^{M_{max}} \{[A^s_M(T_l), A^e_M(T_l)]\}. \qquad (6)$$

The range of M in (6) is such that only intervals overlapping with $[0,1[$ are considered: $M_{min} = 0$ and $M_{max} = floor\left(\frac{1-\beta(T_l)}{\alpha(T_l)}\right)$. Similarly, we find from (4) and (5) with $k = 1$ that $\sigma^2_{HF,1} = 0$ for $1 - r \in R^0_l \bigcap]0,1]$. Clearly, in order that values of r exist that yield both $\sigma^2_{HF,0} = 0$ and $\sigma^2_{HF,1} = 0$, it is necessary that $\beta(T_l) < 1/2$ for each T_l, so $1/T_s > 2(B_{AA} + B_{max})$.

Based upon the above derivations, the range of r that yields $\sigma^2_{HF,0} = \sigma^2_{HF,1} = 0$ for $T \in \{T_{min}, \ldots, T_{max}\}$ can be determined numerically.

3.4.2. Case 2: continuous symbol rates

In this case T can take on any value in the interval $[T_{min}, T_{max}]$. Using a line of reasoning analogous to section (3.4), $\sigma^2_{HF,0}$ is zero for all T in $[T_{min}, T_{max}]$ if $r \in R^0 \bigcap [0,1[$ with

$$R^0 = \bigcup_{M=M_{min}}^{M_{max}} \{[A^s_M(T_{min}), A^e_M(T_{max})]\} \qquad (7)$$

where $M_{min} = 0$ and $M_{max} = floor\left(\frac{1-\beta(T_{min})}{\alpha(T_{min})}\right)$. Similarly, $\sigma^2_{HF,1} = 0$ if $1 - r \in R^0 \bigcap]0,1]$.

A necessary condition for R^0 being non-empty is that $M\alpha(T_{min}) < (M+1)\alpha(T_{max})$ for at least one value of M in the range specified in (7). This condition is equivalent to $T_{max}/T_{min} < (M+1)/M$. Hence, when T_{max}/T_{min} is greater than 2, the only value of M to be considered in (7) is $M = 0$. Assuming $T_{max}/T_{min} \geq 2$, it can be shown that the range for r is non-empty if, with $\Delta_1 > 0$ and $\Delta_2 > 0$ where $\beta(T_{min}) = \frac{1}{2} - \Delta_1$ and $\alpha(T_{max}) - \beta(T_{max}) = \frac{1}{2} + \Delta_2$. Denoting $\Delta = \min(\Delta_1, \Delta_2)$, this leads to the following relationship between the design parameters:

$$\frac{1}{T_s} > 2(B_{AA} + B_{max}) \qquad (8)$$

$$N \geq T_{max}\left(\frac{1}{2T_s} + B_{AA} + B_{min}\right) \qquad (9)$$

$$r \in \left[\frac{1}{2} - \Delta, \frac{1}{2} + \Delta\right]. \qquad (10)$$

The conditions (8) and (9) indicate that both the sampling rate $1/T_s$ of the ADC and the oversampling factor N at the interpolator output must be sufficiently large. Note that condition (8) on the sample rate is the same as in section (3.4.1). In both case 1 and case 2 $|r - 1/2|$ can be no greater than Δ.

4. PERFORMANCE RESULTS

Figs. 2 and 3 show, as a function of $r = rem(2f_{IF}T_s)$, the actual BER degradations, maximized over all considered symbol rates, for a fixed sampling rate, $1/T_s = 11.1$ (which satisfies (8)), and various values of N, assuming

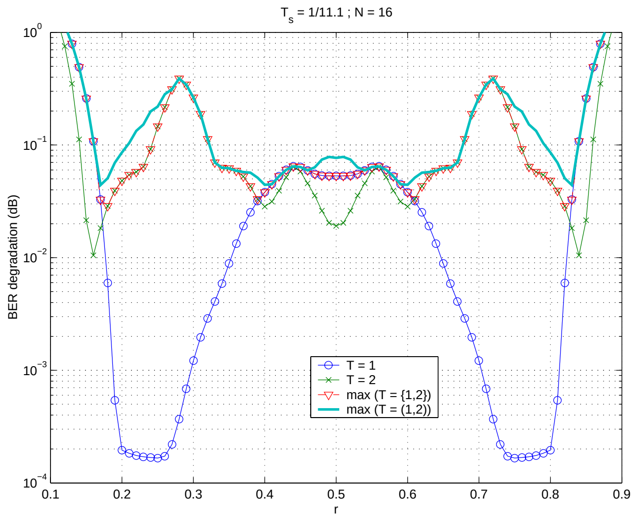

Fig. 2. BER degradations for $N = 16$

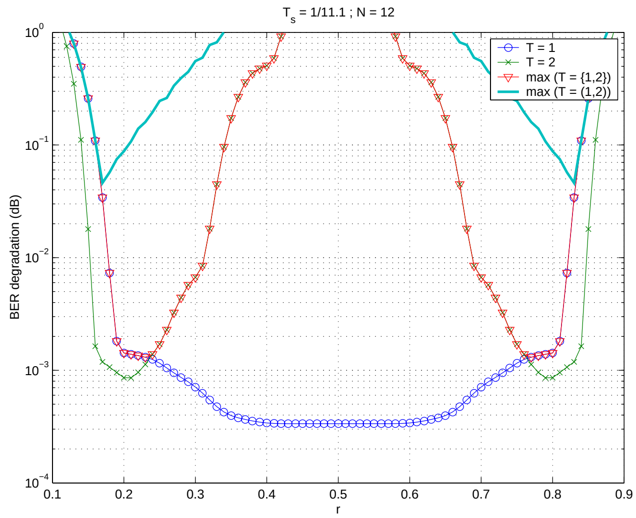

Fig. 3. BER degradations for $N = 12$

BPSK modulation, $(T_{min}, T_{max}) = (1, 2)$, $\alpha_p = 0.5$, $e = 1$ and a reference BER of 10^{-2}. We consider the cases of discrete T (i.e., $T \in \{T_{min}, \ldots, T_{max}\}$) and of continuous T (i.e., $T \in (T_{min}, T_{max})$). For comparison, also the BER degradations for the single-rate systems $T = T_{min}$ and $T = T_{max}$ are included. Note that the degradations are symmetric around $r = 1/2$.

Only for $N = 16$, condition (9) is satisfied, which indicates that there exists a region near $r = 1/2$ where both $\sigma^2_{HF,0}$ and $\sigma^2_{HF,1}$ are zero. The maximum degradations for continuous and discrete T are very similar. According to (10), small degradations are to be expected for $r \in (0.36, 0.64)$. It is observed that within this range, the degradations are not much sensitive to r, and indeed remain fairly low (i.e., less than about $0.1\, dB$)

When $N = 12$, condition (9) is no longer valid. Hence, $\sigma^2_{HF,0}$ and $\sigma^2_{HF,1}$ cannot both be zero for all T in $(T_{min}$, $T_{max})$, so that performance is deteriorated. The BER degradations for the cases of continuous T and discrete T are dissimilar, and are quite sensitive to the value of r.

5. CONCLUSIONS AND REMARKS

In this contribution we have considered a software radio multirate receiver, which makes use of fixed nonsynchronized sampling of the IF signal, digital downconversion to baseband, digital sample rate conversion (SRC) to a multiple N of the actual symbol rate $1/T$, and finally matched filtering. Performing SRC in front of the matched filter has the advantage that the matched filter coefficients depend only on the oversampling factor N, but not on the (variable) data rate. However, we have shown by means of frequency-domain interpretations that in this configuration the SRC gives rise to aliasing within the bandwidth of the matched filter, so that a performance degradation occurs as compared to a receiver with synchronized baseband sampling. We have identified the potentially largest aliasing terms, and have indicated how several system parameters (such as the value f_{IF} of the IF, the fixed sampling frequency $1/T_s$, the oversampling factor N, and the range (T_{min}, T_{max}) of the variable symbol interval) should be selected in order to keep these aliasing terms outside the matched filter bandwidth. This system parameter selection has been validated by computing the BER degradation caused by the aliasing, and observing that the proper selection results in only a small (about $0.1\, dB$) BER degradation.

6. REFERENCES

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