NON-DATA-AIDED CARRIER FREQUENCY OFFSET ESTIMATION OF GMSK SIGNALS IN BURST MODE TRANSMISSION

Hua Peng, Jing Li, Lindong Ge

Department of Information Science, Information Engineering University
P.O.BOX 1001-306, Zhengzhou 450002, China

ABSTRACT

This paper presents a non-data-aided (NDA) carrier frequency offset estimation algorithm for GMSK modulation. A nonlinearity of the GMSK signal is shown to be a sine wave with frequency related to the carrier frequency offset. Based on that, a single frequency estimation method with large estimation range and low computational complexity is used to compute the carrier frequency offset. The estimator has a feedforward structure and is suitable for burst mode transmission. Its estimation accuracy is good and the estimation range is a quarter of the symbol rate, which is illustrated by simulation results.

1. INTRODUCTION

Continuous phase modulation (CPM) is attractive for its high bandwidth efficiency and constant envelope. As a type of CPM, Gaussian minimum shift keying (GMSK) modulation has been adopted in many wireless communication standards such as the global system for mobile (GSM).

Knowledge of carrier frequency offset caused by oscillator instability and Doppler effects is necessary for both coherent demodulation and differential demodulation of GMSK signal[1]. Carrier frequency estimation methods for GMSK modulation are typically categorized in data-aided (DA) methods and non-data-aided (NDA) methods. This paper focuses on the later.

Some NDA methods for carrier frequency offset estimation [1][2][3][4] do not need the aid of timing clock estimation and thus are called non-clock-aided (NCA). The NDA carrier frequency offset detector proposed in reference [1] is based on the Laurent approximation of CPM signals [8] and maximum likelihood (ML) frequency estimation. Reference [2] shows that the change of the continuous signal phase over any one-symbol time interval is not larger than $\pi/2$. Based on that, [2] proposes an NDA frequency difference detector (FDD) and frequency compensation structure that is suitable for one-bit differential demodulation. The estimation range of the FDD is a quarter of the symbol rate. The feed-forward frequency estimator proposed in [3] extracts the carrier frequency offset from the phase of the nonlinearity $x(kT)x^*(kT - DT_s)$, where $x(t)$ is the received GMSK signal, T is the symbol interval, T_s is the sampling time interval and D is a time delay parameter related to the estimation range and variance. The bigger D is, the smaller the estimation range and variance are. In [4], a frequency synchronization algorithm based on fourth order cyclic statistics is proposed. The algorithm works well in a time-selective fading channel.

Some other frequency estimation algorithms [5][6] are clock-aided (CA), i.e. they require timing recovery be previously accomplished. Reference [5] approximately converts GMSK modulation to a linear modulation by means of keeping only the most significant item of the Laurent approximate expression [8]. Based on that, [5] shows that a non-linearity of the GMSK signal can generate a sine wave at frequency related to the carrier frequency offset. Then, the Rife and Boorstyn (R&B) [7] method is used to estimate the frequency of the sine wave. The estimation range of the algorithm is a quarter of the symbol rate. In [6], a frequency offset estimation method based on the fourth order cyclic statistics is proposed.

In this paper, a sine wave is obtained from a GMSK signal in the same way as in [5]. Based on that, a single frequency estimation method with large estimation range and low computational complexity is used to compute the carrier frequency offset.

The paper is organized as follows. Signal model and basic notations are introduced in section 2. Section 3 describes the frequency estimation algorithm. Section 4 gives the simulation results that illustrate the performances of the proposed scheme. Some conclusions are given in Section 5.

2. SIGNAL MODEL

The complex envelope of a GMSK signal can be written as

$$s(t) = \exp\big(j\phi(t, \boldsymbol{\alpha})\big) \quad nT \le t \le (n+1)T \tag{1}$$

where

$$\phi(t, \boldsymbol{\alpha}) = \pi h \sum_i a_i q(t - iT) \tag{2}$$

is the information bearing phase, $\boldsymbol{\alpha}=\{a_i\}$ is a binary information sequence, T is the symbol period. In (2), h denotes the modulation index and it equals 0.5 for GMSK modulation. $q(t)$ is the phase pulse and

$$q(t)=\int_{-\infty}^{t}h(t)dt \qquad (3)$$

The frequency pulse $h(t)$ is limited to the interval [0, LT] and gives

$$\int_{0}^{LT}h(t)dt=1 \qquad (4)$$

where L is an integer.

It is shown in [8] that a GMSK signal can be approximately expressed by

$$s(t)\approx\sum_{n}a_{0,n}h_0(t-nT) \qquad (5)$$

where

$$a_{0,n}=\exp\left\{j\frac{\pi}{2}\sum_{i=1}^{n}a_i\right\} \qquad (6)$$

$$h_0(t)=\prod_{l=0}^{L-1}p(t+lT) \qquad (7)$$

and

$$p(t)=\begin{cases}\sin[q(t)\cdot\pi/2], & 0\le t\le LT\\ p(2LT-t), & LT<t\le 2LT\\ 0, & \text{else}\end{cases} \qquad (8)$$

Assume that $s(t)$ is transmitted over an AGWN channel, the complex envelope of the received GMSK signal in a digital implementation is modeled as

$$x(kT_s)=\exp\{j(2\pi f_e kT_s+\theta)\}\sum_{n}a_{0,n}h_0(kT_s-nT)+n(kT_s) \qquad (9)$$

where f_e and θ denote the carrier frequency offset and phase respectively. The sampling rate $1/T_s$ is a multiple P of the symbol rate $1/T$. $n(kT_s)$ is the filtered Gaussian noise.

Define a non-linearity of the received signal

$$z(k)=(-1)^k x^2(kPT_s+\frac{L+1}{2}PT_s) \qquad (10)$$

Reference [5] shows that $z(k)$ is approximately a discrete time sine wave whose frequency and sampling rate are Δf and $1/T$ respectively. $z(k)$ can be written as

$$z(k)\approx A\exp\{j2\pi\cdot\Delta fkT+2\theta\}+w(k) \qquad (11)$$

where $\Delta f=2f_e$, $A=\{\max\{h_0(t)\}\}^2$ and $w(k)$ is the noise term. Therefore the estimation of carrier frequency offset f_e turns to the estimation of a sine wave's frequency.

3. FREQUENCY ESTIMATION

Consider the m-lag autocorrelation of $z(k)$

$$R(m)=\frac{1}{N-m}\sum_{k=m+1}^{N}z(k)z^*(k-m) \qquad 1\le m\le N-1 \qquad (12)$$

where N is the number of available data. From (11), it can be seen that

$$R(m)=A^2\exp\{j2\pi\cdot\Delta fmT\}+\gamma(m) \qquad (13)$$

where $\gamma(m)$ is the noise term.

Clearly, the frequency Δf can be extracted from the phase of $R(m)$. As is shown in [10], the phase of $R(m)$ with bigger m($m\le N/2$) is less affected by noise. Using the autocorrelation $R(N/2)$, [10] gives an estimator

$$\Delta\hat{f}=\arg[R(N/2)]/\pi TN \qquad (14)$$

The estimator in (14) has a poor estimation range $1/NT$. That is resulted by the phase wrapping introduced by the operation $\arg[\cdot]$. To enlarge the estimation range, phase unwrapping technique has to be adopted.

If the noise term in (13) is ignored, we have

$$\arg\{R(m)\}=2\pi\Delta fmT-2\pi\cdot S \qquad (15)$$

where

$$S=[\Delta fmT] \qquad (16)$$

and $[x]$ denotes the integer that is most close to x. Thus a new estimator of Δf can be written as

$$\Delta\hat{f}_{new}=\arg[R(m)]/2\pi Tm+S/2\pi Tm \qquad (17)$$

The calculation of S is the key of phase unwrapping.

Consider the discrete Fourier transform (DFT) of z(k) ($0\le k<m-1$). Let $\tilde{S}$ ($-m/2<\tilde{S}\le m/2$) denote the sequence number of the largest spectral line. When noise is ignored, we have

$$\tilde{S}=[\Delta f\cdot mT] \qquad (18)$$

From (16) and (18), it can be seen that $S=\tilde{S}$. Let m=N/2. The following estimator of Δf can be derived from (17)

$$\Delta\hat{f}_{new}=\Delta\hat{f}+\hat{f}_{DFT} \qquad (19)$$

where $\hat{f}_{DFT}=\tilde{S}/\pi TN$ is the frequency of the largest spectral line. The carrier frequency offset estimator for GMSK modulation is

$$\hat{f}_e=\Delta\hat{f}_{new}/2=(\Delta\hat{f}+\hat{f}_{DFT})/2 \qquad (20)$$

When noise is ignored, we can see from (11) that $\hat{f}_{DFT}$ can be correctly computed only if the sampling rate of $z(k)$ is larger than $2\Delta f$, which means that the estimation range of the estimator in (19) is $|\Delta f|<T/2$. Hence the estimation range of the estimator in (20) is T/4.

When carrier frequency offset f_e takes some values so that $\Delta f\cdot NT/2$ is very close to $[\Delta f\cdot NT/2]\pm 0.5$, the sequence number of the largest spectral line may be $[\Delta f\cdot NT/2]\pm 1$ instead of $[\Delta f\cdot NT/2]$ because of the existence of noise. That will lead to a wrong phase unwrapping and thus wrong frequency estimation.

As a solution to this problem, we compute the DFT of N samples of $z(k)$ and denote the spectral lines by 0,1,… N-1. Let S' denote the sequence number of the largest spectral line. If S' is even, $\tilde{S}=S'/2$. If S' is odd, we need to determine whether $\tilde{S}$ equals to $(S'+1)/2$ or $(S'-1)/2$. From (15), it can be seen that the sign of

$\arg[R(N/2)]$ indicates which range $\Delta f \cdot NT/2$ is in. If $\arg[R(N/2)] \geq 0$, i.e. $\Delta f \geq 0$, we have $\Delta f \cdot NT/2 \in [\tilde{S}, \tilde{S}+0.5]$ and thus $\tilde{S}=(S'-1)/2$. If $\arg[R(N/2)] < 0$, i.e. $\Delta f < 0$, we have $\Delta f \cdot NT/2 \in (\tilde{S}-0.5, \tilde{S})$ and thus $\tilde{S}=(S'+1)/2$.

As is computed using a high time lag autocorrelation, the sign of $\Delta\hat{f}$ has a reasonable reliability. Hence we take it as a measure to determine the value of $\tilde{S}$. In short, $\tilde{S}$ is computed as

$$\tilde{S} = \begin{cases} (S'-1)/2 & S' \text{ is odd and } \Delta\hat{f} \geq 0 \\ (S'+1)/2 & S' \text{ is odd and } \Delta\hat{f} < 0 \\ S'/2 & S' \text{ is even} \end{cases} \quad (21)$$

In a digital implementation of the GMSK receiver, the received signal is divided into frames. The carrier frequency offset is computed once a frame. If large burst noise exists in a certain frame, the corresponding estimation result will not be credible enough. To reduce the affect of burst noise, the estimation results are filtered as follow.

If $\Delta\hat{f}'_e(i)$ denotes the estimation result computed by (20) in the i^{th} frame and $\Delta\hat{f}_e(i)$ denotes the output of the filter in the i^{th} frame, we have

$$\Delta\hat{f}_e(i) = \eta \cdot \Delta\hat{f}'_e(i) + (1-\eta)[\lambda \cdot \Delta\hat{f}_e(i-1) + (1\text{-}\lambda) \cdot \Delta\hat{f}'_e(i)] \quad (22)$$

where λ is a forgetting factor and is chosen in the range $0 < \lambda < 1$. The variable η is defined as

$$\eta = \begin{cases} 1, & \text{on condition A} \\ 0, & \text{else} \end{cases} \quad (23)$$

where condition A means that $|\Delta\hat{f}_e(j) - \Delta\hat{f}'_e(j)| \geq 4/NT$ for j=i-2, i-1, i and $|\Delta\hat{f}_e(i-3) - \Delta\hat{f}'_e(i-3)| < 4/NT$.

4. SIMULATION RESULTS

This section gives the simulation results that illustrate the performances of the algorithm described in this paper. The frequency estimation algorithms described in [3] [4] [6] are also simulated for comparison. The modified Cramer-Rao bound [9] (MCRB) is given as a benchmark. In the simulations, the length of a frame equals to 128 symbols and the over-sampling factor T/T_s is 4. For MM algorithm in [6] and GSD algorithm in [4], M is chosen to be 10. The performances are described in normalized frequency estimation error variance defined as

$$\sigma^2 = E\left[\left(\left(\hat{f}_e - f_e\right)T\right)^2\right] \quad (23)$$

For the sake of simplicity, we denote the algorithms proposed in this paper, in [3], [4] and [6] by algorithm A, B, C and D respectively.

To illustrate the improvement brought by the frequency computing method defined in (21), algorithms using (21) and without using (21) are compared. Fig.1 shows σ^2 versus E_s/N_0. f_eT is chosen to be 11/256 so that $\Delta f \cdot NT/2$ is very close to $[\Delta f \cdot NT/2] \pm 0.5$. It can be seen that the frequency estimator without using (21) has larger error than that using (21).

Secondly, the algorithms A, B, C and D are simulated in an additive white Gaussian noise (AWGN) channel. Fig.2 illustrates the normalized estimation error variances as functions of E_s/N_0 when f_eT is chosen to be 0. It is shown that algorithm A performs much better than the others. Fig.3 shows σ^2 versus normalized frequency offset f_eT when E_s/N_0 is 15dB. It can be seen that the estimation range of algorithm A is coarsely a quarter of the symbol rate and is larger than the other algorithms.

The algorithms are also simulated in a Rayleigh flat-fading channel. f_eT is chosen to be 0.1. Fig.4 and Fig.5 show σ^2 versus E_s/N_0 when the maximum Doppler frequency shift f_d is 5e-4/T and 1e-3/T respectively. It is shown that the performance of algorithm A degrades a lot in a fading channel yet is the same as the other algorithms.

5. CONCLUSION

A carrier frequency offset estimation algorithm for GMSK modulation is proposed in this paper. The algorithm has low computational complexity and a feed-forward structure and is well suited for burst-mode communications. The estimation accuracy is quite good in an AWGN channel and is the same as the algorithms in [4] and [6] when the channel is Rayleigh flat-fading. Because a phase unwrapping scheme is adopted, the estimation range of the frequency estimator is enlarged to a quarter of symbol rate.

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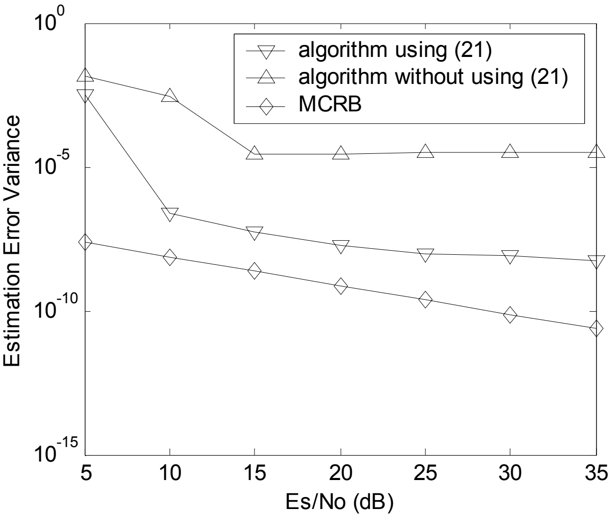

Fig.1. Comparison between using (21) and without using (21)

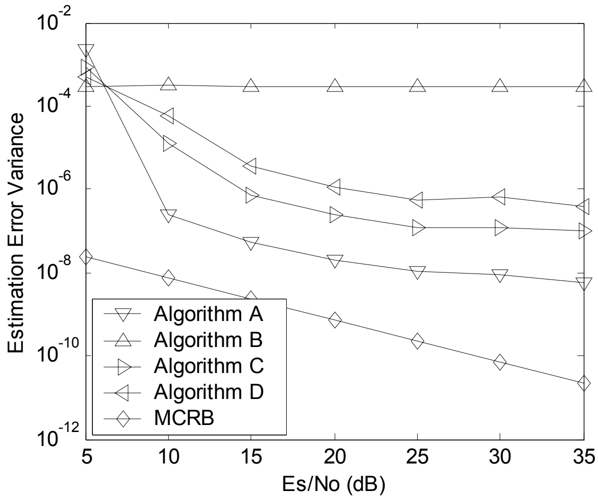

Fig.2. Performances in AGWN channel

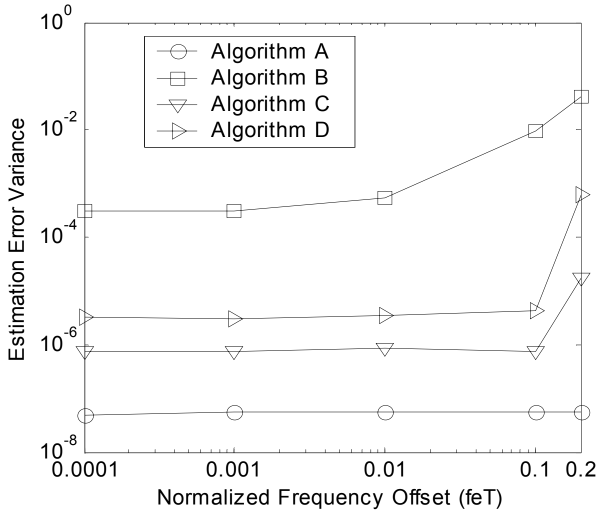

Fig.3. Affect of carrier frequency offset on performances of different algorithms

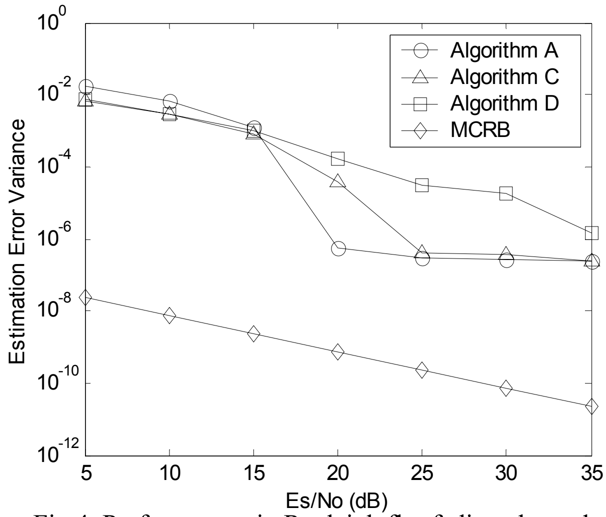

Fig.4. Performances in Rayleigh flat-fading channel (f_dT=5e-4)

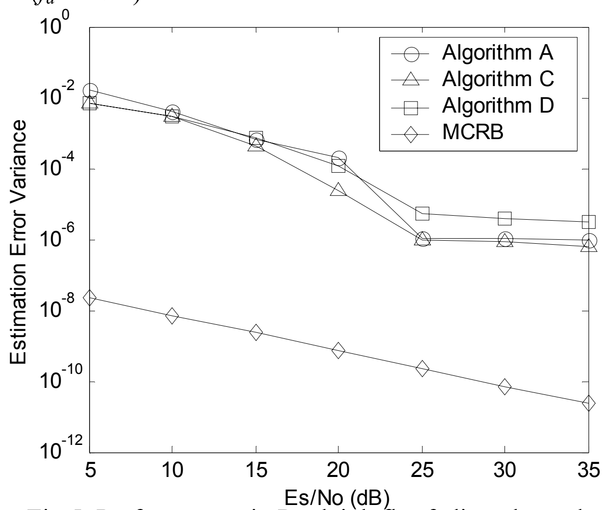

Fig.5. Performances in Rayleigh flat-fading channel (f_dT=1e-3)