SECOND-ORDER CYCLOSTATIONARY APPROACH TO NDA ML SQUARE TIMING ESTIMATION WITH FREQUENCY UNCERTAINTY

José A. López-Salcedo and Gregori Vázquez

Department of Signal Theory and Communications, Universitat Politècnica de Catalunya
D5-117, Campus Nord UPC, c/Jordi Girona 1-3, 08034 Barcelona, SPAIN

ABSTRACT

This paper addresses the problem of Non-Data-Aided (NDA) symbol timing error estimation in presence of unknown deterministic carrier frequency errors. A general framework for the stochastic low-SNR ML estimation is derived considering a *prior* distribution for the carrier frequency error, and exploiting the cyclostationary properties of linear modulations. Finally, the well-known Oerder & Meyr (Square Timing) method is shown to become a particular case of this general solution.

1. INTRODUCTION

In this paper[1] we are addressing a basic problem on digital communications such as the Non-Data Aided (NDA) symbol timing estimation, which constitutes one of the fundamental tasks of a digital receiver. However, despite of the wide range of different approaches to attempt the NDA symbol timing estimation problem, most of them are found to be based on some heuristic or *ad-hoc* reasoning [1]. Conversely, this paper presents an analytical and systematic Maximum Likelihood (ML) approach based on the second order cyclostationarity of linear modulations, to derive a general framework for the symbol timing error estimation problem in presence of a certain carrier frequency uncertainty with some *prior* statistical distribution.

Basically, second order cyclostationarity has been usually reported in recent literature for the purpose of carrier frequency estimation [2]-[3], although [4] presents an interesting study for both frequency and timing estimation. However, most of references assume the processing of the signal at the output of some receive filter, so the carrier frequency error is then assumed to be irremediably constrained within an small interval for not incurring in some mismatch degradation. In this sense, this paper presents a general formulation showing that the received signal Cyclic Autocorrelation Function (CAF) [5] becomes a sufficient statistic for the problem at hand. Finally, this expression will be related to the classical Square Timing recovery method by Oerder & Meyr [6], and its convergence will be shown for the particular cases of both totally unknown and perfect carrier knowledge.

2. DISCRETE-TIME SIGNAL MODEL AND STOCHASTIC ML APPROACH

The complex envelope model that will be used for a linearly modulated received signal corrupted by noise is given by:

$$r\left(k\right)=\sum_{n=-\infty}^{+\infty}x_{n}g\left(kT_{s}-nT-\tau\right)e^{j\left(\omega kT_{s}+\theta_{0}\right)}+w\left(kT_{s}\right) \quad (1)$$

[1]This work was partially funded by the Spanish Government Grants TIC2000-1025, TIC2001-2356-C02-01, TIC2002-04594-C02 and by CIRIT/Generalitat de Catalunya Grant 2001SGR-00268

with x_n the transmitted symbols, $g(kT_s)$ the sampled pulse shape, T_s the sampling period, $T = N_{ss}T_s$ the symbol period with N_{ss} the number of samples per symbol and $w(kT_s)$ the AWGN. Moreover, the signal model in (1) includes the symbol timing error τ constrained within a symbol interval $[-T/2, +T/2)$, the carrier phase θ_0, and the carrier frequency error ω constrained to the Nyquist bandwidth $[-\pi/T_s, +\pi/T_s)$ that will be further on normalized to the symbol rate, that is, $\nu \doteq \frac{\omega}{2\pi}T$. In a more convenient vectorial notation, the received signal can be expressed by means of the shaping matrix $\mathbf{A}\left(\Theta\right)$ as $\mathbf{r} = \mathbf{A}\left(\Theta\right)\mathbf{x} + \mathbf{w}$ [7], with $\Theta = [\tau, \nu]$.

Synchronization methods derived under the *Stochastic* or *Unconditional* Maximum Likelihood approach are the most extended in literature, and they are based on applying the ML principle assuming that the transmitted symbols $\mathbf{x}$ are all random. Assuming a low-SNR scenario, the authors in [7] prove that the sample covariance matrix of the received data given by $\widehat{\mathbf{R}} = \mathbf{r}\mathbf{r}^H$, becomes a sufficient statistic for the NDA parameter estimation problem. Thus, the stochastic ML function depends only on the second order moments. For a long enough observation interval, the log-Likelihood function in low SNR scenarios asymptotically becomes:

$$L\left(\mathbf{r}|\Theta\right)\doteq\ln\left(E_{\mathbf{x}}\left[\Lambda\left(\mathbf{r}|\Theta;\mathbf{x}\right)\right]\right)\approx\frac{C}{\sigma_{w}^{4}}Tr\left(\mathbf{A}(\Theta)\Gamma\mathbf{A}^{H}(\Theta)\widehat{\mathbf{R}}\right) \quad (2)$$

where C is an irrelevant constant, "Tr" stands for the trace operator, and $\Gamma \doteq E\left[\mathbf{x}\mathbf{x}^H\right]$ the autocorrelation matrix for the transmitted symbols, which will be assumed without loss of generality, to be uncorrelated and normalized to the transmitted mean power, that is, $\Gamma = \mathbf{I}$.

3. TIMING ESTIMATION UNDER CARRIER FREQUENCY UNCERTAINTY

3.1. Exploitation of the Second-Order Cyclostationarity

The second order cyclostationarity of the received signal becomes a fundamental part for the NDA ML timing estimation under carrier frequency uncertainty. Assume the signal model presented in (1) in the absence of timing nor carrier frequency errors, to be given by $r(k) = s(k) + w(k)$ with $s(k) = \sum_{n=-\infty}^{+\infty} x_n g\left(kT_s - nT\right)$. With independent uncorrelated symbols, the second-order moment for the cyclostationary signal $s(k)$ can be expressed by means of its time-varying autocorrelation as: $R_s(k;m) = E\left[s(k)s^*\left(k+m\right)\right] = \sigma_x^2 R_g\left(k;m\right)$. Moreover, it is satisfied that $R_s(k;m) = R_s(k + lN_{ss};m)\,, l \in \mathbb{Z}$. Hence, it can be expressed without loss of generality, in terms of it Fourier Series (FS) expansion in k as in [4] and [8]: $R_s(k-\tau;m) = \sum_{\alpha\in\mathcal{A}} R_s^{\alpha}(m)e^{j\alpha(k-\tau)}$, with $\mathcal{A} = \{-\pi \leq \alpha \leq \pi\}$.

A consistent and asymptotical estimator for $R_s^{\alpha}(m)$ is given by $\widehat{R}_s^{\alpha}\left(m\right) = \frac{1}{2M+1}\sum_{k=-M}^{M} s\left(k\right)s^*\left(k+m\right)e^{-j\alpha k}$ as shown in [8], so $R_s^{\alpha}(m)$ is also known as the coefficient for the *Cyclic-Autocorrelation Function* (CAF) evaluated at the *cycle-frequency* α, according to [9].

The FS expansion can be further simplified by noting that for linear modulations, $R_s^{\alpha}(m) \neq 0$ when $|\alpha| = \frac{2\pi}{T}l, l \leq \text{integer}\,(1+\beta)$, and β the pulse shape roll-off factor [10].

Thus, it is seen that the maximum possible value for l is $l = 2$, which implies that the CAF should be evaluated in the worst case, at the two first cycle-frequencies $|\alpha| = \{2\pi/T, 2\pi\,(2/T)\}$. Due to the fact that signal $s(k)$ is shaped by an even pulse shape, we know from the FS theory that its expansion should be given as a weighted sum of *odd* pure tones, so this fact eliminates the second possible cycle-frequency, and just $\alpha = \pm 2\pi/T$ is to be considered. As regards of the cycle-frequency $\alpha = 0$, it will be herein omitted, as it does not provide any information on the timing error parameter τ. Considering the positive and negative cycle-frequency $2\pi/T$, we may conclude that the FS expansion of the time-varying autocorrelation for timing estimation purposes is given by $R_s(k-\tau;m) = 2\sigma_x^2\,\mathrm{Re}\left[R_g^{\frac{2\pi}{T}}(m)e^{j\frac{2\pi}{T}(k-\tau)}\right]$.

3.2. Cyclic-Autocorrelation based NDA Timing Estimation

Herein, we will focus on those applications in which the problem of the marginal estimation of just the symbol timing error τ is required, assuming the carrier frequency error ν to be an *unknown deterministic* parameter constrained within the interval Δ_ν. The lack of knowledge about the carrier frequency error is then reflected by adopting a uniform *prior* distribution, $f_\nu(\nu) = 1/\Delta_\nu$. In this sense, the log-Likelihood function must be averaged in terms of ν, so each of the (p,q) entries of the outer-product matrix $\mathbf{A}\mathbf{A}^H$ are given by:

$$\lim_{K\to\infty} E_\nu\left[\left[\mathbf{A}\mathbf{A}^H\right]_{p,q}\right] = \int_{-N_{ss}/2}^{+N_{ss}/2} f_\nu(\nu)\, e^{j\frac{2\pi}{N_{ss}}\nu(p-q)} d\nu$$

$$\cdot \sum_{n=-\infty}^{+\infty} g\,(pT_s + nT - \tau)\, g^*\,(qT_s + nT - \tau) \qquad (3)$$

thus, it is possible to express (2) as:

$$L\,(\mathbf{r}|\tau) = \frac{C}{\sigma_w^4} Tr\left(\mathbf{M}\widehat{\mathbf{R}}\right) \;\rightarrow\; \mathbf{M} = \left[\left(\mathbf{G}_\tau \mathbf{G}_\tau^H\right) \odot \mathbf{V}\right] \qquad (4)$$

where the "$\odot$" operator stands for the Schur-Hadamard product, $\mathbf{G}_\tau$ is the pulse shaping matrix $\mathbf{A}$ with only dependence on the timing error parameter τ, $\mathbf{G}_\tau = \mathbf{A}\,(\tau, \nu = 0)$, and finally, the entries of the *Doppler spreading matrix* $\mathbf{V}$ result: $[\mathbf{V}]_{p,q} = \mathrm{sinc}\left(\frac{\Delta_\nu}{N_{ss}}(p-q)\right)$. Due to the particular structure of matrices $\mathbf{G}_\tau\mathbf{G}_\tau^H$ and $\mathbf{V}$, the Doppler spreading matrix can be seen to *mold* the Likelihood function according to the *prior* distribution for the carrier frequency error.

Next step is based on recalling the cyclostationary nature of linear modulations. In particuar, each of the m-th diagonal entries in matrix $\mathbf{G}_\tau\mathbf{G}_\tau^H$ are found to be equal to the pulse shape time-varying autocorrelation $R_g\,(k;m)$. Hence, it is straightforward to express $R_g\,(k;m)$ in terms of its Fourier Series expansion (FS), and the m-th diagonal constant entries of $\mathbf{V}$ as $d_{\mathbf{V}}\,(m) \doteq \mathrm{sinc}\left(\frac{\Delta_\nu}{N_{ss}}m\right)$. Due to the particular structure of matrices $\mathbf{M}$ and $\widehat{\mathbf{R}}$, and after some straightforward manipulations, expression (4) can be found to be given by:

$$L\,(\mathbf{r}|\tau) = \frac{C}{\sigma_w^4} Tr\left(\mathbf{M}\widehat{\mathbf{R}}\right) =$$

$$= \frac{C}{\sigma_w^4\sigma_x^2}\,\mathrm{Re}\left[e^{-j\frac{2\pi}{T}\tau}\left(d_{\mathbf{V}}\,(0)\, R_g^{\frac{2\pi}{T}}\,(0) \sum_{k=-M}^{+M} |r(k)|^2\, e^{j\frac{2\pi}{T}k} + \right.\right.$$

$$\sum_{m>0} d_{\mathbf{V}}\,(m) \left|R_g^{\frac{2\pi}{T}}\,(m)\right| \sum_{k=-M}^{+M-m} r^*(k) r(k+m) e^{j\frac{2\pi}{T}(k+\frac{m}{2})} +$$

$$\left.\left.\sum_{m<0} d_{\mathbf{V}}\,(m) \left|R_g^{\frac{2\pi}{T}}\,(m)\right| \sum_{k=-M}^{+M-m} r(k) r^*(k-m) e^{j\frac{2\pi}{T}(k+\frac{m}{2})}\right)\right] \qquad (5)$$

with $|m| \leq L$, and $2L+1$ being the pulse shape duration in samples.

As it can be seen from (5), the log-Likelihood function suggests that the cyclic autocorrelation function (CAF) $R_g^{\frac{2\pi}{T}}\,(m)$ of the received samples should be performed at the frequency lag $\alpha = 2\pi/N_{ss}$ and at all the possible time-lags, in order to collect their spectral components at the symbol rate and then obtaining a robust symbol timing estimate. On the other hand, the window $d_{\mathbf{V}}\,(m)$ is responsible of limiting the number of time-lags to integrate, as the carrier frequency degrades the weighted CAF when increasing m. Grouping common terms and considering some symmetric properties of CAFs, it is found that the optimal ML NDA symbol timing estimate is given as $\widehat{\tau} \rightarrow \max_\tau \left\{Tr\left(\mathbf{M}\widehat{\mathbf{R}}\right)\right\}$

$$\widehat{\tau}_{WCM} = \frac{T}{2\pi}\arg\left\{\sum_{m=-L}^{+L} d_{\mathbf{V}}\,(m)\left[R_g^{\frac{2\pi}{N_{ss}}}\,(m)\right]^* \widehat{R}_r^{\frac{2\pi}{N_{ss}}}\,(m)\right\} \qquad (6)$$

referred as the WCM (Weighted Cyclic Method) timing estimator.

3.3. Reconfigurable NDA Timing Estimation and Tracking

In the previous section it has been shown that a timing estimation can be performed from the ML cost function by computing the argument in the weighted sum of the correlation between the pulse shape CAF and the received signal CAF. This estimate exhibits an estimation range within the whole margin $[-T/2, +T/2)$, which is suitable for working at acquisition mode. Once a coarse timing estimation has been obtained, fine timing can be performed by deriving a NDA timing tracker from the same Likelihood funtion in (5). Assuming the timing error to be small, the simplified version of this timing discriminator is derived by taking the derivative of the ML cost function with respect to the timing parameter:

$$\Delta_{\tau_{WCM}} = \mu\,\mathrm{Im}\left[\sum_{m=-L}^{+L} d_{\mathbf{V}}\,(m)\left[R_g^{\frac{2\pi}{N_{ss}}}\,(m)\right]^* \widehat{R}_r^{\frac{2\pi}{N_{ss}}}\,(m)\right]$$

with μ some normalization factor to force the discriminator S-curve to have unitary slope. Hence, using the same analytical framework, two possible schemes could be employed in a reconfigurable architecture with the purpose of coarse and fine symbol timing estimation without any increase in complexity.

4. RELATIONSHIP WITH THE SQUARE TIMING - OERDER & MEYR ALGORITHM

The Square Timing recovery method (SQT) presented in [6] is based on exploiting the cyclostationary nature of linear modulations by taking the argument of the input signal Fourier Transform at the symbol rate. An analytical development is presented in [7] that shows the SQT optimality for maximum carrier frequency uncertainty. However, no exact nor formal reasoning can be found to prove whether the SQT is optimum or not when there exists perfect knowledge of the carrier frequency. In this section, two main results are presented: on one hand, it is proved that the WCM converges to the SQT for maximum carrier frequency uncertainty, and on the other, that the WCM converges to the SQT at the output of the matched filter when perfect carrier knowledge exists. Therefore, as the WCM is the optimum ML timing estimator for low-SNR, the SQT at the output of a matched filter becomes so.

4.1. Scenarios with Maximum Carrier Frequency Uncertainty

It is straightforward to prove that when there exists maximum carrier frequency uncertainty, the WCM converges to the optimum ML solution of the classical SQT based on received samples. In this case, the interval for the uniform prior in (3) comprises the whole Nyquist bandwidth, so the diagonal entries of the Doppler spreading matrix result in $d_{\mathbf{V}}(0) = 1$, $d_{\mathbf{V}}(m) = 0$ for $m \neq 0$, as $\Delta_\nu \rightarrow N_{ss}$ Therefore, expression (6) just takes into consideration the central time-lag of the CAF $\widehat{R}_r^{\frac{2\pi}{N_{ss}}}(m)$, thus resulting in the well-known SQT method. $\widehat{\tau}_{WCM} \rightarrow \frac{T}{2\pi} \arg\left\{ R_g^{\frac{2\pi}{N_{ss}}}(0) \sum_{k=-\infty}^{+\infty} |r(k)|^2 e^{j\frac{2\pi}{N_{ss}}k} \right\}$ with $R_g^{\frac{2\pi}{N_{ss}}}(0)$ a real valued constant that does not affect the argument computation, as it is evaluated off-line assuming $\tau = 0$.

4.2. Scenarios with Perfect Carrier Frequency Knowledge

In order to derive the equivalence between cyclic-moments and time-linear filtering, it is convenient to express the CAF in terms of its frequency domain representation. For that purpose, the Fourier transforms (FT) for the Nyquist square-root pulse shape $g(t)$, the received signal $r(t)$ and the output of the matched filtering $y(t) = r(t) * g(-t)$ are further on expressed as $G(\omega)$, $R(\omega)$, and $Y(\omega)$. Using Parseval's theorem, we find that $R_g^{\frac{2\pi}{N_{ss}}}(m) = \frac{1}{2M+1} \int_{-\pi}^{+\pi} G(\omega) G^*\left(-\omega + \frac{2\pi}{N_{ss}}\right)$ $e^{-j\omega m} d\omega$. Substituing in (6), assuming C_1 an irrelevant constant, and the argument in (6) to be denoted as $\Phi\left(\frac{2\pi}{N_{ss}}\right)$, with $\tau_{WCM} = \frac{T}{2\pi} \arg\left\{\Phi\left(\frac{2\pi}{N_{ss}}\right)\right\}$, it is found that

$$\Phi\left(\frac{2\pi}{N_{ss}}\right) = C_1 \sigma_x^2 \left[\int_{-\pi}^{+\pi} \int_{-\pi}^{+\pi} G(v) G^*(-u) R\left(u - \frac{2\pi}{N_{ss}}\right) \right.$$

$$\left. R^*\left(-v + \frac{2\pi}{N_{ss}}\right) du dv \right] \sum_{m=-L}^{+L} e^{-j(v-u)m} \qquad (7)$$

where the weighting window $d_{\mathbf{V}}(m)$ in (6) results to be an all ones sequence for the case of perfect carrier frequency knowledge ($\Delta_\nu = 0$). Moreover, note that $\sum_{m=-L}^{+L} e^{-j(v-u)m} = 0$, $u \neq v$ for a sufficiently large L. Thus, expression (7) results in $\Phi\left(\frac{2\pi}{N_{ss}}\right) = C_2 \int_{-\pi}^{+\pi} Y(u)$ $Y\left(\frac{2\pi}{N_{ss}} - u\right) du$ with $Y(\omega) = R(\omega) G^*(-\omega) \overset{FT}{\longleftrightarrow} r(k) * g(-k)$. Finally, the timing estimator in (6) can be rewritten as follows:

$$\widehat{\tau}_{WCM} = \frac{T}{2\pi} \arg\left\{ \sum_{k=-M}^{+M} |r(k) * g(-k)|^2 e^{-j\frac{2\pi}{N_{ss}}k} \right\} \qquad (8)$$

In the absence of carrier frequency error, this last expression shows that the optimal ML timing estimation derived in (6) based on the CAF correlation between the pulse shape and the received signal, can be equivalently achieved by passing the received signal through its matched filter $g_{MF}(k) = g(-k)$, squaring the result, and observing the argument of its Fourier Transform at the symbol rate.

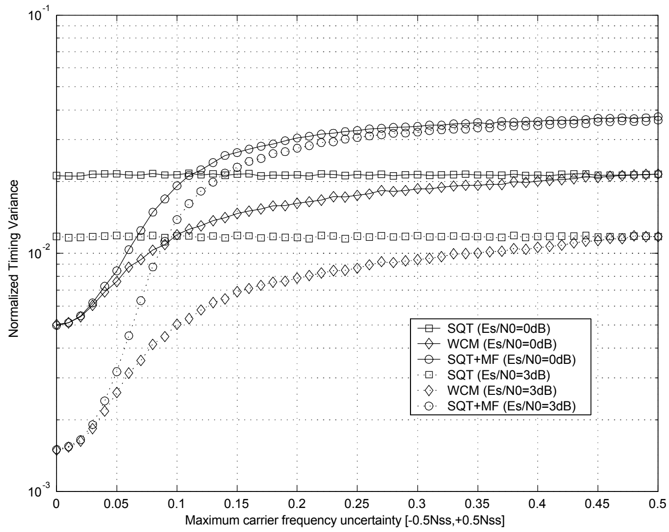

Fig. 1. Timing error variance for $|\Delta_\nu/2| \in [0, 0.5)\, N_{ss}$, with 50 % roll-off square root Nyquist pulse shape and $E_s/N_0 = \{0, 3\}$ dB.

5. LOW-SNR CRAMER RAO BOUND

The Cramer-Rao bound (CRB) is a useful benchmark for testing the performance of any unbiased estimator, as it constitutes a lower bound of the error variance: $CRB(\tau) = \left(E_r\left[-T^2 \frac{\partial^2}{\partial \tau^2} L(\mathbf{r}|\tau)\right]\right)^{-1}$ [11]. From (5)-(6), and noting that, $E_r\left[\widehat{R}_r^{\frac{2\pi}{N_{ss}}}(m)\right] = e^{-j\frac{2\pi}{T}} \mathrm{sinc}\left(\frac{\Delta_\nu}{N_{ss}} m\right)$ $\cdot R_s^{\frac{2\pi}{N_{ss}}}(m)$, the CRB for the problem at hand is given by:

$$CRB(\tau) = \left(\frac{16\pi^2}{\sigma_w^4 \sigma_x^2} \sum_{m=-L}^{+L} \mathrm{sinc}^2\left(\frac{\Delta_\nu}{N_{ss}} m\right) \left| R_g^{\frac{2\pi}{N_{ss}}}(m) \right|^2 \right)^{-1}$$

where it has been used the fact that for the noise CAF, $R_w^{\frac{2\pi}{N_{ss}}}(m) = 0$ as it is evaluated at a cycle-frequency different from $\alpha = 0$ [5]. This bound is useful for operation at very low EbN0 values, such as the required for coded transmission (e.g. Turbo Codes). As seen in figure 2, for EbN0 below this margin, outliers become predominant and the variance performance for this non-linear estimator collapses. Moreover, this limit is shown to best suit for small values of Δ_ν, as higher Δ_ν moves the variance curves to higher EbN0 values away from the low-SNR assumption.

6. SIMULATION RESULTS

Computer simulations have been carried out in order to demonstrate the performance of the Cyclic-Autocorrelation based NDA timing estimator in scenarios with some carrier frequency uncertainty. The simulation parameters include the employment a 16-QAM modulation with square root Nyquist pulses in an additive Gaussian noise channel, where $N_{ss} = 4$ samples per symbol have been selected, and an observation interval of typically $N = 64$ symbols. In addition, some results are also presented for the particular case of using a rectangular pulse shape, although in this case, some of the statements presented in section 3.1 may not completely apply due to the pulse bandwidth. That is, the harmonic decomposition of matrix $\mathbf{GG}^H$ in (4) into a FS can not be approximated by just taking the main term ($n = 1$) when using a rectangular pulse shape. However, the rectangular pulse shape

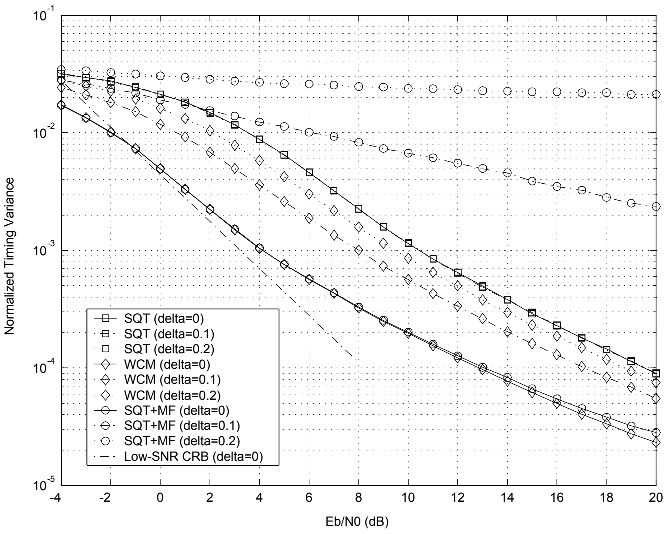

Fig. 2. Timing error variance as a function of E_b/N_0 for $\Delta_\nu = \{0, 0.1, 0.2\}\ N_{ss}$ and 50 % roll-off.

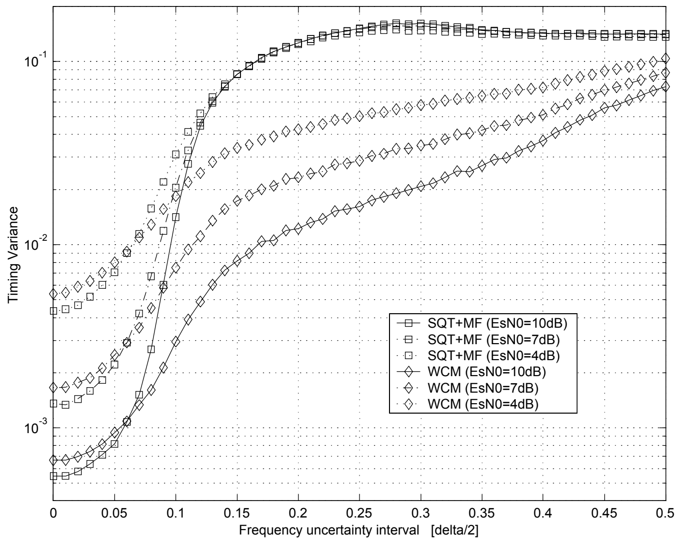

Fig. 3. Timing error variance as a function of the frequency error uncertainty interval $|\Delta_\nu/2| \in [0, 0.5)\ N_{ss}$, with rectangular pulse.

is commonly used in a wide range of applications, and the classical Square Timing recovery fails to provide any timing estimate because of the signal constant envelope, in contrast with the WCM.

- Figure 1 shows the variance evolution for the symbol timing error as a function of the carrier frequency error uncertainty. When the frequency uncertainty increases, the SQT at the output of the matched filter (SQT+MF) is seriously degraded due to the mismatch between the incoming signal and the receiving filter, but the WCM is able to overcome this effect. For values approaching the maximum uncertainty, the WCM timing estimator inherently becomes the classical Square Timing method, as discussed in section 4.1.

- Figure 2 shows the evolution for the timing error variance as a function of the E_b/N_0. The curves have also been plot for scenarios with carrier frequency uncertainty $\Delta_\nu = \{0, 0.1, 0.2\}\ N_{ss}$, and as in figure 1, it can be seen the convergence of the WCM towards the SQT.

- Figure 3 presents some results for the timing error estimation problem with rectangular pulse shape. A severe degradation is shown to appear in presence of carrier frequency errors due to the mismatch between the incoming signal and the receiving filter in the SQT+MF. This degradation is shown in figure 3 to be drastically diminished by using the WCM timing estimator, especially for carrier frequency uncertainty intervals with $|\Delta_\nu/2| > 0.1$.

7. CONCLUSIONS

This paper presented a new result to the basic synchronization problem of NDA symbol timing estimation. Following a Maximum Likelihood approach, it has been derived an optimal timing estimator for low-SNR in presence of a uniform *prior* distribution for the carrier frequency uncertainty. The resulting timing estimator is shown to include as a particular case, the classical Square Timing (Oerder&Meyr) algorithm. For the case of maximum carrier uncertainty, the CAF based timing estimator with $\Delta_\nu \to N_{ss}$ was proved to converge towards the Square Timing recovery with the received signal. Similarly, for the case of perfect carrier frequency knowledge, the resulting CAF with $\Delta_\nu \to 0$ attained the performance of the Square Timing applied at the output of a matched filter. Finally, for intermediate cases, the CAF based estimator provides the optimal ML NDA timing estimation.

8. REFERENCES

[1] U. Mengali and A. N. D'Andrea, *Synchronization Techniques for Digital Receivers*, Plenum Press, 1997.

[2] P. Ciblat et Al., "Non-data aided feedforward cyclostationary statistics based carrier frequency offset estimators for linear modulations," *Proceedings of Globecom 2001*, pp. 1386–1390.

[3] U. Mengali M. Morelli and G. M. Vitetta, "Further results in carrier frequency estimation for transmissions over flat fading channels," *IEEE Commun. Letters*, vol. 2, no. 12, December 1998.

[4] F. Gini and G. B. Giannakis, "Frequency offset and symbol timing recovery in flat-fading channels: A cyclostationary approach," *IEEE Trans. Commun*, vol. 46, no. 3, pp. 400–411, March 1998.

[5] W. A. Gardner, *Statistical Spectral Analysis. A Non Probabilistic Theory*, Prentice-Hall, 1988.

[6] M. Oerder and H. Meyr, "Digital filter and square timing recovery," *IEEE Transactions on Communications*, vol. 36, no. 5, pp. 605–612, May 1988.

[7] G. Vazquez and J. Riba, *Signal Processing Advances in Wireless and Mobile Communications,* by G.B. Giannakis, Y. Hua, P. Stoica and L. Tong, vol. 2 (Trends in Single- and Multi-User Systems), Prentice Hall, July 2000, Chapter 9, pp. 357-402.

[8] A. V. Dandawaté and G. B. Giannakis, "Asymptotic theory of mixed time averages and kth-order cyclic moment and cumulant statistics," *IEEE Trans. Inform. Theory*, vol. 41, no. 1, pp. 216 – 232, January 1995.

[9] W. A. Gardner, "Exploitation of spectral redundancy in cyclostationary signals," *IEEE Signal Processing Magazine*, pp. 14 – 36, April 1991.

[10] J. Riba and G. Vàzquez, "Bayesian recursive estimation of frequency and timing exploiting the cyclostationarity property," *EURASIP Signal Processing*, vol. 40, no. 1, pp. 21–37, 1994.

[11] H. Steendam and M. Moeneclaey, "Low-SNR limit of the cramer-rao bound for estimating the time delay of a PSK, QAM or PAM waveform," *IEEE Commun. Letters*, vol. 5, no. 1, pp. 31–33, January 2001.