EFFECT OF PARTIAL BAND JAMMING ON OFDM-BASED WLAN IN 802.11G

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ABSTRACT

One WLAN cell results in unintentional interference to the adjacent cells using the channel of which center frequency is insufficiently separated. The interference affects the adjacent cells as partial band jamming (PBJ). This paper presents the effect of partial band jamming on orthogonal frequency division multiplexing (OFDM) systems. PBJ is modeled on the rectangular spectrum with uniformly distributed random phase over [0, 2π). An upper bound of bit error probability (BEP) performance based on the effective signal to noise power ratio (SNR) is provided for OFDM systems in Rayleigh fading channels in addition to AWGN. Finally, coding gain over partial band jamming is analyzed in OFDM-based wireless local area network (WLAN).

1. INTRODUCTION

Recently OFDM-based WLAN is adopted as a mandatory mode in IEEE 802.11g for enabling the transmission of higher data rate. Unlike 802.11a occupying 5GHz frequency band, 802.11g operates in 2.4GHz industrial, scientific and medical (ISM) band. Therefore OFDM-based WLAN in 802.11g inevitably undergo many interferences - collocated WPANs, microwave ovens, and existing WLANs. Previous works have been studied the need of coexistence [1] [2] and the performance degradation of frequency hopping (FH) systems resulting from band or multitone jamming [3] [4].

Whole frequency band from 2401MHz to 2483MHz is available for 802.11g. This frequency band divides into the 13 channels with the center frequency of 5MHz interval-2412MHz, 2417MHz and so on. One channel for 802.11g occupies approximately 20MHz. In one WLAN cell, the channel not overllaped with other WLANs is assigned. In the adjacent cells, however, WLANs using insufficiently separated frequency channels cause unintentional interference. This interference can be regarded as partial band jamming (PBJ), which has the spectrum with uniformly distributed random phase over [0, 2π). In this paper, the effect of PBJ on OFDM systems is analyzed in Rayleigh channels in addition to AWGN. Analytical aspects of BEP performance is obtained based on effective SNR. Moreover coding gain over PBJ is investigated for OFDM-based WLAN.

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2. PBJ MODELING AND EFFECTIVE SNR

One cell using the channel overlapped with neighbor cells causes unintentional interference, which can be regarded as partial band jamming. The signal causing interference is assumed to use single carrier modulation, which shaping filter is sinc function in time domain. Fig.1 shows the power spectral density (PSD) of OFDM signal, which center frequency is f_d apart from that of interference. The shaded area indicates the collision in frequency domain which can be modeled as PBJ. Note that not all jamming power but part of them is considered from the viewpoint of OFDM signal, and that the effective jamming power on each OFDM subcarrier is different. Additionally, the phase of jamming signal at each subcarrier is the uniformly distributed random variable over [0, 2π). Assuming one OFDM symbol is rectangular, the PSD of OFDM signal is represented by

$$G_S(f) = \sum_{k=-N/2}^{N/2-1} \frac{P_{sub}}{f_s} sinc^2\Big(f/f_s - k\Big), \qquad (1)$$

where N is the number of subcarriers, P_{sub} is the power of one OFDM subcarrier and f_s is the subcarrier spacing. The PSD of interference signal is represented by

$$G_J(f) = \frac{P_J}{f_W} rect\Big((f - f_d)/f_W\Big), \qquad (2)$$

where P_J and f_W are the power and the bandwidth of interference signal. The distance of center frequency between

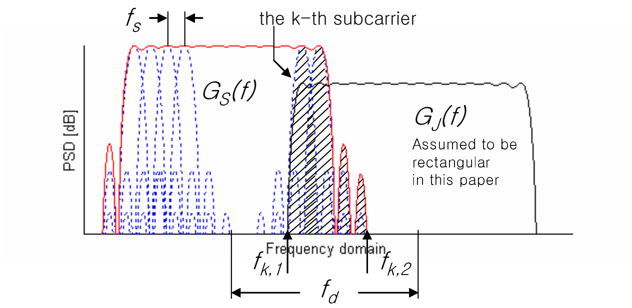

Fig. 1. Frequency spectrum of OFDM and interference signal insufficiently separated

OFDM signal and interference is f_d. In this paper it is assumed that the bandwidth of interference is identical to that of OFDM sigal, $f_W = Nf_s$.

For the k-th subcarrier the effective signal to noise power ratio is defined as

$$\begin{aligned}\tilde{\gamma}_k &= \frac{|\int_{-\infty}^{\infty} S_k(f)H(f)e^{j2\pi ft}df|^2}{\int_{-\infty}^{\infty} G_J(f)|H(f)|^2df + \int_{-\infty}^{\infty} G_N(f)|H(f)|^2df} \\ &= \frac{|\int_{-\infty}^{\infty} S_k(f)H(f)e^{j2\pi ft}df|^2}{\frac{P_{sub}P_J}{f_W f_S}\int_{f_{k,1}}^{f_{k,2}} \frac{1}{f_s} sinc^2(f/f_s)df + \frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2df},\end{aligned} \tag{3}$$

where $S_k(f)$ is the frequency response of the k-th subcarrier and $H(f)$ is the frequency response of the matched filter of $S_k(f)$. For the k-th subcarrier, PBJ affects it from $f_{k,1}$ to $f_{k,2}$. Consequently the effective SNR of the k-th subcarrier is bounded by using the Cauchy-Schwarz inequality

$$\begin{aligned}\tilde{\gamma}_k &\le \frac{\int_{-\infty}^{\infty} |S_k(f)|^2df \int_{-\infty}^{\infty} |H(f)|^2df}{\frac{P_{sub}P_J}{f_W f_S}\rho_k + \frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2df} \\ &= \frac{1}{\left(\frac{P_S}{\rho_k P_J}\right)^{-1} + \left(\frac{2E_b}{No}\right)^{-1}},\end{aligned} \tag{4}$$

where P_S is the total power of OFDM signal, $P_S = NP_{sub}$, and ρ_k is the fraction of the PBJ power overlapped with the k-th subcarrier,

$$\rho_k = \int_{f_{k,1}}^{f_{k,2}} \frac{1}{f_s} sinc^2(f/f_s)df. \tag{5}$$

3. EFFECT OF PARTIAL BAND JAMMING ON OFDM SYSTEMS

3.1. AWGN channels

As shown in Fig.1 the effective SNR, $\tilde{\gamma}_k$, is different for all subcarriers $k = 0, ..., N-1$. For OFDM systems, therefore, the BEP performance is obtained by averaging BEP for all N subcarriers. When BPSK modulation is assumed, the BEP performance of the k-th subcarrier in AWGN channels is obtained by

$$P_{b,k} \ge Q\left(\sqrt{\gamma_k}\right), \tag{6}$$

where γ_k denotes the maximum effective SNR. Consequently the average BEP performance of OFDM systems in AWGN channels is obtained by

$$P_b = \frac{1}{N}\sum_{k=0}^{N-1} P_{b,k}. \tag{7}$$

3.2. Rayleigh fading channels

In Rayleigh fading channels the OFDM signal and jamming signal are both independently attenuated. Considering the attenuation of both signals the effective SNR is achieved by

$$\gamma_k \le \frac{1}{\left(\frac{\alpha^2 P_S}{\beta^2\rho_k P_J}\right)^{-1} + \left(\frac{2\alpha^2 E_b}{N_0}\right)^{-1}}, \tag{8}$$

where α and β are independent random variables with the probability density function (PDF) of Rayleigh distribution. Denoting the conditional BEP with the instantaneous effective SNR of the k-th subcarrier by $P_b(\gamma)$, the average BEP in the presence of Rayleigh fading is obtained from

$$P_{b,k} = \int_0^{\infty} p_k(\gamma)P_b(\gamma)d\gamma, \tag{9}$$

where $p_k(\gamma)$ is the PDF of the instantaneous effective SNR. In Eq.(8) let us define $c_1 \equiv \frac{P_S}{\rho_k P_J}$ and $c_2 \equiv \frac{E_b}{N_0}$. Then the PDF of the instantaneous effective SNR is obtained from the transformation of the random variables, α and β.

$$\begin{aligned}p_k(\gamma) &= \int_{-\infty}^{\infty} |\gamma_b| p_{\gamma_a,\gamma_b}(\gamma\gamma_b, \gamma_b)d\gamma_b \\ &= \int_0^{\infty} \frac{\gamma_b}{2\sigma_1^2}e^{-\gamma\gamma_b/2\sigma_1^2}\frac{c_1}{2\sigma_2^2}e^{-c_1(\gamma_b-1/c_2)/2\sigma_2^2}d\gamma_b \\ &= \frac{c_1 e^{c_1/(2c_2\sigma_2^2)}}{4\sigma_1^2\sigma_2^2}\int_0^{\infty} \gamma_b e^{-\left(\frac{\gamma}{2\sigma_1^2}+\frac{c_1}{2\sigma_2^2}\right)\gamma_b}d\gamma_b,\end{aligned} \tag{10}$$

where $\gamma_a \equiv \alpha^2$ and $\gamma_b \equiv \frac{\beta^2}{c_1} + \frac{1}{c_2}$, which PDF $p_{\gamma_a}(\gamma_a)$ and $p_{\gamma_b}(\gamma_b)$ are shown in the APPENDIX. Using the equality of $\int_0^{\infty} x^n e^{-ax}dx = \frac{\Gamma(n+1)}{a^{n+1}}$, Eq.(10) can be induced as

$$\begin{aligned}p_k(\gamma) &= \frac{c_1 e^{\frac{c_1}{2c_2\sigma_2^2}}}{4\sigma_1^2\sigma_2^2}\frac{1}{\left[\frac{1}{2}\frac{\gamma\sigma_2^2+c_1\sigma_1^2}{\sigma_1^2\sigma_2^2}\right]^2} \\ &= \frac{c_1\sigma_1^2\sigma_2^2}{(\gamma\sigma_2^2+c_1\sigma_1^2)^2}e^{\frac{c_1}{2c_2\sigma_2^2}},\end{aligned} \tag{11}$$

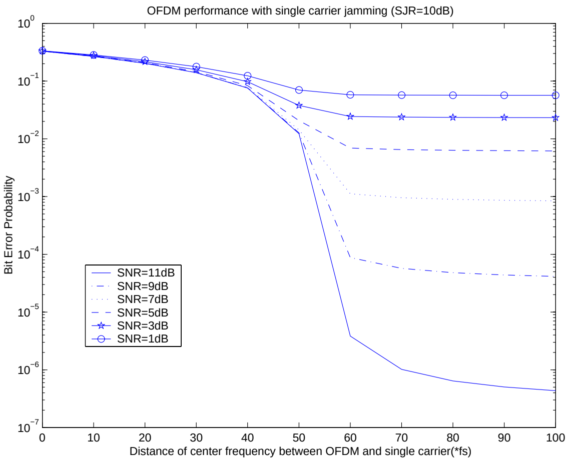

Fig. 2. BEP performance of OFDM systems with PBJ in AWGN channels (SJR=10dB)

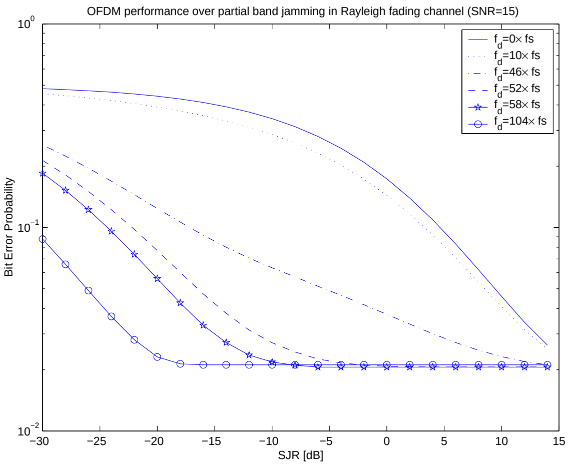

Fig. 3. BEP performance of OFDM systems with PBJ in Rayleigh fading channels (SNR=15dB)

where σ_1^2 and σ_2^2 are the variances of α and β, respectively, which can be assumed to be unity without any loss of generality. Therefore the average BEP for the k-th subcarrier in Rayleigh fading channels is obtained by

$$\begin{aligned} P_{b,k} &\geq \int_0^\infty p_k(\gamma)P_b(\gamma)d\gamma \\ &= \int_0^\infty \frac{c_1 e^{c_1/2c_2}}{(\gamma+c_1)^2} Q\left(\sqrt{\gamma}\right)d\gamma \\ &= \frac{c_1}{2}e^{c_1/2c_2}\sqrt{\frac{1}{c_1}}\left(\sqrt{\frac{1}{c_1}}+Q\left(\sqrt{c_1}\right)\right), \end{aligned} \quad (12)$$

The effective SJR constant c_1 is different for all subcarriers. Consequently the BEP performance of OFDM systems in the Rayleigh faded PBJ environments is obatined by

$$P_b \geq \frac{1}{N}\sum_{k=0}^{N-1} P_{b,k}. \quad (13)$$

Fig.2 and Fig.3 show the BEP performance of OFDM systems using the 52 subcarriers with the PBJ environments in AWGN and Rayleigh fading channels, respectively. In Fig.2 it is shown that the effect of PBJ decreases as the distance of the center frequency, f_d, increases. The performance is poor before f_d arrives at around $52 \times f_s = f_W$. On the other side, the effect of PBJ is barely found over $60\times f_s$ and the performance approaches that in only AWGN channel. This is because that the main lobe of the PSD of any subcarrier is not under the influence of PBJ. Fig.3 illustrates the results for the Rayleigh faded PBJ environments. The value of f_d lower $52 \times f_s$ shows very poor results. As SJR increases and f_d increases, and the BEP performance is mainly dependent on SNR. Finally the BEP performance converges on the performance dependent on only SNR.

4. CODING GAIN OVER PBJ

To get insight into coding gain for OFDM systems over PBJ, the BEP performance of OFDM-based WLAN is simply analyzed by the computer simulations. Fig.4 shows the overall block diagram of OFDM-based WLAN. The BPSK modulation and the 1/2 rate convolutional encoder with the generator ploynomials of $g_0 = 133_8$ and $g_1 = 171_8$ are used according to the 6Mbps mode in OFDM-based 802.11g. In this paper any frequency offset is not considered. As our earlier discussion PBJ is modeled on the rectangular spectrum with uniformly distributed random phase over $[0, 2\pi)$ and PBJ has the total power of P_J. Rayleigh faded PBJ is added to OFDM signal at the output of IFFT. In 802.11g 64 subcarriers are adopted in total. Only 52 subcarriers, however, are actually used to modulate the input data and pilots. Therefore $52 \times f_s$ is assigned to PBJ for the bandwidth. Fig.5 shows the BEP performance of OFDM-based WLAN in the Rayleigh fading PBJ environments. When the distance of center frequency f_d becomes around $52 \times f_s$, the BEP decreases rapidly. This means that OFDM systems can overcome the PBJ, which is not bring about the burst errors, by coding gain. On the reverse, coding gain cannot be expected to overcome PBJ when PBJ occupies some part of the OFDM bandwidth so that the burst errors may appears. Note that PBJ affects the channel estimation, also. The noticeable improvement of BEP performance is not found for the case of $f_d = 46\times f_s$ because of channel estimation error from PBJ.

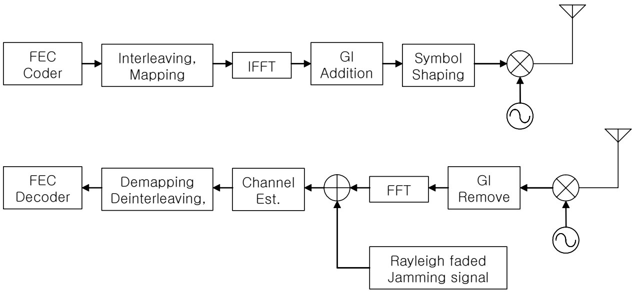

Fig. 4. The block diagram of OFDM-based WLAN in Rayleigh faded PBJ environments

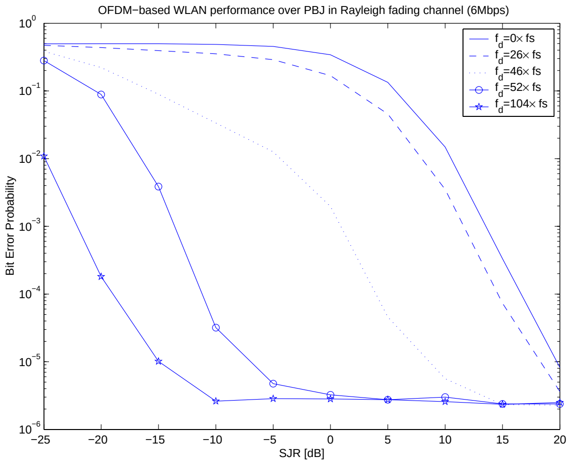

Fig. 5. OFDM-based WLAN performance over PBJ in Rayleigh fading channels (6Mbps)

5. CONCLUSION

Unintentional interference from the overlapped channel in WLANs was modeled as PBJ. The effect of PBJ was numerically analyzed for OFDM systems in AWGN and Rayleigh fading channels. The BEP performance of OFDM systems was obtained by averaging the BEP performance of N subcarriers because the effective SNR is different for all subcarriers. In Rayleigh fading channels, both OFDM signal and jamming signal are independently attenuated. The effective SNR, therefore, was achieved through the transform of two independent random variables. Consequently the compact expression of BEP performance of OFDM systems was presented in Rayleigh faded PBJ environments. Additionally coding gain over PBJ is simply analyzed by computer simulations. Remarkable coding gain was achieaved when PBJ doesn't result in the burst errors.

APPENDIX

Derivation of PDF expression of γ_a and γ_b

Let us define $\gamma_a \equiv \alpha^2$ and $\gamma_b \equiv \frac{\beta^2}{c_1} + \frac{1}{c_2}$. Two independent random variables, α and β, have the PDF of Rayleigh distribution,

$$p_\alpha(\alpha) = \alpha/\sigma_1^2 e^{-\alpha^2/2\sigma_1^2}$$
$$p_\beta(\beta) = \beta/\sigma_2^2 e^{-\beta^2/2\sigma_2^2}. \quad (14)$$

For the transform of $y = ax^2$ the PDF of y is obtained by

$$p_y(y) = \frac{1}{2a\sqrt{y/a}}\left[p_x\left(\sqrt{\frac{y}{a}}\right) + p_x\left(-\sqrt{\frac{y}{a}}\right)\right] \quad y > 0, \quad (15)$$

and for the transform of $y = ax + b$ the PDF of y is obtained by

$$p_y(y) = \frac{1}{|a|} p_x\left(\frac{y-b}{a}\right). \quad (16)$$

Consequently the PDF of γ_a and γ_b are

$$p(\gamma_a) = \frac{1}{2\sigma_1^2} e^{-\frac{\gamma_a}{2\sigma_1^2}}$$
$$p(\gamma_b) = \frac{c_1}{2\sigma_2^2} e^{-\frac{c_1(\gamma_b - 1/c_2)}{2\sigma_2^2}}. \quad (17)$$

6. REFERENCES

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