OPTIMUM BIORTHOGONAL DMT SYSTEMS FOR MULTI-SERVICE COMMUNICATION

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ABSTRACT

This paper considers the design of biorthogonal DMT multicarrier transceiver systems supporting multiple services. The supported user services may have differing quality of service (QoS) requirements, quantified in this paper by bit rate and symbol error rate specifications. To reflect their service priorities, different users on the system can be potentially assigned different number of subchannels. Our goal is to minimize the transmitted power given the QoS specifications for the different users, subject to the knowledge of colored interference at the receiver input of the DMT system. In particular we find an optimum bit loading scheme that distributes the bit rate transmitted across the various subchannels belonging to the different users, and subject to this bit allocation, determine an optimum transceiver. This work differs from our prior work where the same number of subchannels were assigned to each user.

1. INTRODUCTION

Future broadband communication systems will be expected to deliver multiple services, such as voice, data, video, with multiple-stream support. Because delivery of these streams will be under differing requirements such as information rate and error performance, allocation of critical resources like power would have a significant impact on the overall performance of the communication system. Discrete Multitone (DMT) transmission involves a channel coding technique to achieve reliable, high data rate communications in such systems. It is a current standard in various wireline applications like ADSL, VDSL, [11], and in the form of Orthogonal frequency division multiplexing (OFDM) has been proposed for fixed wireless standards like IEEE 802.11a. This paper considers transceiver optimization for such multicarrier transmission systems operating in a multiuser environment.

More specifically, we assume that a single DMT system supports r users, each having its own QoS specification quantified by its bit rate and symbol error rate (SER). The k-th user is assumed to have been assigned n_k subchannels, and requires a bit rate of t_k, and an SER of no more than η_k. The number of subchannels assigned to each user is fixed *a priori* according to some priorities determined by the user service, and *may vary from user to user.* As proposed in several recent papers, [7], [6], [10], we consider general DMT transceivers which are more general than the traditional DFT based systems in that the input and output transforms are general block transforms. We consider biorthogonal systems employing zero padding redundancy with the redundancy removal at the receiver being a general linear operation. Our goal is to select the input and output block transforms G_0, S_0 (see fig. 1), the linear operation reflecting redundancy removal, the number of bits/symbol assigned to each subchannel, and the subchannels assigned to each user to achieve the QoS specifications, under a zero intersymbol interference (ISI) condition with the minimum possible transmitted power. We assume that the channel and equalizer are known and so is the interference autocorrelation.

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We thus generalize our earlier result reported in [6], where the same optimization problem was considered with an *orthonormal transceiver* under the assumption that each user is assigned the *same number of subchannels*. In [8], the same problem as [6] was considered without the assumption of orthonormality. The novelty of this paper lies in both relaxing the orthonormality condition, i.e. considering instead the *biorthogonal case* and permitting different users to be allocated potentially *unequal number of subchannels*. We shall see in the following sections how the extension to [6], [8] considered here nontrivially modifies the optimization problem. We note that the asymmetric subchannel allocation considered here is more realistic as service priorities may cause certain users to receive greater number of subchannels than others. For example, one may allocate more subchannels to video services than to audio services.

Figure 1 depicts a DMT system. An incoming data stream is converted into M-parallel data streams of lower rate. An M-point block transformation G_0, of these streams of data is followed by a parallel-to-serial conversion, prior to transmission through the communication channel. An equalizer is employed to shorten the dispersive effects of the transmission channel. The *equalized* channel $C(z)$ is assumed to be FIR of length κ. For an FIR equalized channel of length κ, extra redundancy of length κ in the form of zero padding is added at the channel input to infuse resistance to channel induced ISI. At the channel output, one performs in succession the operations of redundancy removal, serial-to-parallel conversion, and the application of an inverse block transform, S_0.

Past treatment of optimum resource allocation, [1], [2], [3], has been restricted mostly to bit loading and power allocation algorithms. Some authors have studied the optimum transceiver design in the single user case, [7], [10]. While [7] was concerned with optimizing the transmitted power, [10] focussed on the maximization of the mutual information between the transmitted and received signals. In [5] the authors consider the problem considered here for the single user case of $r = 1$, and with orthonormality condition enforced. In [7] the single user case is considered with orthogonality removed. Both [7] and [8] show in their respective cases, that biorthogonality leads to no improvement in the transmitted power. Likewise a major conclusion of this paper is to show that even in the multiuser environment with potentially asymmetric subchannel allocations, optimal performance is acheived by orthonormal transformations.

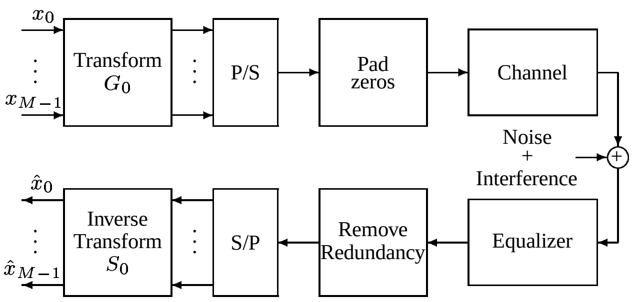

Fig. 1. DMT communication system.

2. FORMULATION

2.1. Preliminaries

Barring [7] and [8], most papers assume that G_0 is unitary, i.e.

$$G_0^H G_0 = I. \tag{2.1}$$

In the *biorthogonal case* considered here we *relax (2.1)* and simply assume that G_0 and S_0 can be *arbitrary nonsingular* $M \times M$ *matrices*. Denote the blocks of M input and output symbols respectively by $\mathbf{x}(n) = [x_0(n), \cdots, x_{M-1}(n)]^T$, and $\hat{\mathbf{x}}(n) = [\hat{x}_0(n), \cdots, \hat{x}_{M-1}(n)]^T$. With $v(n)$, the noise and interference effect at the output of the equalizer, denote $\mathbf{v}(n) = [v(Nn), v(Nn+1), \cdots, v(Nn+N-1)]^T$, as the N-fold blocked version of $v(n)$, with $N = M+\kappa$. Then one can show, [5] that with $\mathbf{C_L}$ an $N \times M$ constant matrix characterized by the κ order FIR equalized channel, and S_1, an $M \times N$ matrix, representing the linear redundancy removal operation, the blocked input-output relation of the system is given by

$$\hat{\mathbf{x}}(n) = S_0 S_1 \mathbf{C_L} G_0 \mathbf{x}(n) + S_0 S_1 \mathbf{v}(n). \tag{2.2}$$

We impose the perfect reconstruction (PR) condition, i.e., in the absence of noise/interference, $\hat{\mathbf{x}}(n) = \mathbf{x}(n)$ for all n. In other words,

$$S_0 S_1 \mathbf{C_L} G_0 = I, \tag{2.3}$$

and the DMT system has no ISI. To obtain a more useful characterization of PR, consider the singular value decomposition of $\mathbf{C_L}$

$$\mathbf{C_L} = U_c \begin{bmatrix} \Lambda_c \\ 0 \end{bmatrix} V_c^H = U_0 \Lambda_c V_c^H \tag{2.4}$$

where U_c and V_c are respectively $N \times N$ and $M \times M$ unitary matrices and Λ_c is a $M \times M$ real, positive definite diagonal matrix. Then, because of (2.3), given G_0, the class of all $S_0 S_1$ enforcing PR is completely characterized by

$$S_0 = G_0^{-1}. \tag{2.5}$$

and

$$S_1 = V_c \Lambda_c^{-1} \begin{bmatrix} I_M & A \end{bmatrix} U_c^H, \tag{2.6}$$

where A is any arbitrary $M \times \kappa$ matrix. In the sequel it will be useful to partition U_c as $U_c = [U_0 \quad U_1]$, where U_0 is $N \times M$ and U_1 is $N \times \kappa$.

Note, as V_c, U_c and Λ_c are supplied by the channel, the only quantities that need to be found to determine the transceiver completely are G_0 and A.

2.2. Problem formulation

As mentioned earlier the M subchannels are distributed among the r users with the k-th user allocated n_k subchannels. Thus consider disjoint subsets $\mathcal{I}_k \subset \{0, \ldots, M-1\}$ with $|\mathcal{I}_k| = n_k > 1$, and $\mathcal{I}_k \cap \mathcal{I}_j = \emptyset, k \neq j$. Subchannel assignment to the k-th user constitutes determining $\mathcal{I}_k$. We assume that the j-th subchannel of the k-th user is assigned $b_{j,k}$ bits per symbol. To meet the bit rate specification for the k-th user one requires that

$$\frac{1}{N} \sum_{j \in \mathcal{I}_k} b_{j,k} = t_k. \tag{2.7}$$

Let the input power in the j-th subchannel of the k-th user be $\sigma^2_{x_{j,k}}$. Assume that $\sigma^2_{e_{j,k}}$ is the noise power in this subchannel. Under high SNR most modulation schemes, [9], require that to achieve a given SER the required SNR is proportional to $2^{b_{j,k}}$. More precisely,

$$\sigma^2_{x_{j,k}} = d_k 2^{b_{j,k}} \sigma^2_{e_{j,k}}, \tag{2.8}$$

where the constant d_k depends on the desired SER, η_k, for the k-th user. For example, for QAM, $d_k = \frac{1}{3}[Q^{-1}(\frac{\eta_k}{4})]^2$. Under this framework, the transmitted power for the biorthogonal DMT system is given by

$$P_B = \sum_{k=1}^{r} \sum_{j \in \mathcal{I}_k} \sigma^2_{x_{j,k}} [G_0^H G_0]_{jj} \tag{2.9}$$

$$= \sum_{k=1}^{r} \sum_{j \in \mathcal{I}_k} d_k 2^{b_{j,k}} \sigma^2_{e_{j,k}} [G_0^H G_0]_{jj}. \tag{2.10}$$

Define R_v denoting the *known* autocorrelation matrix of the noise vector $\mathbf{v}(n)$, and

$$R_e = S_0 R_w S_0^H \quad \text{and} \quad R_w = S_1 R_v S_1^H. \tag{2.11}$$

Then $\sigma^2_{e_{j,k}}$ are the diagonal elements of R_e, the autocorrelation matrix of the output noise vector $\mathbf{e}(n)$. Thus, because of (2.5), (2.10) can be rewritten as

$$P_B(S_0) = \sum_{k=1}^{r} \sum_{j \in \mathcal{I}_k} d_k 2^{b_{j,k}} [S_0^{-H} S_0^{-1}]_{jj} [S_0 R_w S_0^H]_{jj}. \tag{2.12}$$

Thus the optimization problem becomes: Given R_v, n_k, η_k, t_k, minimize (2.12) subject to (2.7) by selecting $b_{j,k}$ (bit loading), selection of $\mathcal{I}_k$ (subchannel assignment), S_0 (transformation selection) and because of (2.6), A (redundancy removal selection).

We show that there is a conceptual separation between the three selections, i.e. the optimizing A is determined exclusively by R_v, provided by the knowledge of the interference and equalizer characteristics; S_0 is determined entirely by A and the channel characteristics; $\mathcal{I}_k$ are determined entirely by R_e, in turn provided by S_1 and S_0, and the bit allocations are determined once the above quantities are found. Further as noted in the introduction, we will show that without loss of generality, the optimizing S_0, G_0 are unitary.

3. OPTIMUM SELECTIONS

In this section we consider the selection of the various variables.

3.1. Optimum Bit Loading

From the Arithmetic Mean-Geometric Mean (AM-GM) inequality that states that the Arithmetic Mean, exceeds the Geometric mean, with equality if all samples are equal, we have that *for a given choice of $\mathcal{I}_k$ and S_0, under (2.7),*

$$P_B \geq P_{BOPT} = \sum_{k=1}^{r} d_k \left[2^{Nt_k} \prod_{j \in \mathcal{I}_k} [S_0^{-H} S_0^{-1}]_{jj} [S_0 R_w S_0^H]_{jj} \right]^{1/n_k} \quad (3.13)$$

with equality iff for all k and i, j

$$2^{b_{j,k}} [S_0^{-H} S_0^{-1}]_{jj} [S_0 R_w S_0^H]_{jj} = 2^{b_{i,k}} [S_0^{-H} S_0^{-1}]_{ii} [S_0 R_w S_0^H]_{ii}. \quad (3.14)$$

This is in turn equivalent to the optimum bit loading rule:

$$b_{j,k} = \frac{Nt_k}{n_k} - \log_2 \left[\frac{\sigma^2_{e_{j,k}} [G_0^H G_0]_{jj}}{(\prod_{j \in \mathcal{I}_k} \sigma^2_{e_{j,k}} [G_0^H G_0]_{jj})^{1/n_k}} \right]. \quad (3.15)$$

Note that P_{BOPT} is much more complicated than its specializations, $r = 1$, studied in [5] and, $n_k = L$ for all k, studied in [6]. Thus, under optimum bit loading the remaining variables must be selected to minimize P_{BOPT}. *Observe, that while the choice of these other variables impacts the selection of $b_{j,k}$, P_{BOPT} itself is independent of $b_{j,k}$. This underscores the fact that the remaining variables can be selected regardless of the precise values of $b_{j,k}$ obtained through (3.15).*

3.2. Selection of S_0, $\mathcal{I}_k$ and A

Assume for the moment that A and hence S_1 has been selected and that the resulting positive definite Hermitian R_w has the SVD:

$$R_w = U \Lambda^2 U^H \quad (3.16)$$

with Λ real, diagonal and U unitary. The goal is to select S_0 and $\mathcal{I}_k$ to minimize P_{BOPT}.

For convenience we first work with the minimization of

$$J(S_0) = \sum_{i=0}^{M-1} \alpha_i [S_0 R_w S_0^H]_{ii} [S_0^{-H} S_0^{-1}]_{ii} \quad (3.17)$$

given positive α_i. Note $J(S_0)$ has the form of P_B.

It is noteworthy that in all the papers [5]-[8], the $S_0 = PU^H$, with P a permutation matrix miminimizes P_{BOPT}. If S_0 is restricted to be unitary, then [12] shows that this choice of S_0 also minimizes (3.17). Consider, however, the example where $\alpha_i = 1$, $M = 2$ and $R_w =$ diag $\{9, 1\}$. Then observe that $J(I) = 10$ but with

$$B = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{3} & 0 \\ 0 & 1 \end{bmatrix}$$

$J(B) = 8$. Thus in general $S_0 = U^H$ does not minimize (3.17). However, we will show in the sequel that it *does minimize P_{BOPT}*.

The following result shows that the search space of S_0 can be restricted to a particular form.

Lemma 3.1 *For some unitary V, (3.17) is minimized by*

$$S_0 = V \Lambda^{-1/2} U^H \quad (3.18)$$

and (3.17) becomes

$$J(S_0) = \sum_{i=0}^{M-1} \alpha_i \left[V \Lambda V^H \right]^2_{ii} \quad (3.19)$$

Denote $\beta_k = d_k 2^{Nt_k/n_k}$ and $\mu_j = \left[V \Lambda V^H \right]_{jj}$. Then under optimum bit loading it suffices to restrict the search of S_0 to (3.18) and to seek to minimize under unitary V:

$$P_B^\star = \sum_{k=1}^{r} \beta_k \prod_{j \in \mathcal{I}_k^\star} \mu_k^{2/n_k} \quad (3.20)$$

with $\mathcal{I}_k^\star$ defining the optimal arrangement of the sequence of μ_k.

Before proceeding, we need a few results from the theory of majorization, [4].

Definition 3.1 *Consider two sequences $x = \{x_i\}_{i=1}^n$ and $y = \{y_i\}_{i=1}^n$ with $x_i \geq x_{i+1}$ and $y_i \geq y_{i+1}$. Then we say that y majorizes x, denoted as $x \prec y$, if $\sum_{i=1}^k x_i \leq \sum_{i=1}^k y_i$ holds for $1 \leq k \leq n$, with equality at $k = n$. We say that y weakly supermajorizes x, denoted $x \prec^W y$, if $\sum_{i=j}^n x_i \geq \sum_{i=j}^n y_i, 1 \leq j \leq n$.*

Fact 1 *If H is an $n \times n$ Hermitian matrix with diagonal elements $h = \{h_i\}_{i=1}^n$ and eigenvalues $\lambda = \{\lambda_i\}_{i=1}^n$, then $h \prec \lambda$.*

Definition 3.2 *A real valued function $\phi(z) = \phi(z_1, \ldots, z_n)$ defined on a set $\mathcal{A} \subset R^n$ is said to be Schur concave on $\mathcal{A}$ if $x \prec y$ on $\mathcal{A} \Rightarrow \phi(x) \geq \phi(y)$. ϕ is strictly Schur concave on $\mathcal{A}$ if strict inequality $\phi(x) > \phi(y)$ holds when x is not a permutation of y. Further if $x \prec^W y$ then also $\phi(x) > \phi(y)$.*

We will now state a theorem that results in a test for strict Schur concavity. We denote $\phi_{(k)}(z) = \frac{\partial \phi(z)}{\partial z_k}$.

Lemma 3.2 *Let $\phi(z)$ be a scalar real valued function defined and continuous on $\mathcal{D} = \{(z_1, \ldots, z_n) : z_1 \geq \ldots \geq z_n\}$, and twice differentiable on the interior of $\mathcal{D}$. Then $\phi(z)$ is Schur concave on $\mathcal{D}$ if $\phi_{(k)}(z)$ is increasing in k.*

The following Lemma provides an important property of $\mathcal{I}_k^\star$ the *optimum arrangement of the subchannels*.

Lemma 3.3 *Consider for integers $p, q \geq 2$,*

$$f = (\alpha \prod_{k=0}^{p-1} a_k)^{2/p} + (\beta \prod_{l=0}^{q-1} b_l)^{2/q}$$

with $\alpha, \beta, a_i, b_j > 0$, $a_i \geq a_{i+1}$ and $b_i \geq b_{i+1}$. Suppose for some i, j

$$a_i > b_j \text{ and } \frac{\partial f}{\partial a_i} > \frac{\partial f}{\partial b_j}. \quad (3.21)$$

Then $g = (\alpha \prod_{k=0, k \neq i}^{p-1} a_k . b_j)^{1/p} + (\beta \prod_{l=0, l \neq j}^{q-1} b_l . a_i)^{1/q} < f$. Further $\partial f / \partial a_i \leq \partial f / \partial a_{i+1}$ and $\partial f / \partial b_i \leq \partial f / \partial b_{i+1}$.

Thus from Lemma 3.3, any optimum arrangement for (3.20) requires that for all $\mu_k > \mu_l$, $\frac{\partial P_B^\star}{\partial \mu_k} < \frac{\partial P_B^\star}{\partial \mu_l}$. Under this condition, $P_B^\star$ is Schur concave. Thus from Fact 1, as $\{\left[V \Lambda V^H \right]_{ii}\}_{i=0}^{M-1} \prec \{\lambda_0, \ldots, \lambda_{M-1}\}$, the choice of V as a permutation matrix that enforces an optimum arrangement of subchannels, minimizes (3.20). Thus to within a permutation matrix P, under optimum bit allocation, one can choose as an optimizing $S_0 = P \Lambda^{-1/2} U^H$. Now note that for any diagonal nonsingular matrix Ω, $J(S_0) = J(\Omega S_0)$, and that for some diagonal matrix $\tilde{\Lambda}$,

$$S_0 = P \Lambda^{-1/2} U^H = \tilde{\Lambda}^{-1/2} P U^H.$$

Thus as in [5]- [8], the minimizing $S_0 = PU^H$ with P enforcing the optimum arrangement. Under these conditions

$$[S_0^{-H} S_0^{-1}]_{jj} [S_0 R_w S_0^H]_{jj}$$

and indeed $\sigma^2_{e_{j,k}}$ are the eigenvalues of R_w.

Thus, regardless of A the best S_0 is a Karhunen Loeve Transform of R_w, and the $\sigma^2_{e_{j,k}}$ equal the eigenvalues of R_w. From the comment on supermajorization made at the end of Definition 3.2, it follows that the optimizing A must be such that the set of resulting eigenvalues of R_w weakly supermajorize all possible sets of attainable eigenvalues. The optimizing A can then be shown to be given by, [6],

$$A = -U_0^H R_v U_1 \left(U_1^H R_v U_1 \right)^{-1}. \quad (3.22)$$

4. SIMULATION RESULTS

In this section, we compare the transmitting power of the DFT based DMT under no bit allocation and optimum bit allocation with an optimum unitary transceiver. We assume the equalized channel to be $C(z) = 1 + 0.5z^{-1}$, and a noise source $v(n)$ whose power spectral density is shown in fig. 2. We assume the DMT system supports two user services. The (i, j) on the x-axis of the plot indicates that user 1 and 2 were respectively allocated i, j number of channels. The plot shows that there is a 10 dB saving in transmit power with our design over the DFT based DMT under optimum bit allocation, and a 14 dB improvement over the conventional DMT with no optimum bit allocation.

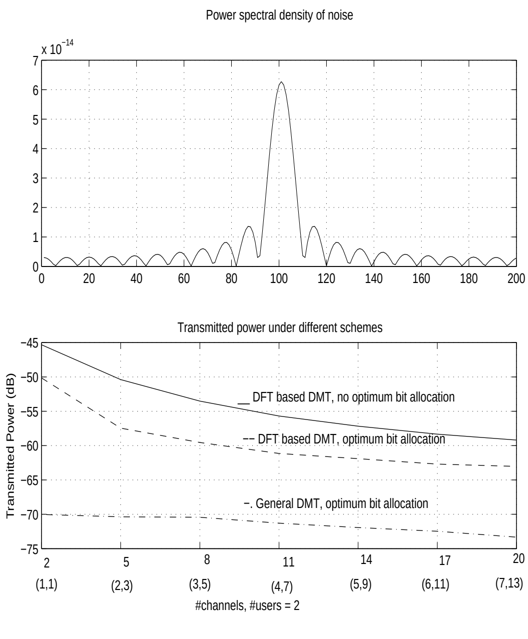

Fig. 2. Comparison of transmit power levels.

5. CONCLUSIONS

In this paper, an optimum bit allocation strategy and design of a general biorthogonal DMT multicarrier transceiver system employing zero padding redundancy were presented, for minimizing the transmit power when different users with varied QoS requirements are supported and are assigned potentially different number of subchannels. We showed that no gains in transmit power can be obtained by considering biorthogonal transceivers over orthogonal transceivers. These results also show that the optimum transceiver depends *only on the channel and interference conditions and not on the QoS requirements*. Indeed to within a permutation of subchannels, the optimum transceiver obtained here is identical to that obtained in [5]-[8]. Equally should the channel/interference remain invariant after the initial connection is established, then only bit loading and subchannel selection need be updated in response to changing traffic needs.

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