PERFORMANCE ANALYSIS OF MULTIUSER DETECTORS FOR MC-CDMA IN THE PRESENCE OF FREQUENCY OFFSET

Mashury

Curtin University of Technology
GPO Box U1987, Perth 6845, Australia
e-mail: mashury@ece.curtin.edu.au

ABSTRACT

A performance analysis of multiuser detectors for MC-CDMA over a fading channel in the presence of carrier frequency offset is presented in this paper. A theoretical analysis of the bit error probability of the detectors is derived. The theoretical analysis shows a close agreement to simulation results for a low value of the frequency offset and a high number of subcarriers. Based on the performance evaluation, multiuser detectors can tolerate only a small amount of frequency offset.

1. INTRODUCTION

Multicarrier code-division multiple access (MC-CDMA) has been proposed as one of the candidates for the next generation of wireless communication systems. MC-CDMA system is a combination of direct-sequence CDMA and multicarrier modulation systems. A drawback of MC-CDMA, which is due to the use of multicarrier modulation, is its sensitivity to a frequency offset. This frequency offset results from a Doppler shift, due to mobile movement, and a mismatch between the carrier frequencies at the transmitter and receiver [1, 2]. As a result of the offset, the subcarriers' orthogonality of one user is lost, causing inter-carrier interference (ICI). The frequency offset also causes a rotation of the constellation of the desired signal at each subcarrier, hence introducing inter-rail interference between the I- and Q-rails, further reducing the carrier to interference ratio (CIR) [3].

A performance analysis of the maximal-ratio combining (MRC) receiver on the MC-CDMA was conducted with the presence of frequency offset in [2] and without the frequency offset in [5]. In this paper, we develop a bit error rate (BER) analysis of the multiuser detectors for MC-CDMA in the presence of frequency offset. An alternative Gaussian approximation (AGA) [6] was applied in order to derive the BER of the detectors employing a maximal-ratio combining (MRC) equalisation.

The rest of this paper is organised as follows. In section II, the MC-CDMA system model in the presence of frequency offset is introduced. Section III contains performance analysis for the implemented multiuser detectors. Numerical and simulation results are presented in section IV. Section V contains conclusions of this paper.

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2. SYSTEM MODEL

2.1. Channel Model

In the evaluated MC-CDMA system, each user undergoes independent frequency-selective fading channel. It is assumed that each subcarrier has independent frequency non-selective Rayleigh (flat) fading. The equivalent time- varying complex low-pass impulse response of the fading channel for the kth user, nth subcarrier is given as:

$$H_{k,n}(t;\tau) = \beta_{k,n}(t)e^{j\theta_{k,n}(t)}\delta(\tau - \tau_k) \tag{1}$$

where $H_{k,n}(t;\tau)$ is a zero mean complex Gaussian random variable, τ_k is the propagation delay for the kth user and $\delta(.)$ is the Dirac delta function. The amplitudes $\beta_{k,n}(t)$ are independent and identical distributed (i.i.d.) Rayleigh random variables (r.v.'s) and the phase offsets due to channel fading $\theta_{k,n}(t)$ are identical r.v.'s uniformly distributed over $[0, 2\pi)$. For the downlink, $H_{k,n}$ is the same for all k, $k = 0, 1, \ldots, K-1$.

2.2. Receiver Model

The MC-CDMA received signal model for the performance analysis is described as follows.

$$\begin{aligned} r(t) \quad = \quad & \sqrt{2P/N}\sum_{k=0}^{K-1}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\beta_{k,n}(t)c_k(n)b_k(m) \\ & \times\cos(2\pi(f_c + n/T_s)t + \theta_{k,n}(t)) \\ & \times p_{T_s}(t - mT_s - \tau_k) + n(t) \end{aligned} \tag{2}$$

where P is the power of data bit, M is the packet size, $c_k[0], c_k[1], \ldots, c_k[N-1]$ represents the spreading code of the kth user, p_{T_s}(t) is the rectangular pulse defined over $[0, T_s]$. T_s is the symbol duration, $b_k(m)$ represents the input data bits, where m denotes the mth bit interval. The subcarrier spacing, Δ_f, is $1/T_s$. $c_k \in \{-1, +1\}$ and $b_k \in \{-1, +1\}$. $n(t)$ is the additive white Gaussian noise (AWGN) with a double-sided power spectral density of $N_o/2$.

Due to the assumption of flat fading on each subcarrier, the channel fading and phase shift variables are considered to be constant over a symbol duration and denoted by $\beta_{k,n}(m)$ and $\theta_{k,n}(m)$. Thus, $H_{k,n}(m) = \beta_{k,n} e^{j\theta_{k,n}(m)}$.

In the MC-CDMA system, an equaliser can be inserted in the frequency domain in order to improve the performance of the system. This can be done by using a one-tap frequency domain equaliser, which multiplies each subcarrier by a factor $G_{k,n}(m)$ in the mth bit interval. The Maximal ratio combining (MRC) scheme is applied for equalisation. This scheme is based on correcting the phase shift and weighting the received signal with the attenuation of the channel fading. The equalisation coefficient is given as:

$$G_{k,n}(m) = \hat{H}^*_{k,n}(m) = \beta_{k,n} e^{-j\theta_{k,n}(m)} \qquad (3)$$

In this paper, without the loss of generality, the signal from the first user is considered to be the desired signal and the signals from all other users are interfering signals. The carrier frequency offset, f_o, is introduced at the receiver because of the nonideal frequency down conversion. The decision variable v_0 of the mth data bit of the first user is given by:

$$\begin{aligned} v_0 &= \frac{1}{T_b} \int_{mT_b}^{(m+1)T_b} r(t) \sum_{n=0}^{N-1} c_0(n)\beta_{0,n}(m) \\ &\quad \times \cos(2\pi(f_c + n/T_s + f_o)t - \theta_{0,n}(m))dt \qquad (4) \end{aligned}$$

where T_b is the bit duration and, on the MC-CDMA system, one data bit occupies all subcarriers and the receiver is synchronised with the desired user $(k = 0)$.

3. PERFORMANCE ANALYSIS

In this paper, an alternative Gaussian approach (AGA) [6] is applied to evaluate the BER of the detectors on MC-CDMA. In this approach, the Q-function is presented as:

$$Q(x) = \frac{1}{\pi} \int_0^{\pi/2} \exp\left(-\frac{x^2}{2\sin^2\alpha}\right) d\alpha, \quad x \geq 0 \qquad (5)$$

Orthogonal Walsh-Hadamard codes are employed as spreading codes. Assuming that the users are time synchronous after demodulation and combining subcarrier signals, the decision variable in (4) can be written as

$$v_0 = S + I + \eta \qquad (6)$$

where S represents the desired signal term, I is the multiple access interference (MAI) from other users, and η is the AWGN term.

From (4) and (6), the desired signal is obtained as

$$S = \sqrt{(P/2N)} \sum_{n=0}^{N-1} b_0(m)\beta_{0,n}^2 \cos(2\pi f_o) \qquad (7)$$

where the frequency offset f_o is normalised by the subcarrier spacing Δ_f. Also from (4) and (6), it can obtained that η is a Gaussian random variable with zero mean and variance

$$\sigma_\eta^2 = \frac{N_o}{4T_b} \sum_{n=0}^{N-1} \beta_{0,n}^2 \qquad (8)$$

Then, the MAI term can be expressed as

$$\begin{aligned} I &= \sqrt{(P/2N)} \sum_{k=1}^{K-1} \sum_{n=0}^{N-1} b_k(m)c_k(n)c_0(n)\beta_{k,n}\beta_{0,n} \\ &\quad \times \cos(2\pi f_o + \tilde{\theta}_{k,n}) \qquad (9) \end{aligned}$$

where $\tilde{\theta}_{k,n} = \theta_{0,n} - \theta_{k,n}$. $\theta_{0,n}$ and $\theta_{k,n}$ are i.i.d. r.v.'s, uniformly distributed over $[0, 2\pi)$. The term $2\pi f_o$ only rotates the phase $\tilde{\theta}_{k,n}$. If we set $\phi_{k,n} = 2\pi f_o + \tilde{\theta}_{k,n}$ and $\phi_{k,n} \in [0, 2\pi)$. Then, $E[\cos(\phi_{k,n})] = 0$. Since $\beta_{k,n}$ and $\theta_{k,n}(k = 1, 2, \ldots, K-1)$ are i.i.d. r.v.'s, all $(K-1) \times N$ terms in the summation of (9) are uncorrelated with zero means. Without the presence of the near-far effect, the MAI can be approximated by a Gaussian r.v. with zero mean and variance

$$\sigma_I^2 = E[I^2] = \frac{P(K-1)}{4N} E[\beta_{k,n}^2] \sum_{n=0}^{N-1} \beta_{0,n}^2 \qquad (10)$$

where $E[\cos^2(\phi_{k,n})] = 1/2$. It can be seen that v_0 is a conditional Gaussian conditioned on $\beta_{0,n}$. Since η and I are mutually independent, the probability of error conditioned on $\beta_{0,n}$ is given by

$$Pr(error \mid \beta_{0,n}) = Q\left(\frac{S}{\sqrt{(\sigma_\eta^2 + \sigma_I^2)}}\right) \qquad (11)$$

To compute the average BER, the equation in (11) must be averaged statistically over the joint probability density function of the fading amplitudes. With the assumption of independent fading among different subcarriers, the average BER is derived using the alternative Q-function (5) as follows [6]

$$\begin{aligned} P_e &= \int_0^\infty \cdots \int_0^\infty \frac{1}{\pi} \int_0^{\pi/2} \exp\left(-\frac{\frac{S^2}{(\sigma_\eta^2+\sigma_I^2)}}{2\sin^2\alpha}\right) p_{\beta_{0,0}}(\beta_{0,0}) \\ &\quad \ldots, p_{\beta_{0,N-1}}(\beta_{0,N-1}) d\beta_{0,0} \ldots d\beta_{0,N-1} d\alpha \\ &= \frac{1}{\pi} \int_0^{\pi/2} \Pi_{n=0}^{N-1} I_{0,n}(\bar{\gamma}_{0,n}, \alpha) d\alpha \qquad (12) \end{aligned}$$

where $\bar{\gamma}_{0,n}$ is the average signal to interference plus noise ratio (SINR) for the nth subcarrier of the first user. In a Rayleigh fading channel, $I_{0,n}(\bar{\gamma}_{0,n}, \alpha)$ is given as [6]

$$I_{0,n}(\bar{\gamma}_{0,n}, \alpha) = \left(1 + \frac{\bar{\gamma}_{0,n}}{sin^2\alpha}\right)^{-1} \quad (13)$$

The average SINR $\bar{\gamma}_{0,n}$ can be obtained as follows

$$\bar{\gamma}_{0,n} = \frac{S^2}{\sigma_\eta^2 + \sigma_I^2} \quad (14)$$

When all N subcarrier are identically distributed with the same average SINR per bit, then (12) can be simplified into [5]

$$P_e = \frac{1}{\pi}\int_0^{\pi/2} I_{0,n}(\bar{\gamma}_{0,n}, \alpha)^N d\alpha \quad (15)$$

For a multiuser system, the average BER of the system is given by

$$BER = \frac{1}{K}\sum_{k=0}^{K-1} P_e(k) \quad (16)$$

3.1. Decorrelating Detector

The decorrelating detector applies the inverse of the correlation matrix of the users' spreading codes to the decision variable v_0 in (4) in order to decorrelate the data. The correlation matrix $\mathbf{R}$ is given by

$$\boldsymbol{R} = \boldsymbol{C}^T\boldsymbol{C} \quad (17)$$

where $\boldsymbol{C} = [\boldsymbol{c}_1, \ldots, \boldsymbol{c}_K]$ and $\boldsymbol{c}_1 = [c_1(0), c_1(1), \ldots, c_1(N-1)]^T$. Due to the use of an orthogonal Walsh-Hadamard code as the spreading code, the correlation matrix $\mathbf{R}$ contains a value of one along its diagonal and zero on each off-diagonal element, i.e., $\mathbf{R} = diag(1, 1, \ldots, 1)$. Because $\mathbf{R}$ is an identity matrix, the inverse of $\mathbf{R}$ is the same as $\mathbf{R}$, i.e., $\boldsymbol{R}^{-1} = \boldsymbol{R}$.

For this detector, the desired signal component is given in (7) while the multiuser interference (MAI) in (9) becomes 0, i.e., $I = 0$. Using the same way as given in (12)-(16), the average BER can be obtained by replacing the average SINR (14) with

$$\bar{\gamma}_{0,n} = \frac{\frac{2P}{N}E[\beta_{0,n}^2]\cos^2(2\pi f_o)}{N_o} \quad (18)$$

This detector completely eliminates the MAI.

3.2. Minimum Mean-Square Error Detector

The minimum mean-squared error (MMSE) detector is a linear detector which takes into account the background noise and utilises knowledge of the received signal powers. This detector implements the linear mapping, which is a partial or modified inverse of the correlation matrix, to the decision variable v_0. For this detector, the mapping coefficient, $\mathbf{L}$, is given by [4]

$$\mathbf{L} = [\boldsymbol{R} + (No/2)\boldsymbol{A}^{-2}]^{-1} \quad (19)$$

where $\boldsymbol{A} = diag[A_0, A_1, \ldots, A_{K-1}]$ and $A_k = \sqrt{(2P/N)}$ for the k-th user. The mapping coefficient, $\mathbf{L}$, is a $K \times K$ matrix. The matrix $\mathbf{L}$ can be written as

$$\mathbf{L} = \begin{pmatrix} L_{0,0} & L_{0,1} & \ldots & L_{0,K-1} \\ L_{1,0} & L_{1,1} & \ldots & L_{1,K-1} \\ \vdots & \vdots & \vdots & \vdots \\ L_{K-1,0} & L_{K-1,1} & \ldots & L_{K-1,K-1} \end{pmatrix} \quad (20)$$

From the above equation, the elements of matrix $\mathbf{L}$ for the desired user are $\mathbf{L}_0 = [L_{0,0}, L_{0,1}, \ldots, L_{0,K-1}]$. The desired signal component, S, for this detector is given by

$$S = L_{0,0}\sqrt{(P/2N)}\sum_{n=0}^{N-1} b_0(m)\beta_{0,n}^2\cos(2\pi f_o) \quad (21)$$

While the interfering component, I, is expressed as follows

$$I = \sqrt{(P/2N)}\sum_{k=1}^{K-1}\sum_{n=0}^{N-1} L_{0,k}b_k(m)c_k(n) \times c_0(n)\beta_{k,n}\beta_{0,n}\cos(2\pi f_o + \tilde{\theta}_{k,n}) \quad (22)$$

We consider the case without the near-far effect, thus it can be assumed that $L_{0,1} \simeq L_{0,2} \simeq \cdots \simeq L_{0,K-1}$. Due to the use of Walsh-Hadamard code as the spreading code, $L_{0,0} \gg L_{0,k},\ k = 1, \ldots, K-1$. Then, the variance of the interference in (10) becomes

$$\sigma_I^2 = \frac{P(K-1)L_{0,k}^2}{4N}E[\beta_{k,n}^2]\sum_{n=0}^{N-1}\beta_{0,n}^2 \quad (23)$$

The variance of the Gaussian random variable is given as

$$\sigma_\eta^2 = L_{0,0}^2\frac{N_o}{4T_b}\sum_{n=0}^{N-1}\beta_{0,n}^2 \quad (24)$$

The average SINR in (14) becomes

$$\bar{\gamma}_{0,n} = \frac{\frac{2P}{N}L_{0,0}^2E[\beta_{0,n}^2]\cos^2(2\pi f_o)}{L_{0,0}^2N_o + \frac{P(K-1)}{N}L_{0,k}^2E[\beta_{k,n}^2]} \quad (25)$$

then, this average SINR is injected into (13) in order to derive the BER for the desired user as given in (15).

4. NUMERICAL AND SIMULATION RESULTS

Performance evaluation of the multiuser detectors for MC-CDMA system in the presence of carrier frequency offset is performed using Monte Carlo simulations and the BER analysis. The users' spreading codes are Walsh-Hadamard codes. The number of users is set to 4 and all users are time synchronous. The presented results in this paper are for user 1 as the desired user. For the uplink transmission simulations, the signals are transmitted through a multipath fading

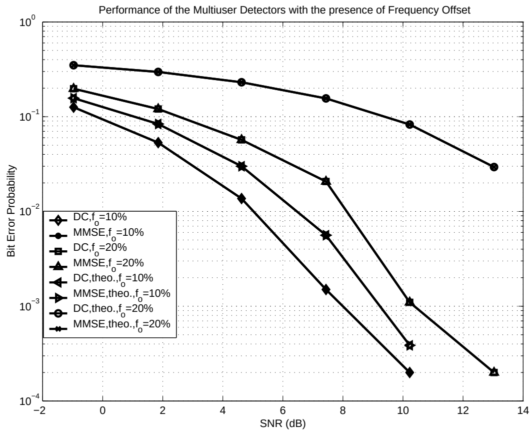

Fig. 1. Performance of the multiuser detectors with the presence of the frequency offset.

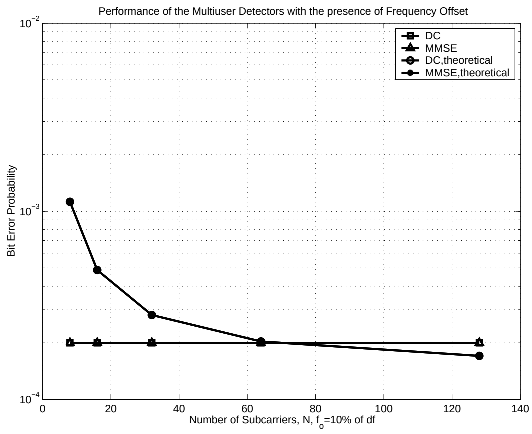

Fig. 2. Performance of the multiuser detectors for a variety of number of subcarriers.

channel and also corrupted by the additive white Gaussian noise. The channel is a slowly time-varying channel, where the fading coefficients do not change within a symbol duration, T_s. The number of paths for the multipath fading channel is set to 4. The channel coefficients are assumed to be known at the receiver. All users experience the same amount of frequency offset during uplink transmission. For the chip waveform, a rectangular pulse is used.

Figure 1 shows the performance of the multiuser detectors for two different values of the frequency offset f_o. The values of the frequency offset were set to 10% (0.1) and 20% (0.2) of the subcarrier spacing Δ_f. It can be seen from this figure that the performance of the detectors gets worse as the value of frequency offset increases. The performance of the decorrelating and the MMSE detectors are similar, where the plots of these detectors are overlaid. There is a small gap, which is about 1 dB, between the theoretical and the simulation results for $f_o = 0.1\Delta_f$. While, the gap becomes larger for $f_o = 0.2\Delta_f$, see Figure 1.

The performance of the detectors with a variety of number of subcarriers is depicted in Figure 2. In this evaluation, the value of the frequency offset is set to 10% of the Δ_f. It is shown that the gap between the theoretical and simulation results becomes generally smaller for a higher number of subcarriers. This results from the assumption that the subcarriers are independent becomes more accurate as the number of subcarriers increases.

5. CONCLUSIONS

A performance analysis of multiuser detectors for MC-CDMA in the presence of carrier frequency offset has been presented. A theoretical BER expression for the multiuser detectors was derived. The performance of the detectors is limited by the amount of the frequency offset in the system. The theoretical results closely follow the simulation results for a low value of the frequency offset and a high number of subcarriers.

6. REFERENCES

[1] Luciano Tomba and Witold A. Krzymien, "Sensitivity of the mc-cdma access scheme to carrier phase noise and frequency offset," *IEEE Transactions on Vehicular Technology*, vol. 48, no. 5, pp. 1657–1665, September 1999.

[2] Jiho Jang and Kwang Bok Lee, "Effects of frequency offset on mc/cdma system performance," *IEEE Communications Letters*, vol. 3, no. 7, pp. 196–198, July 1999.

[3] Matthijs A. Visser and Yeheskel Bar-Ness, "Joint multiuser detection and frequency offset correction for downlink mc-cdma," in *Proceedings of the IEEE Global Telecommunications Conference 1999*, 1999, pp. 2400–2404.

[4] Sergio S. Verdu, *Multiuser Detection*, Cambridge, 1998.

[5] Zexian Li and Matti Latva-aho, "Simple analysis of mrc receivers for mc-cdma systems in fading channels," in *International Conferences on Info-Tech & Info-Net (ICII)*, October 2001.

[6] Marvin K. Simon and Mohamed-Slim Alouini, "A unified approach to the performance analysis of digital communication over generalized fading channels," in *Proceedings of the IEEE*, vol. 86, pp. 1860–1877. 1998.