TIME DELAY ESTIMATION IN DIGITAL PRE-DISTORTION

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ABSTRACT

OFDM communications require extreme linearity of High Power Amplifiers (HPA) to avoid Inter-Carrier Interference (ICI) and subsequent degradation of the BER and Quality of Service (QoS). Amplitude (AM/AM) and Phase (AM/PM) amplifier non-linearities can be compensated in a number of ways. Nevertheless, digital adaptive schemes based on least squares optimization criteria need estimate the delay introduced by the analog chains in the generation of the error signal. A sufficiently precise HPA input-output time delay estimate (TDE) is necessary before attempting pre-distortion of the amplifier itself. This paper introduces statistical criteria for this initial acquisition stage.

1. INTRODUCTION

Increased capacity requirements of wireless communication systems demand bandwidth efficient and multipath resistant modulation schemes. OFDM has been the subject of wide interest in this respect. Although many advantages can be reaped from multicarrier signals, they still prove to be very sensitive to non-linear distortion. Its non-constant amplitude nature is critical in the radio frequency (RF) amplification stage. RF amplifiers introduce a type of non-linear multiplicative distortion [1] dependent on the modulus of the input baseband signal $u_x(t) = |b_x(t)|$ as,

$$b_y(t) = b_x(t-\Delta)G(u_x(t-\Delta)) \tag{1}$$

with $b_y(t)$ the baseband signal at the HPA output and Δ the time delay introduced by the baseband to RF up-conversion (U/C) chain. This effect introduces spectral re-growth (very high spectral components outside the OFDM signal passband) and in-band distortion. The bit-error rate of the link is thus degraded. Pre-distortion (PD) methods have been considered in the literature ([2],[3],[4] and others) to invert the non-linear characteristic of the amplifier. A depiction of an amplification chain can be observed in figure (1), where the corresponding RF signals $s_*(t)$ are expressed in terms of their baseband equivalent signals as,

$$s_*(t) = \mathrm{Re}\{b_*(t)e^{j2\pi f_c t}\} \tag{2}$$

with the subindex $*$ denoting either x (input to the HPA) or y (output from the HPA). The central carrier frequency is denoted with f_c (Hz). The phase term will be denoted as $\alpha_*(t)$ for either signal.

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The multiplicative gain in (1) is expressed in terms of modulus and phase non-linearities as,

$$|G(u)| = A(u) \tag{3}$$
$$\arg(G(u)) = \alpha_x + \Phi(u) \tag{4}$$

with $A(u)$ and $\Phi(u)$ the AM/AM and AM/PM distortion curves, respectively. The unknown time delay Δ constitutes a nuisance parameter when digital baseband PD is intended to compensate for the non-linear effects of the amplifier. In particular, the compensation of the phase response $\Phi(u)$ does require the estimation of Δ, while complete information on $A(u)$ can be gathered from the probability density function of the input and output modulus u_x and u_y. The approximation of the memory effect present in the up-conversion (U/C) plus HPA plus down-conversion (D/C) chain by a pure time delay is reasonable provided that the filters response in U/C and D/C be sufficiently flat in the band of interest. This assumption, which is technologically feasible, leads to a useful simplification of the signal model. The objective of the TDE algorithm herein described is the robust estimation of the analog time delay Δ in (1) in the presence of the non-linear gain $G(u)$. Correct identification of the inverse multiplicative factor $G^{-1}(u)$ for PD of the input signal necessitates the estimation of this model parameter. Thus, time alignment has been devised to operate independently from the adaptive PD algorithm. PD takes place then as,

$$b_y(t) = b_z(t-\Delta)G(u_z(t-\Delta)) \tag{5}$$
$$= b_x(t-\Delta)G^{-1}(u_x(t-\Delta))G(u_z(t-\Delta)) \tag{6}$$

such that $b_z(t)$ is the baseband output of the PD function and $G^{-1}(u_x)G(u_z) = 1$ is the necessary condition to achieve PD within the input dynamic range. Note that this TDE problem is not governed by the presence of noise in the signal model, which is inexistent (it takes place in the modulator). Difficulties are rather related with unknown distortion and the probability distribution of the input signal. The estimation of Δ in the presence of unknown AM/AM and AM/PM distortion is formulated in terms of an invariance equation as follows: note that equation (1) can be recast into,

$$b_y(t+\Delta) = b_x(t)G(u_x(t)) \tag{7}$$

Now, multiplication of this equality with $b_x^*(t)$ yields,

$$b_y(t+\Delta)b_x^*(t) = u_x^2(t)G(u_x(t)) \tag{8}$$

So that the following invariance equation may be established,

$$\nabla_{u_x(t)} b_y(t+\Delta)b_x^*(t) = 0 \tag{9}$$

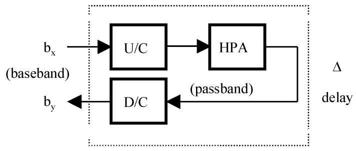

Fig. 1. baseband model of the RF non-linear chain: analog up-converter (U/C) plus HPA plus analog down-converter (D/C). The input/output delay is denoted with Δ.

for the derivation of robust TDE algorithms in the presence of memoryless amplifier non-linearities. This equation states in fact the functional independence between the terms $c(t,\Delta) = b_y(t+\Delta)b_x^*(t)$ and $u_x(t)$. This invariance property has been derived assuming negligible bandlimitation and aliasing effects. In the following we will assume that processing is done in the discrete domain. Hence, $t = nT_s$ with $f_s = 1/T_s$ a sufficiently high sampling frequency and T_s the corresponding sampling period. For reasons of convenience, the time delay Δ can be recast into a fixed rough estimate (an integer number of samples) Δ_0 plus a finer fractional estimate ε:$\Delta = \Delta_0 + \varepsilon$. Then, the search is performed in ε rather than in the wider range Δ and the invariance equation becomes,

$$b_y(t+\varepsilon)b_x^*(t-\Delta_0) = u_x^2(t-\Delta_0)G(u_x(t-\Delta_0))$$

The modified cross-correlation can be re-expressed now as,

$$c(t,\varepsilon) = b_y(t+\varepsilon)b_x^*(t-\Delta_0)$$

2. TDE ALGORITHM

TDE algorithms will be based on tests of the invariance principle. The method seeks the model that best fits $c(t,\varepsilon)$ with the sole dependence on u_x. The N-sample long data record is defined as vectors,

$$\mathbf{u}_x = [u_x(1), u_x(2) \cdots u_x(N)]^{\mathrm{T}} \quad (10)$$

$$\mathbf{c}_{\widehat{\varepsilon}} = [c(1,\widehat{\varepsilon}), c(2,\widehat{\varepsilon}) \cdots c(N,\widehat{\varepsilon})]^{\mathrm{T}} \quad (11)$$

where for convenience we have assumed $T_s = 1$. A regression equation using a set of N_b interpolation functions $\{h_k(u), 1 \le k \le N_b\}$ may be established on the coordinates defined by these vectors: $(\mathbf{u}_x|_i, \mathbf{c}_{\widehat{\varepsilon}}|_i)$ with $|_i$ the corresponding i-th component. Evaluation of the parameter $\widehat{\varepsilon}$ for which the regression error is minimum establishes a test for invariance on $u_x(nT_s)$ of the quantity $c(nT_s,\widehat{\varepsilon})$. That is, influence of random sources other than u_x lead to model misfit and to a large regression error. The error consistent with this invariance property can be defined in scalar and vector forms as,

$$\mathrm{e}(i,\widehat{\varepsilon},\mathbf{a}) = c(i,\widehat{\varepsilon}) - \sum_{k=1}^{L} a_k h_k(u_x(i)) \quad (12)$$

$$\mathrm{e}(\widehat{\varepsilon},\mathbf{a}) = \mathbf{c}_{\widehat{\varepsilon}} - \mathbf{H}(\mathbf{u}_x)\mathbf{a} \quad (13)$$

in terms of the regression coefficients $\mathbf{a}$. The regression error σ_{e}^2 is then evaluated as,

$$\sigma_{\mathrm{e}}^2(\widehat{\varepsilon},\mathbf{a}) = \mathrm{e}(\widehat{\varepsilon},\mathbf{a})^{\mathrm{H}}\mathrm{e}(\widehat{\varepsilon},\mathbf{a}) \quad (14)$$

so that minimization with respect to $\mathbf{a}$ yields,

$$\nabla_{\mathbf{a}^{\mathrm{H}}}\sigma_{\mathrm{e}}^2(\widehat{\varepsilon},\mathbf{a}) = \mathbf{H}^{\mathrm{H}}(\mathbf{u}_x)(\mathbf{c}_{\widehat{\varepsilon}} - \mathbf{H}(\mathbf{u}_x)\mathbf{a}) \quad (15)$$

with the optimum (classical least squares) solution $\mathbf{a}_{\mathrm{opt}}$,

$$\mathbf{a}_{\mathrm{opt}} = (\mathbf{H}^{\mathrm{H}}(\mathbf{u}_x)\mathbf{H}(\mathbf{u}_x))^{-1}\mathbf{H}^{\mathrm{H}}(\mathbf{u}_x)\mathbf{c}_{\widehat{\varepsilon}} \quad (16)$$

Hence, the error evaluated at the optimum regression coefficients $\sigma_{\mathrm{e}}^2(\widehat{\varepsilon}) = \sigma_{\mathrm{e}}^2(\widehat{\varepsilon},\mathbf{a}_{\mathrm{opt}})$ is,

$$\begin{aligned} \sigma_{\mathrm{e}}^2(\widehat{\varepsilon}) &= \mathbf{c}_{\widehat{\varepsilon}}^{\mathrm{H}}\left(\mathbf{I} - \mathbf{H}(\mathbf{u}_x)(\mathbf{H}^{\mathrm{H}}(\mathbf{u}_x)\mathbf{H}(\mathbf{u}_x))^{-1}\mathbf{H}(\mathbf{u}_x)^{\mathrm{H}}\right)\mathbf{c}_{\widehat{\varepsilon}} \\ &= \mathbf{c}_{\widehat{\varepsilon}}^{\mathrm{H}}\mathbf{P}_{\mathbf{H}}^{\perp}\mathbf{c}_{\widehat{\varepsilon}} \quad (17) \\ &= \|\mathbf{P}_{\mathbf{H}}^{\perp}\mathbf{c}_{\widehat{\varepsilon}}\|_2^2 \quad (18) \end{aligned}$$

in terms of the orthogonal projector $\mathbf{P}_{\mathbf{H}}^{\perp}$, which is independent of the estimation variable $\widehat{\varepsilon}$. Hence, this criterion searches for that $\widehat{\varepsilon}$ such that the projection of $\mathbf{c}_{\widehat{\varepsilon}}$ is most contained in the null space of $\mathbf{P}_{\mathbf{H}}^{\perp}$. $\|\cdot\|_2^2$ denotes the Euclidean norm. The compressed cost function must now be minimized in terms of $\widehat{\varepsilon}$. We define,

$$\mathbf{c}_{\widehat{\varepsilon}} = \mathbf{b}_x^* \bullet \mathbf{b}_y^{\widehat{\varepsilon}} = \mathbf{D}_x^{\mathrm{H}}\mathbf{b}_y^{\widehat{\varepsilon}} \quad (19)$$

with $\mathbf{D}_x = \mathrm{diag}(\mathbf{b}_x)$. Therefore,

$$\sigma_{\mathrm{e}}^2(\widehat{\varepsilon}) = \mathbf{b}_y^{\widehat{\varepsilon},\mathrm{H}}\mathbf{D}_x\mathbf{P}_{\mathbf{H}}^{\perp}\mathbf{D}_x^{\mathrm{H}}\mathbf{b}_y^{\widehat{\varepsilon}} \quad (20)$$

where the term $\mathbf{b}_y^{\widehat{\varepsilon}}$ can be obtained from the output of an interpolator. Equation (20) can be used to perform a rough search in ε. A more precise estimate of ε can then be tracked using steepest descent methods. The S-curve of the synchronizer is obtained as the derivative of $\sigma_{\mathrm{e}}^2(\widehat{\varepsilon})$,

$$\begin{aligned} \nabla_{\widehat{\Delta}}\sigma_{\mathrm{e}}^2(\widehat{\varepsilon}) &= 2\mathrm{Re}\left[\mathbf{b}_y^{\widehat{\varepsilon},\mathrm{H}}\mathbf{D}_x\mathbf{P}_{\mathbf{H}}^{\perp}\mathbf{D}_x^{\mathrm{H}}\left(\nabla_{\widehat{\varepsilon}}\mathbf{b}_y^{\widehat{\varepsilon}}\right)\right] \quad (21) \\ &= 2\mathrm{Re}\left[\mathbf{b}_y^{\widehat{\varepsilon},\mathrm{H}}\mathbf{D}_x\mathbf{P}_{\mathbf{H}}^{\perp}\mathbf{D}_x^{\mathrm{H}}\dot{\mathbf{b}}_y^{\widehat{\varepsilon}}\right] \quad (22) \end{aligned}$$

with the term $\dot{\mathbf{b}}_y^{\widehat{\varepsilon}} = \nabla_{\widehat{\varepsilon}}\mathbf{b}_y^{\widehat{\varepsilon}}$ the output of the derivative interpolator. For stochastic gradient updates, the TDE is modified in accordance with the equation,

$$\widehat{\varepsilon}_{k+1} = \widehat{\varepsilon}_k - \mu\nabla_{\widehat{\varepsilon}}\sigma_{\mathrm{e}}^2(\widehat{\varepsilon}) \quad (23)$$

Let the impulse response of a derivative interpolator be denoted as $h'_d(n;\widehat{\varepsilon})$. Then,

$$\dot{\mathbf{b}}_y^{\widehat{\Delta}} = \sum_{k=-\infty}^{+\infty} h'_d(n;\widehat{\varepsilon})b_y(n-k)$$

where the samples of the derivative interpolator depend on the interpolation time. Perfect derivative interpolation (of a bandlimited signal) requires an infinitely long impulse response. Hence, in practice, approximate finite length derivative interpolation is used. In this way, the synchronizer can operate continuously and with independence of the pre-distorter.

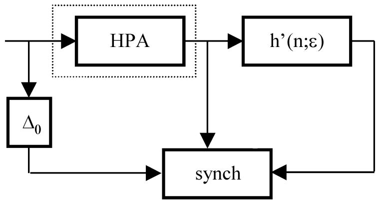

Fig. 2. fine timing estimation is achieved via derivative interpolation in $h'(n;\varepsilon)$ in a steepest descent algorithm. For simplicity we assume a fixed delay Δ_0 on the HPA baseband input to compensate for the equivalent delay of the analog chain. The resulting delay $\varepsilon = \Delta - \Delta_0$ will have much a smaller range. An initial rough search for ε is performed substituting the derivative interpolator $h'(n;\varepsilon)$ for a direct interpolator $h(n;\varepsilon)$ or via oversampling.

3. INTERPOLATION FUNCTIONS

There exist several choices for the set of interpolation functions $\{h_k(u), 1 \leq k \leq N_b\}$. Either non-overlapping rectangular functions covering the whole modulus interval are used, in which case the computation of the projector $\mathbf{P}^{\perp}_{\mathbf{H}}$ is straightforward,

$$h_k(u) = \Pi\left(\frac{u - c_k}{d_k}\right)$$

or more precise linear interpolation (triangular) functions are chosen. In this case, the evaluation of the projector is computationally more involved as it requires inversion of a square tri-diagonal matrix. The eigenvalue spread of the matrix $\mathbf{H}^{\mathrm{H}}(\mathbf{u}_x)\mathbf{H}(\mathbf{u}_x)$ is directly related to the probability density function of the modulus. For rectangular interpolation functions, this is a diagonal matrix such that,

$$\mathbf{H}^{\mathrm{H}}(\mathbf{u}_x)\mathbf{H}(\mathbf{u}_x) = \mathbf{H}(\mathbf{u}_x)\mathbf{H}^{\mathrm{H}}(\mathbf{u}_x) = \mathbf{D}_H(\mathbf{u}_x) \quad (24)$$

$$= N\mathrm{diag}\left[p_1, p_2, ..., p_{N_b}\right] \quad (25)$$

with $p_i = \mathrm{Prob}\left[u_i \leq \mathbf{u}_x|_i \leq u_{i+1}\right]$. A previous examination of the data, can help determine the most suitable set of bin centroids $c_i = \frac{1}{2}(u_i + u_{i+1})$ to guarantee that all entries of the diagonal matrix $\mathbf{H}^{\mathrm{H}}(\mathbf{u}_x)\mathbf{H}(\mathbf{u}_x)$ be equal for the rectangular function case. That is, each interval $[u_i, u_{i+1}]$ is activated with equi-probability with $p_i = 1/N_b$ and N_b the number of bins of the activation range. Then, the computation of the projector is amenable to direct implementation,

$$\mathbf{D}_H(\mathbf{u}_x) = \frac{N}{N_b}\mathbf{I} \quad (26)$$

$$\mathbf{P}^{\perp}_{\mathbf{H}} = \mathbf{I} - \frac{N_b}{N}\mathbf{H}(\mathbf{u}_x)\mathbf{H}(\mathbf{u}_x)^{\mathrm{H}} \quad (27)$$

Note that as the cost function is $\sigma^2_{\mathrm{e}}(\widehat{\varepsilon}) = \|\mathbf{P}^{\perp}_{\mathbf{H}}\mathbf{c}_{\widehat{\varepsilon}}\|^2_2$, using $\mathbf{H} = \mathbf{H}(\mathbf{u}_x)$ for a more compact expression, we can establish that,

$$\sigma^2_{\mathrm{e}}(\widehat{\varepsilon}) = \mathbf{b}^{\widehat{\varepsilon},\mathrm{H}}_y\mathbf{D}_x\mathbf{P}^{\perp}_{\mathbf{H}}\mathbf{D}^{\mathrm{H}}_x\mathbf{b}^{\widehat{\varepsilon}}_y \quad (28)$$

$$\mathbf{D}_x\mathbf{P}^{\perp}_{\mathbf{H}}\mathbf{D}^{\mathrm{H}}_x = \mathbf{D}_x\mathbf{D}^{\mathrm{H}}_x - \frac{N_b}{N}\mathbf{D}_x\mathbf{H}\mathbf{H}^{\mathrm{H}}\mathbf{D}^{\mathrm{H}}_x \quad (29)$$

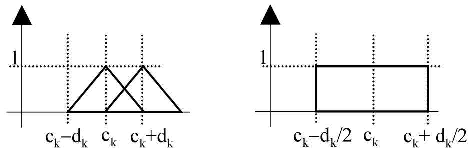

Fig. 3. (left) triangular and (right) rectangular interpolation functions

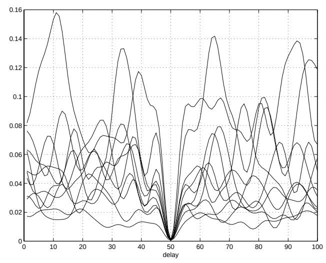

Fig. 4. 10 realizations of $\sigma^2_{\widehat{\varepsilon}}$ over a 200-sample block. The normalized signal bandwidth is $B = 0.08$. The number of modulus bins of u_x is $N_b = 4$. The correct time delay is $\Delta = 50$. Observe the large variance off the correct delay for small block size.

with $\mathbf{H}\mathbf{H}^{\mathrm{H}}$ a rank-defficient square matrix with components either one or zero. A one entry in component (l, l') meaning that the l-th and l'-th entries of $\mathbf{u}_x$ belong to the same bin. Hence, each component of matrix $\mathbf{D}_x\mathbf{H}\mathbf{H}^{\mathrm{H}}\mathbf{D}^{\mathrm{H}}_x$ contains either a zero or the product $b_x(l)b^*_x(l')$, if both samples $b_x(l)$ and $b^*_x(l')$ of $\mathbf{b}_x$ are in the same modulus bin. Hence, $\sigma^2_{\mathrm{e}}(\widehat{\varepsilon})$ can be expressed as,

$$\sigma^2_{\mathrm{e}}(\widehat{\varepsilon}) = \sum_{l,l'} b^*_y(l+\varepsilon)b_y(l'+\varepsilon)b_x(l)b^*_x(l')\left(\delta_{l,l'} - \frac{N_b}{N}h(l,l')\right) \quad (30)$$

with $h(l, l') \in \{0, 1\}$ the coefficient indicating whether $b_x(l)$ and $b^*_x(l')$ share the same modulus bin and $\delta_{l,l'}$ the Kronecker delta.

4. SIMULATIONS

The simulations in figures (4) to (9) show the behaviour of the cost function for several values of the block length N. More detailed descriptions are provided in the caption of each particular figure. Saleh's model [1] has been used for the non-linear HPA. The complexity in evaluating equation (30) for all candidate $\widehat{\varepsilon}$'s, is quadratic in the length of the sample block. Averaging of $\sigma^2_{\mathrm{e}}(\widehat{\varepsilon})$ over a number of short sample blocks provides important savings in complexity without noticeable degradation in the final result. Thus, the poor discrimination that can be observed in figure (4) for some realizations is averaged out in figure (5).

5. CONCLUSIONS

A TDE algorithm has been presented to aid in input/output signal alignment of a non-linear amplification chain. PD algorithms require that this time delay be compensated in the absence of knowledge of the non-linear characteristic. A simple algorithm based on

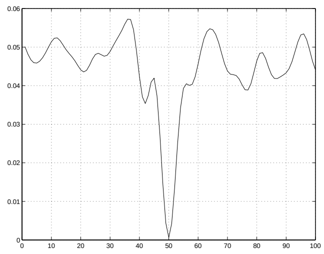

Fig. 5. average of the 10 realizations shown in figure (4).Note that the discrimination is noticeably improved. The width of the zero lobe is inversely proportional to the bandwidth of the input signal b_x. A number of bins as low as 4 already provides good discrimination for the correct time delay.

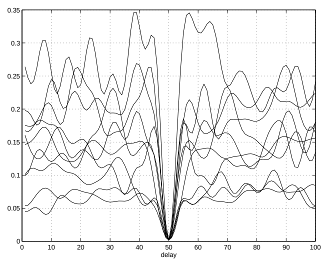

Fig. 6. 10 realizations of $\sigma^2_{\hat{\varepsilon}}$ in the same context of figure (4) obtained with a larger block size: 400. Observe the better discrimination for each particular realization.

derivative interpolation and selective modulus activation functions has been shown to achieve this purpose. The amplifier characteristic can be assumed time-invariant over large sample records. Hence, the convergence rate of time delay estimation and predistortion itself does not constitute a critical parameter for this type of application.

6. REFERENCES

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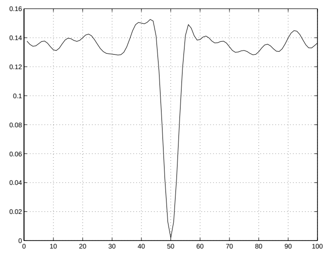

Fig. 7. average of the 10 realizations shown in figure (6).Note that the discrimination with respect to that obtained in (5) is significantly better. Nevertheless, the behaviour observed in figure (5) already guarantees the estimation of the correct time delay to within one sample resolution.

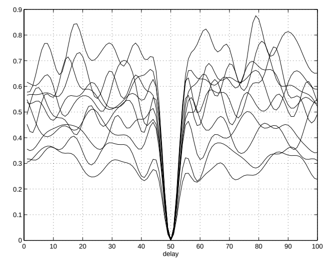

Fig. 8. 10 realizations of $\sigma^2_{\hat{\varepsilon}}$ obtained for a block size of 1000 samples and a number of bins $N_b = 12$. For such a high number of samples, averaging of $\sigma^2_{\hat{\varepsilon}}$ over different realizations is not already necessary as discrimination is sufficiently precise.

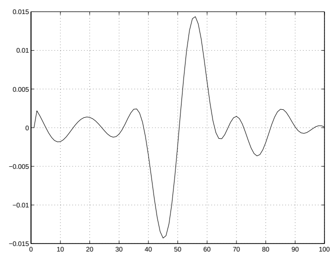

Fig. 9. S-function obtained for a signal normalized bandwidth $B = 0.04$, using 1000-sample blocks and 5-block averaging. The number of bins is $N_b = 4$.