EGRESS REDUCTION BY INTRA-SYMBOL WINDOWING IN DMT-BASED TRANSMITTERS

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ABSTRACT

Discrete multi tone (DMT) uses an inverse discrete fourier transform (IDFT) to modulate data on the carriers. The rather high side lobes of the IDFT filter bank can lead to spurious emissions in unauthorised frequency bands. Applying a window function within the DMT symbol alleviates this, but introduces distortions that are generally not easy to compensate. In this paper a special class of window functions is derived for which this compensation only requires a modest amount of processing at the receiver.

1. INTRODUCTION

Discrete multitone (DMT) offers an elegant way to achieve a high spectral efficiency, and is very suitable for digital subscriber line (DSL), such as asymmetric DSL (ADSL) or very high bitrate DSL (VDSL). The available bandwidth is divided into small frequency bands centered around carriers (often called tones) that are individually QAM modulated in the digital domain. Modulation is done through the inverse discrete fourier transform (IDFT), while demodulation is done using the discrete fourier transform (DFT). Before transmission, a cyclic prefix (CP) is added to each symbol. In case the order of the channel impulse response does not exceed the CP length, the linear convolution with the channel impulse response can be described as a circular one. Equalization can than be done very easily, using a one-tap frequency domain equalizer (FEQ), correcting the phase shift and attenuation at each tone invididually. In case the channel impulse response's length exceeds the CP by more than one sample, it needs to be shortened using a time domain equalizer (TEQ). Alternatively, one can use a per-tone equalizer (PTEQ) [1], which forms an upper performance bound for any combination of TEQ+FEQ of equal length. Although the proposed technique is irrespective of the equalization, for the remainder of the text, a PTEQ is assumed.

While DMT is highly welcomed because of its flexibility towards spectrum control, the high sidelobe levels associated with the DFT filter bank form a serious impediment. This problem is two-sided. On the one hand the receiver is rather susceptible to narrow-band interference (such as AM broadcast stations) being spread over a broad tone range. This problem has been recognized, and various schemes have been developed to tackle it ([2], [3], [4]).

On the other hand, the poor spectral containment makes it difficult to meet egress norms, e.g. the ITU-norm [5] specifies that the transmit power of VDSL should be lowered 20dB in the amateur radio bands. Controlling egress is usually done in the frequency domain, combining neighbouring IDFT-inputs (such as in [6]), or equivalently, by abandoning the DFT altogether and reverting to other filter banks, such as e.g. in [7].

Another approach would be to apply an appropriate time domain window (see [8] for a nice overview). In [9], a VDSL system is proposed, where the window is applied to additional cyclic continuations of the DMT symbol to prevent distorting the symbol itself.

The technique proposed in this article avoids the overhead resulting from such symbol extension by applying the window directly to the DMT symbol. Obviously, this alters the frequency content at each carrier, such that a correction at the receiver is needed. While this compensation is generally nontrivial, we construct a class of windows that can be compensated for with only a minor amount of additional computations at the receiver.

In section 2, we derive the novel windowing system. Section 3 covers the simulation results. Finally, in section 4, conclusions and possible extensions are given.

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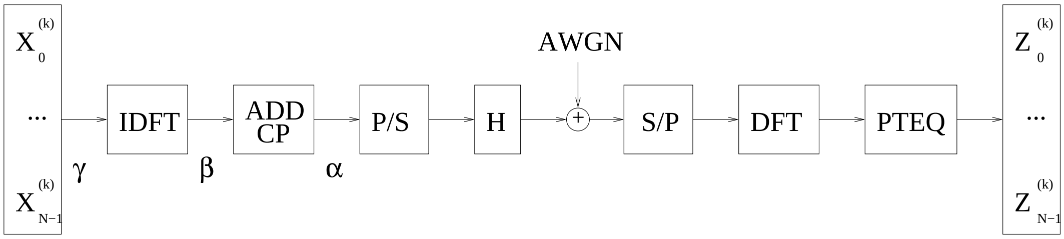

Fig. 1. The basic DMT system (refer to text for α to γ)

2. TRANSMITTER WINDOWING

2.1. Notation

The basic DMT system is shown in fig. 1. At time k, $\mathbf{X}^{(k)} = [X_0^{(k)} ... X_{N-1}^{(k)}]^T$ holds the complex subsymbols at tones $i, i = 0 : N-1$. The DMT modulator performs the N-point IDFT and adds the CP of length ν, resulting in the time-domain sample vector of length $s = N + \nu$,

$$\mathbf{x}^{(k)} = [x(ks), ..., x((k+1)s-1)]^T,$$

$$x(ks+n) = \begin{cases} x(ks+n+N) & n=0,...,\nu-1 \\ \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} X_i^{(k)} e^{j\frac{2\pi}{N}(n-\nu)} & n=\nu,...,N+\nu-1 \end{cases}$$

Note that in a baseband system, such as ADSL, the time-domain signal contains no imaginary component, requiring that $X_i(k) = X^*_{N-i}(k)$.

After parallel-to-serial conversion, the samples enter the channel, which is modeled as a finite impulse response (FIR) filter $h(t)$ of order L, such that the received samples $y(n)$ are equal to

$$y(n) = \sum_{\tau=0}^{L} h(\tau) x(n-\tau) + v(n),$$

where $v(n)$ represents an additive white gaussian noise (AWGN) contribution. The PTEQ-based equalizer can be described as the cascade of a (sliding) DFT and a tone-dependent filter operating on each tone-output, resulting in the equalized subsymbols $Z_i^{(k)}$.

To implement transmit windowing, the sample vector $\mathbf{x}^{(k)}$ is weighted with a window vector

$$\mathbf{w} = [w(0) ... w(N+\nu-1)]^T$$

at point α in fig. 1. In the next paragraph, we impose constraints on $\mathbf{w}$, to construct a class of window functions that are easy to compensate for at the receiver.

2.2. Derivation of the window structure

To ensure the cyclic structure of the transmitted symbols, needed for the easy equalization, we impose the cyclic constraint:

$$w(n) = w(n+N), n = 0, ..., \nu - 1 \tag{1}$$

Now, instead of applying the window $\mathbf{w}$ at point α (fig. 1), one can also apply the window

$$\begin{aligned} \mathbf{g} &= [g(0) ... g(n-1)]^T \\ &= [w(\nu) ... w(N+\nu-1)]^T \end{aligned}$$

at point β. After defining $\mathcal{I}_N$ the IDFT-matrix of size N, the vector of windowed samples $\mathbf{x}_w^{(k)}$ at point β, (before the application of the CP) can be written as:

$$\mathbf{x}_w^{(k)} = \begin{bmatrix} g(0) & 0 & \dots & 0 \\ 0 & g(1) & \ddots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & \dots & 0 & g(N-1) \end{bmatrix} \mathcal{I}_N \cdot \mathbf{X}^{(k)} \tag{2}$$

Recalling that the product of a diagonal matrix and the IDFT-matrix can be written as the product of the IDFT-matrix and a circulant matrix, we can rewrite (2) as:

$$\mathbf{x}_w^{(k)} = \mathcal{I}_N \underbrace{\begin{bmatrix} G(0) & G(1) & \dots & G(N-1) \\ G(N-1) & G(0) & \ddots & G(N-2) \\ \ddots & \ddots & \ddots & \ddots \\ G(1) & \dots & & G(0) \end{bmatrix}}_{\mathbf{C}} \cdot \mathbf{X}^{(k)}, \tag{3}$$

with

$$\mathbf{G} = [G(0) ... G(N-1)]^T = \mathcal{I}_N \cdot \mathbf{g}$$

the IFFT of $\mathbf{g}$. The transition from (2) to (3) is more than mathematical trickery. Looking at the DMT-scheme incorporating transmitter windowing of fig. 2, it becomes clear that the weighting operation in the time domain is equivalent to the multiplication of the subsymbol vector with a (pre-)coding matrix $\mathbf{C}$. Compensating for the window at the receiver is now identical to decoding in the frequency domain by multiplication with the decoding matrix $\mathbf{D} = \mathbf{C}^{-1}$, leaving the rest of the signal path (equalization etc.) unaltered. Thus, appealing windows should not only satisfy the

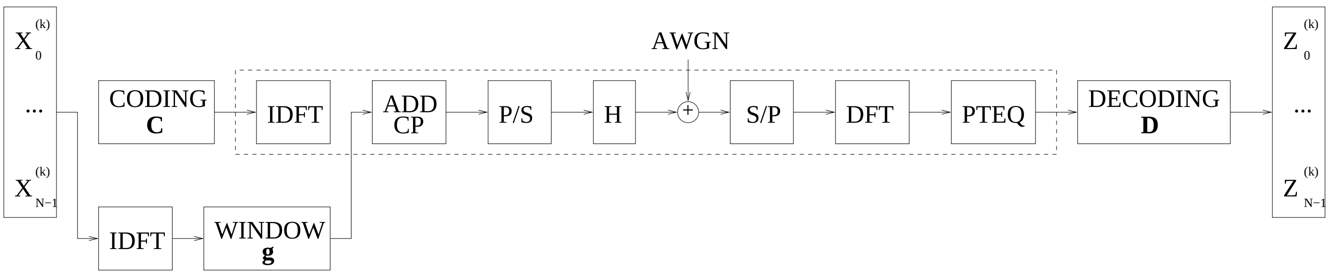

Fig. 2. Transmitter windowing translates to symbol precoding

constraint (1), but preferably also give rise to a sparse decoding matrix $\mathbf{D}$. We will now further investigate the nature of such windows.

Being the inverse of a circulant matrix, $\mathbf{D}$ is also circulant. If we denote the first row of $\mathbf{D}$ as

$$\mathbf{F}^T = [F(0)...F(N-1)],$$

and define $\mathcal{F}_N$ the DFT-matrix of size N, and

$$\mathbf{f} = [f(0), ...f(N-1)]^T = \mathcal{F}_N \cdot \mathbf{F},$$

we can write $\mathbf{g}$ as

$$g(n) = f(n)^{-1}, n = 0, ..., N-1. \tag{4}$$

Since $\mathbf{g}$ is real-valued, so is $\mathbf{f}$. Thus in the simplest case, $\mathbf{F}$ can be described as:

$$F(n) = \begin{cases} a & n=0 \\ b.e^{j\phi} & n=p \\ b.e^{-j\phi} & n=N-p \\ 0 & n \notin \{0, p, N-p\} \end{cases}, \tag{5}$$

with

$$\begin{array}{ll} a, b & \text{real} \\ \phi & \text{real} \in \left[\, -\pi \quad \pi \,\right] \\ p & \text{integer} \in \left[\, 1 \quad N-1 \,\right] \end{array}$$

In practice, this means that $\mathbf{f}$ takes the form of a generalized *raised cosine* function. The different parameters influencing $\mathbf{f}$ are the pedestal height a, the frequency and amplitude of the sinusoidal part p and b, and ϕ determining the position of the peak(s).

3. RESULTS

Returning to the original goal of egress reduction, we now need to choose the parameters a, b, p, and ϕ, such that an improved side-lobe characteristic of $\mathbf{w}$ is obtained. We propose $p = 1$, and ϕ such that $\mathbf{w}$ is symmetrical (i.e. $\phi = \frac{-\nu\pi}{N}$). As a design criterion, we specify that the power outside the main lobe should be as low as possible.

More rigourously, this problem can be stated as the minimization of ρ_S

$$\rho_S = \int_{\frac{\pi}{N+\nu}}^{\pi} |W(e^{j\omega})|^2 \frac{d\omega}{\pi}, \tag{6}$$

$$\text{with } W(z) = \sum_{n=0}^{N+\nu-1} w(n) z^{-n}$$

under unit-energy constraint for $\mathbf{w}$ [10]. Minimizing ρ_S is equivalent to maximizing ρ:

$$\rho = \mathbf{w}^T \cdot \mathbf{P} \cdot \mathbf{w}, \tag{7}$$

where $\mathbf{P}$ has (m, n)th entry

$$p_{mn} = \int_0^{\frac{\pi}{N+\nu}} \cos(m-n)\omega \frac{d\omega}{\pi}, \qquad 0 \le m, n \le N+\nu-1.$$

In fig. 3, ρ has been plotted as a function of b (with a determined from $\mathbf{w}^T \cdot \mathbf{w} = 1$).

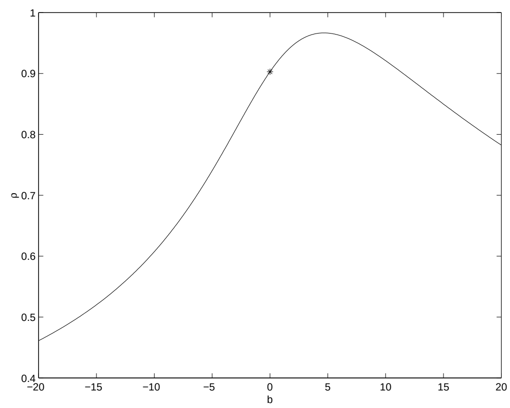

Fig. 3. ρ as a function of b (assuming $\mathbf{w}^T \cdot \mathbf{w} = 1$). The rectangular window is represented by '*'.

Although the rectangular window (marked with '*') is a good contender, clearly it is not optimal. The resulting optimal window function $\mathbf{w}_{opt}$ for the ADSL case ($N =$

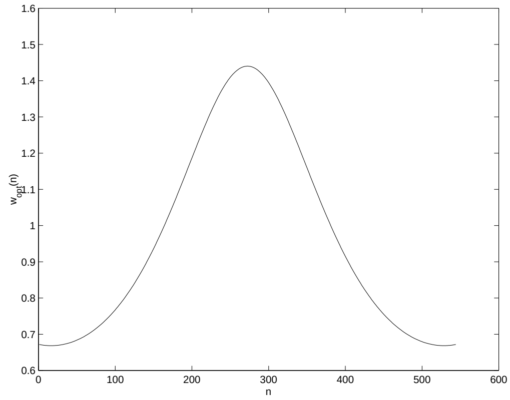

Fig. 4. The optimal window for the ADSL case

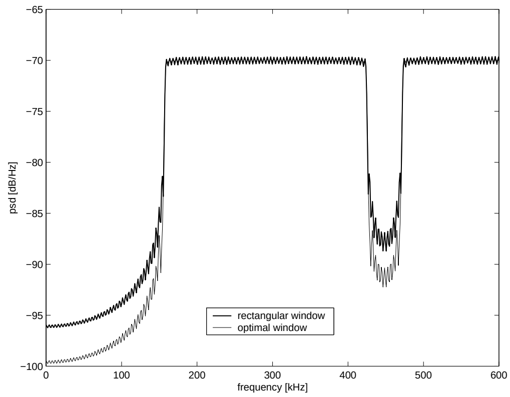

Fig. 5. Egress comparison for ADSL (tones 38-99 and 111-255 used)

512, $\nu = 32$) is shown in fig. 4. To illustrate the egress-reduction, in fig. 5, the spectrum of a classic non-windowed ADSL system is compared to a system using $\mathbf{w}_{opt}$. Tones 38 to 99 and 111 to 255 carry information, while the others are left blank.

4. CONCLUSION AND FURTHER WORK

A novel transmitter windowing technique for DMT has been proposed, which does not rely on an additional cyclic extension of the symbol. Windowing within the symbol inevitably introduces a distortion of the signal. For a special class of windows, this distortion can be described as a coding for which the decoding at the receiver can be done easily. Essentially, these window functions can be described as the pointwize inversion of a raised cosine window.
Future work will focus on a selective windowing of the tones in the vicinity of unauthorised bands. Also the trade-off between decoder complexity and egress should be further studied, as well as the interaction between the transmitter window and the equalizer.

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