MMSE TIME-VARYING FIR EQUALIZATION OF DOUBLY-SELECTIVE CHANNELS

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ABSTRACT

In this paper, we propose a time-varying (TV) finite impulse response (FIR) equalizer for doubly-selective (time- and frequency-selective) channels. We use the basis expansion model (BEM) to approximate the doubly-selective channel and to design the TV FIR equalizer. This structure allows us to turn a large design problem into an equivalent small design problem, containing only the BEM coefficients of both the doubly-selective channel and the TV FIR equalizer. Focus is on the minimum mean-square error (MMSE) solution, but the zero-forcing (ZF) solution is also discussed. Comparisons with the linear block equalizer (LBE) are made. Through computer simulations we show that the performance of the MMSE TV FIR equalizer approaches the one of the MMSE LBE, while the design as well as the implementation complexity are much lower.

1. INTRODUCTION

The need for high data rates and high mobility in future wireless communication systems introduces doubly-selective (time- and frequency-selective) channel effects. To combat these effects, equalizers are needed. For frequency-selective channels, such equalizers have been extensively studied in literature. We can distinguish between block equalizers and serial equalizers. Linear block equalizers (LBEs) for frequency-selective channels only require a single receive antenna for the zero-forcing (ZF) solution to exist [1]. They are usually complex to design and implement. However, since a frequency-selective channel can be diagonalized by means of the Fast Fourier Transform (FFT), the design and implementation complexity can be reduced, at the cost of a slight decrease in performance. On the other hand, linear serial equalizers (LSEs), more specifically finite impulse response (FIR) equalizers, for frequency-selective channels require at least two receive antennas for the ZF solution to exist, but are simpler to design and implement [2, 3].

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Recently, equalizers have also been developed for doubly-selective channels. Similar to the frequency-selective case, LBEs for doubly-selective channels only require a single receive antenna for the ZF solution to exist. However, since a doubly-selective channel can not be diagonalized, they can not be simplified and are always complex to design and implement. This motivates us to look at LSEs, more specifically FIR equalizers, for doubly-selective channels, which should be simpler to design and implement. Up till now, only time-invariant (TIV) FIR equalizers for doubly-selective channels have been introduced [4]. However, such a TIV FIR equalizer requires many receive antennas for the ZF solution to exist. In this paper, we introduce time-varying (TV) FIR equalizers for doubly-selective channels. We use the basis expansion model (BEM) to approximate the doubly-selective channel and to design the TV FIR equalizer. This structure allows us to turn a large design problem into an equivalent small design problem, containing only the BEM coefficients of both the doubly-selective channel and the TV FIR equalizer. A TV FIR equalizer requires at least two receive antennas for the ZF solution to exist. However, this is much lower than the number of receive antennas a TIV FIR equalizer requires.

Notation: We use upper (lower) bold face letters to denote matrices (column vectors). Superscripts *, ^{T}, and ^{H} represent conjugate, transpose, and Hermitian, respectively. We denote the 1- and 2-dimensional Kronecker delta as δ_n and $\delta_{n,m}$, respectively. We denote the $N \times N$ identity matrix as $\mathbf{I}_N$ and the $M \times N$ all-zero matrix as $\mathbf{0}_{M\times N}$. Finally, diag$\{\mathbf{x}\}$ denotes the diagonal matrix with $\mathbf{x}$ on the diagonal.

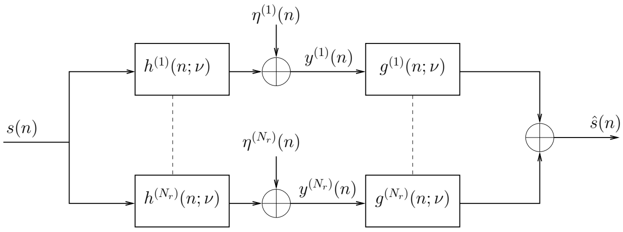

Fig. 1. System Model

2. SYSTEM MODEL

The system under consideration is depicted in Figure 1. We assume a single-input multiple-output (SIMO) system, where N_r receive antennas are used. Focusing on a baseband-equivalent description, when transmitting a symbol sequence $s(n)$ and sampling each receive antenna at the symbol rate, the received sample sequence at the rth receive antenna can be written as

$$y^{(r)}(n) = \sum_{\nu=-\infty}^{\infty} h^{(r)}(n;\nu)s(n-\nu) + \eta^{(r)}(n), \quad (1)$$

where $\eta^{(r)}(n)$ is the additive noise at the rth receive antenna, and $h^{(r)}(n;\nu)$ is the doubly-selective (time- and frequency-selective) channel to the rth receive antenna.

In this paper, we model a doubly-selective channel using a basis expansion model (BEM). Many BEMs exist. In this paper, we use the BEM of [5], which is shown to accurately approximate the well-known Jakes' model. In this BEM (which we simply call *the BEM* from now on), a doubly-selective channel is modeled as an FIR filter where the taps are expressed as a superposition of complex exponential basis functions with frequencies on an FFT grid. Let us first make the following assumptions:

A1) The delay-spread is bounded by τ_{max};

A2) The Doppler-spread is bounded by f_{max}.

Under assumptions A1) and A2), it is possible to model the doubly-selective channel $h^{(r)}(n;\nu)$ for $n \in \{0, 1, \ldots, N-1\}$ as

$$h^{(r)}(n;\nu) = \sum_{l=0}^{L} \delta_{\nu-l} \sum_{q=-Q/2}^{Q/2} h_{q,l}^{(r)} e^{j2\pi qn/N}, \quad (2)$$

where L and Q satisfy the following conditions:

C1) $LT \geq \tau_{max}$; **C2)** $Q/(NT) \geq 2f_{max}$,

and T is the symbol period. In this expansion model, L represents the delay-spread (expressed in multiples of T, the delay resolution of the model), and Q represents the Doppler-spread (expressed in multiples of $1/(NT)$, the Doppler resolution of the model).

Assuming that $s(n) = 0$ for $n \notin \{0, 1, \ldots, M-1\}$, where $M = N - L_{zp}$ and $L_{zp} \geq L$, we can rewrite (1) as

$$y^{(r)}(n) = \sum_{l=0}^{L} \sum_{q=-Q/2}^{Q/2} e^{j2\pi qn/N} h_{q,l}^{(r)} s(n-l) + \eta^{(r)}(n).$$

For convenience, we assume that $y^{(r)}(n) = 0$ for $n \notin \{0, 1, \ldots, N-1\}$.

The above input-output relation can also be written in block form. Defining the $M \times 1$ symbol block as $\mathbf{s} := [s(0), \ldots, s(M-1)]^T$, the received sample block at the rth receive antenna $\mathbf{y}^{(r)} := [y^{(r)}(0), \ldots, y^{(r)}(N-1)]^T$ can be written as

$$\mathbf{y}^{(r)} = \mathbf{H}^{(r)}\mathbf{T}_{zp}\mathbf{s} + \boldsymbol{\eta}^{(r)}, \quad (3)$$

where $\boldsymbol{\eta}^{(r)}$ is similarly defined as $\mathbf{y}^{(r)}$, $\mathbf{T}_{zp} := [\mathbf{I}_M, \mathbf{0}_{M \times L_{zp}}]^T$, and $\mathbf{H}^{(r)}$ is an $N \times N$ lower triangular matrix. Using (2), $\mathbf{H}^{(r)}$ can be written as

$$\mathbf{H}^{(r)} = \sum_{l=0}^{L} \sum_{q=-Q/2}^{Q/2} h_{q,l}^{(r)} \mathbf{D}_q \mathbf{Z}_l, \quad (4)$$

where $\mathbf{D}_q := \text{diag}\{[1, \cdots, e^{j2\pi q(N-1)/N}]^T\}$, and $\mathbf{Z}_l$ is the $N \times N$ lower triangular Toeplitz matrix with first column $[\mathbf{0}_{1\times l}, 1, \mathbf{0}_{1\times N-l-1}]^T$. Substituting (4) in (3), the $N \times 1$ received sample block at the rth receive antenna can be written as:

$$\mathbf{y}^{(r)} = \sum_{l=0}^{L} \sum_{q=-Q/2}^{Q/2} h_{q,l}^{(r)} \mathbf{D}_q \mathbf{Z}_l \mathbf{T}_{zp}\mathbf{s} + \boldsymbol{\eta}^{(r)}. \quad (5)$$

Stacking the N_r received sample blocks: $\mathbf{y} := [\mathbf{y}^{(1)T}, \ldots, \mathbf{y}^{(N_r)T}]^T$, we obtain $\mathbf{y} = \mathbf{H}\mathbf{T}_{zp}\mathbf{s} + \boldsymbol{\eta}$, where $\boldsymbol{\eta}$ is similarly defined as $\mathbf{y}$, and $\mathbf{H} := [\mathbf{H}^{(1)T}, \ldots, \mathbf{H}^{(N_r)T}]^T$. For simplicity, we will assume that the data and the noise are zero-mean white, with variances σ_s^2 and σ_η^2, respectively. We now review linear block equalization, and then investigate TV FIR equalization. We assume perfect channel knowledge at the receiver. In practice, the BEM coefficients have to be estimated. This can be done blindly [6] or by training [7].

3. LINEAR BLOCK EQUALIZATION

We apply a linear block equalizer (LBE) $\mathbf{G}^{(r)}$ on the rth receive antenna. Hence, an estimate of $\mathbf{s}$ is computed as

$$\hat{\mathbf{s}} = \sum_{r=1}^{N_r} \mathbf{G}^{(r)}\mathbf{y}^{(r)} = \left(\sum_{r=1}^{N_r} \mathbf{G}^{(r)}\mathbf{H}^{(r)}\right)\mathbf{T}_{zp}\mathbf{s} + \sum_{r=1}^{N_r} \mathbf{G}^{(r)}\boldsymbol{\eta}^{(r)}. \quad (6)$$

Defining $\mathbf{G} := [\mathbf{G}^{(1)}, \ldots, \mathbf{G}^{(N_r)}]$, we obtain

$$\hat{\mathbf{s}} = \mathbf{G}\mathbf{y} = \mathbf{G}\mathbf{H}\mathbf{T}_{zp}\mathbf{s} + \mathbf{G}\boldsymbol{\eta}.$$

Let us focus on the minimum mean-square error (MMSE) LBE, which minimizes the quadratic cost function $\mathcal{E}\{\|\mathbf{s} - \hat{\mathbf{s}}\|^2\}$. The solution is well-known and given by

$$\begin{aligned}\mathbf{G}_{MMSE} &= (\mathbf{H}\mathbf{T}_{zp})^H(\mathbf{H}\mathbf{T}_{zp}(\mathbf{H}\mathbf{T}_{zp})^H + \sigma_\eta^2/\sigma_s^2\mathbf{I}_N)^{-1} \\ &= ((\mathbf{H}\mathbf{T}_{zp})^H\mathbf{H}\mathbf{T}_{zp} + \sigma_\eta^2/\sigma_s^2\mathbf{I}_M)^{-1}(\mathbf{H}\mathbf{T}_{zp})^H.\end{aligned}$$

The corresponding zero-forcing (ZF) LBE is obtained by setting $\sigma_\eta = 0$:

$$\mathbf{G}_{ZF} = ((\mathbf{H}\mathbf{T}_{zp})^H\mathbf{H}\mathbf{T}_{zp})^{-1}(\mathbf{H}\mathbf{T}_{zp})^H.$$

4. TV FIR EQUALIZATION

In this section, we will apply a TV FIR equalizer $g^{(r)}(n;\nu)$ to the rth receive antenna, as depicted in Figure 1. Hence, an estimate of $s(n-d)$ is computed as

$$\hat{s}(n-d) = \sum_{r=1}^{N_r} \sum_{\nu=-\infty}^{\infty} g^{(r)}(n;\nu)y^{(r)}(n-\nu), \quad (7)$$

where d represents the synchronization delay. Since the doubly-selective channel $h^{(r)}(n;\nu)$ was modeled by the BEM, it is also convenient to design the TV FIR equalizer $g^{(r)}(n;\nu)$ using the BEM. This structure will allow us to turn a large design problem into an equivalent small design problem, containing only the BEM coefficients of both the doubly-selective channel and the TV FIR equalizer.

Using the BEM, we design each TV FIR equalizer $g^{(r)}(n;\nu)$ to have $L'+1$ taps, where the time-variation of each tap is modeled by $Q'+1$ complex exponential basis functions with frequencies on the same FFT grid as the FFT grid for the channel:

$$g^{(r)}(n;\nu) = \sum_{l'=0}^{L'} \delta_{\nu-l'} \sum_{q'=-Q'/2}^{Q'/2} g_{q',l'}^{(r)} e^{j2\pi q'n/N}. \quad (8)$$

Instead of continuing to work on the sample level, it's easier to switch to the block level at this point. On the block level, (7) corresponds to estimating $\mathbf{s}$ as in (6) but with $\mathbf{G}^{(r)}$ constrained to

$$\mathbf{G}^{(r)} = \mathbf{R}_{zp} \sum_{l'=0}^{L'} \sum_{q'=-Q'/2}^{Q'/2} g_{q',l'}^{(r)} \mathbf{D}_{q'} \mathbf{Z}_{l'}, \tag{9}$$

where $\mathbf{R}_{zp} := [\mathbf{0}_{M\times d}, \mathbf{I}_M, \mathbf{0}_{M\times(L_{zp}-d)}]$. It is clear that this requires $0 \leq d \leq L_{zp}$. In the simulation results, we will always take $L_{zp} = \max\{L, d\}$. An estimate of $\mathbf{s}$ is now obtained as

$$\begin{aligned}\hat{\mathbf{s}} = \mathbf{R}_{zp} \sum_{r=1}^{N_r} \left(\sum_{q'=-Q'/2}^{Q'/2} \sum_{l'=0}^{L'} g_{q',l'}^{(r)} \mathbf{D}_{q'} \mathbf{Z}_{l'} \right. \\ \left. \times \sum_{q=-Q/2}^{Q/2} \sum_{l=0}^{L} h_{q,l}^{(r)} \mathbf{D}_q \mathbf{Z}_l \right) \mathbf{T}_{zp} \mathbf{s} \\ + \mathbf{R}_{zp} \sum_{r=1}^{N_r} \sum_{q'=-Q'/2}^{Q'/2} \sum_{l'=0}^{L'} g_{q',l'}^{(r)} \mathbf{D}_{q'} \mathbf{Z}_{l'} \boldsymbol{\eta}^{(r)}.\end{aligned} \tag{10}$$

Defining $p := q + q'$, $k := l + l'$, and using the property $\mathbf{Z}_{l'} \mathbf{D}_q = e^{-j2\pi q l'/N} \mathbf{D}_q \mathbf{Z}_{l'}$, (10) can be rewritten as

$$\begin{aligned}\hat{\mathbf{s}} = \mathbf{R}_{zp} \sum_{p=-(Q+Q')/2}^{(Q+Q')/2} \sum_{k=0}^{L+L'} f_{p,k} \mathbf{D}_p \mathbf{Z}_k \mathbf{T}_{zp} \mathbf{s} \\ + \mathbf{R}_{zp} \sum_{r=1}^{N_r} \sum_{q'=-Q'/2}^{Q'/2} \sum_{l'=0}^{L'} g_{q',l'}^{(r)} \mathbf{D}_{q'} \mathbf{Z}_{l'} \boldsymbol{\eta}^{(r)},\end{aligned} \tag{11}$$

where

$$f_{p,k} := \sum_{r=1}^{N_r} \sum_{q'=-Q'/2}^{Q'/2} \sum_{l'=0}^{L'} e^{-j2\pi(p-q')l'/N} g_{q',l'}^{(r)} h_{p-q',k-l'}^{(r)}. \tag{12}$$

Next, we rewrite (11) as

$$\begin{aligned}\hat{\mathbf{s}} &= (\mathbf{f}^T \otimes \mathbf{I}_M) \tilde{\mathbf{A}} \mathbf{s} + \sum_{r=1}^{N_r} (\mathbf{g}^{(r)T} \otimes \mathbf{I}_M) \tilde{\mathbf{B}} \boldsymbol{\eta}^{(r)} \\ &= (\mathbf{f}^T \otimes \mathbf{I}_M) \tilde{\mathbf{A}} \mathbf{s} + (\mathbf{g}^T \otimes \mathbf{I}_M)(\mathbf{I}_{N_r} \otimes \tilde{\mathbf{B}}) \boldsymbol{\eta},\end{aligned} \tag{13}$$

where we have $\mathbf{f} := [f_{-Q/2-Q'/2,0}, \ldots, f_{-Q/2-Q'/2,L+L'}, \ldots, f_{Q/2+Q'/2,L+L'}]^T$, $\mathbf{g}^{(r)} := [g_{-Q'/2,0}^{(r)}, \ldots, g_{-Q'/2,L'}^{(r)}, \ldots, g_{Q'/2,L'}^{(r)}]^T$, and $\mathbf{g} := [\mathbf{g}^{(1)T}, \ldots, \mathbf{g}^{(N_r)T}]^T$. Defining the matrices $\mathbf{A}$ and $\mathbf{B}$ as

$$\mathbf{A} := \begin{bmatrix} \mathbf{D}_{-Q/2-Q'/2} \mathbf{Z}_0 \\ \vdots \\ \mathbf{D}_{-Q/2-Q'/2} \mathbf{Z}_{L+L'} \\ \vdots \\ \mathbf{D}_{Q/2+Q'/2} \mathbf{Z}_{L+L'} \end{bmatrix}, \quad \mathbf{B} := \begin{bmatrix} \mathbf{D}_{-Q'/2} \mathbf{Z}_0 \\ \vdots \\ \mathbf{D}_{-Q'/2} \mathbf{Z}_{L'} \\ \vdots \\ \mathbf{D}_{Q'/2} \mathbf{Z}_{L'} \end{bmatrix},$$

the matrices $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{B}}$ in (13) are $\tilde{\mathbf{A}} := (\mathbf{I}_{(Q+Q'+1)(L+L'+1)} \otimes \mathbf{R}_{zp}) \mathbf{A} \mathbf{T}_{zp}$ and $\tilde{\mathbf{B}} := (\mathbf{I}_{(Q'+1)(L'+1)} \otimes \mathbf{R}_{zp}) \mathbf{B}$, respectively. Note that the term in $f_{p,k}$ corresponding to the rth receive antenna is related to a 2-dimensional convolution of the BEM coefficients of the doubly-selective channel for the rth receive antenna and the BEM coefficients of the TV FIR equalizer for the rth receive antenna. This allows us to derive a linear relationship between $\mathbf{f}$ and $\mathbf{g}$. We first define the $(L'+1) \times (L'+L+1)$ Toeplitz matrix

$$\mathcal{T}_{l,L'+1}(h_{q,l}^{(r)}) := \begin{bmatrix} h_{q,0}^{(r)} & \ldots & h_{q,L}^{(r)} & & 0 \\ & \ddots & & \ddots & \\ 0 & & h_{q,0}^{(r)} & \ldots & h_{q,L}^{(r)} \end{bmatrix}.$$

We then introduce the notation $\boldsymbol{\mathcal{H}}_q^{(r)} := \boldsymbol{\Omega}_q \mathcal{T}_{l,L'+1}(h_{q,l}^{(r)})$, where $\boldsymbol{\Omega}_q := \text{diag}\{[1, e^{-j2\pi q/N}, \ldots, e^{-j2\pi q L'/N}]^T\}$, and define the $(Q'+1)(L'+1) \times (Q+Q'+1)(L+L'+1)$ block Toeplitz matrix

$$\mathcal{T}_{q,Q'+1}(\boldsymbol{\mathcal{H}}_q^{(r)}) := \begin{bmatrix} \boldsymbol{\mathcal{H}}_{-Q/2}^{(r)} & \ldots & \boldsymbol{\mathcal{H}}_{Q/2}^{(r)} & & \mathbf{0} \\ & \ddots & & \ddots & \\ \mathbf{0} & & \boldsymbol{\mathcal{H}}_{-Q/2}^{(r)} & \ldots & \boldsymbol{\mathcal{H}}_{Q/2}^{(r)} \end{bmatrix}.$$

Introducing the notations $\boldsymbol{\mathcal{H}}^{(r)} := \mathcal{T}_{q,Q'+1}(\boldsymbol{\mathcal{H}}_q^{(r)})$ and $\boldsymbol{\mathcal{H}} := [\boldsymbol{\mathcal{H}}^{(1)T}, \ldots, \boldsymbol{\mathcal{H}}^{(N_r)T}]^T$, we can then derive from (12) that

$$\mathbf{f}^T = \mathbf{g}^T \boldsymbol{\mathcal{H}}. \tag{14}$$

Let us focus on the MMSE TV FIR equalizer, which minimizes the quadratic cost function $\mathcal{E}\{\|\mathbf{s} - \hat{\mathbf{s}}\|^2\}$. Using (13) and (14), the MSE can be written as:

$$\begin{aligned}\mathcal{E}\{\|\mathbf{s} - \hat{\mathbf{s}}\|^2\} = \sigma_s^2 \text{tr}\{(\mathbf{g}^T \boldsymbol{\mathcal{H}} \otimes \mathbf{I}_M) \tilde{\mathbf{A}} \tilde{\mathbf{A}}^H (\boldsymbol{\mathcal{H}}^H \mathbf{g}^* \otimes \mathbf{I}_M)\} \\ + \sigma_\eta^2 \text{tr}\{(\mathbf{g}^T \otimes \mathbf{I}_M)(\mathbf{I}_{N_r} \otimes \tilde{\mathbf{B}} \tilde{\mathbf{B}}^H)(\mathbf{g}^* \otimes \mathbf{I}_M)\} \\ - 2\sigma_s^2 \Re\{\text{tr}\{(\mathbf{g}^T \boldsymbol{\mathcal{H}} \otimes \mathbf{I}_M) \tilde{\mathbf{A}}\}\} + \sigma_s^2 M.\end{aligned} \tag{15}$$

Introducing the properties

$$\begin{aligned}\text{tr}\{(\mathbf{x}^T \otimes \mathbf{I}_M) \mathbf{X}\} = \mathbf{x}^T \text{red}\{\mathbf{X}\}, \\ \text{tr}\{(\mathbf{x}^T \otimes \mathbf{I}_M) \mathbf{X} (\mathbf{x}^* \otimes \mathbf{I}_M)\} = \mathbf{x}^T \text{red}\{\mathbf{X}\} \mathbf{x}^*,\end{aligned}$$

where red$\{\cdot\}$ splits the matrix up into $M \times M$ submatrices and replaces each submatrix by its trace, the MSE can be rewritten as:

$$\begin{aligned}\mathcal{E}\{\|\mathbf{s} - \hat{\mathbf{s}}\|^2\} = \sigma_s^2 \mathbf{g}^T \boldsymbol{\mathcal{H}} \text{red}\{\tilde{\mathbf{A}} \tilde{\mathbf{A}}^H\} \boldsymbol{\mathcal{H}}^H \mathbf{g}^* \\ + \sigma_\eta^2 \mathbf{g}^T (\mathbf{I}_{N_r} \otimes \text{red}\{\tilde{\mathbf{B}} \tilde{\mathbf{B}}^H\}) \mathbf{g}^* \\ - 2\sigma_s^2 \Re\{\mathbf{g}^T \boldsymbol{\mathcal{H}} \text{red}\{\tilde{\mathbf{A}}\}\} + \sigma_s^2 M,\end{aligned} \tag{16}$$

where we have used the fact that red$\{\mathbf{I}_{N_r} \otimes \tilde{\mathbf{B}} \tilde{\mathbf{B}}^H\} = \mathbf{I}_{N_r} \otimes$ red$\{\tilde{\mathbf{B}} \tilde{\mathbf{B}}^H\}$. Defining $\mathbf{r}_A :=$ red$\{\tilde{\mathbf{A}}\}$, $\mathbf{R}_A :=$ red$\{\tilde{\mathbf{A}} \tilde{\mathbf{A}}^H\}$, and $\mathbf{R}_B :=$ red$\{\tilde{\mathbf{B}} \tilde{\mathbf{B}}^H\}$, and solving $\partial \mathcal{E}\{\|\mathbf{s} - \hat{\mathbf{s}}\|^2\} / \partial \mathbf{g} = \mathbf{0}$, we then obtain

$$\begin{aligned}\mathbf{g}_{MMSE}^T &= \mathbf{r}_A^H \boldsymbol{\mathcal{H}}^H (\boldsymbol{\mathcal{H}} \mathbf{R}_A \boldsymbol{\mathcal{H}}^H + \tfrac{\sigma_\eta^2}{\sigma_s^2} (\mathbf{I}_{N_r} \otimes \mathbf{R}_B))^{-1} \\ &= \mathbf{e}_d^T (\boldsymbol{\mathcal{H}}^H (\mathbf{I}_{N_r} \otimes \mathbf{R}_B)^{-1} \boldsymbol{\mathcal{H}} + \sigma_\eta^2 / \sigma_s^2 \mathbf{R}_A^{-1})^{-1} \\ &\quad \times \boldsymbol{\mathcal{H}}^H (\mathbf{I}_{N_r} \otimes \mathbf{R}_B)^{-1},\end{aligned}$$

where we have used the fact that $\mathbf{r}_A^H \mathbf{R}_A^{-1} = \mathbf{e}_d^T$, with $\mathbf{e}_d$ a $(Q+Q'+1)(L+L'+1) \times 1$ unit vector with the 1 in position $d(Q+Q'+1) + (Q+Q')/2 + 1$. The corresponding ZF TV FIR equalizer is obtained by setting $\sigma_\eta = 0$ (see also [8]):

$$\begin{aligned}\mathbf{g}_{ZF}^T = \mathbf{e}_d^T (\boldsymbol{\mathcal{H}}^H (\mathbf{I}_{N_r} \otimes \mathbf{R}_B)^{-1} \boldsymbol{\mathcal{H}})^{-1} \\ \times \boldsymbol{\mathcal{H}}^H (\mathbf{I}_{N_r} \otimes \mathbf{R}_B)^{-1},\end{aligned}$$

When $M \gg L_{zp}$, i.e., when edge effects can be ignored, it is not too difficult to show that $\mathbf{R}_A \approx M\mathbf{I}_{(Q+Q'+1)(L+L'+1)}$ and $\mathbf{R}_B \approx M\mathbf{I}_{(Q'+1)(L'+1)}$, which simplifies the expressions. These simplified but approximate expressions become exact expressions when a cyclic prefix (CP) is used to handle the edge effects (such as in CP-Only or OFDM [1]). Note that in OFDM one can even interchange the TV FIR equalizer with the FFT, and apply the equalizer in the frequency-domain instead of in the time-domain.

5. COMPARISON

Existence: The existence of a ZF LBE requires that $\mathbf{HT}_{zp}$ is of full column rank, which happens with probability one (for i.i.d. BEM coefficients), regardless of N_r. On the other hand, the existence of a ZF TV FIR requires that $\boldsymbol{\mathcal{H}}$ is of full column rank, which happens with probability one (for i.i.d. BEM coefficients), if $N_r(Q'+1)(L'+1) \geq (Q+Q'+1)(L+L'+1)$, which implies that $N_r \geq 2$. For more detailed identifiability results, we refer the interested reader to [9].
Complexity: The design of a TV FIR equalizer requires $\mathcal{O}\{(Q+Q'+1)^3(L+L'+1)^3\}$ flops, instead of the $\mathcal{O}\{M^3\}$ flops needed to design an LBE. Hence, provided that $(Q+Q'+1)(L+L'+1) \leq M$ (which usually is the case), a TV FIR equalizer has a lower design complexity than an LBE. The implementation of a TV FIR equalizer requires $\mathcal{O}\{N_r(L'+1)\}$ flops, instead of the $\mathcal{O}\{N_r N\}$ flops needed to implement an LBE. Hence, provided that $L'+1 \leq N$ (which always is the case), a TV FIR equalizer has a lower implementation complexity than an LBE.

6. SIMULATIONS

In the simulations, we consider a system with $N_r = 1$ and $N_r = 2$ receive antennas. Further, we consider $N = 800$, $T = 25\mu$s, $\tau_{max} = 75\mu$s, $f_{max} = 100$Hz, $L = \lceil \tau_{max}/T \rceil = 3$, $Q = 2\lceil f_{max}NT \rceil = 4$. The channel taps are simulated as i.i.d., correlated in time with a correlation function according to Jakes' model $r_{hh}(\tau) = J_0(2\pi f_d \tau)$. The doubly-selective channel is approximated using the BEM. The resulting BEM coefficients are used to determine the equalizer (serial or block). We use both Jakes' model and the approximated BEM to simulate propagation. In all simulations, QPSK signaling is used and the processing delay is chosen to be $d = \lfloor \frac{L+L'}{2} \rfloor + 1$. The performance is measured in terms of BER vs. SNR. We compare the performance of the MMSE TV FIR equalizer with the performance of the MMSE LBE for both Jakes' model and the BEM. For $N_r = 1$, we take $Q' = L' = 20$, whereas for $N_r = 2$, we take $Q' = L' = 12$. The results are shown in Figure 2. We see that the performance of the MMSE TV FIR equalizer approaches the one of the MMSE LBE, for both Jakes' model and the BEM, while the design as well as the implementation complexity are much lower.

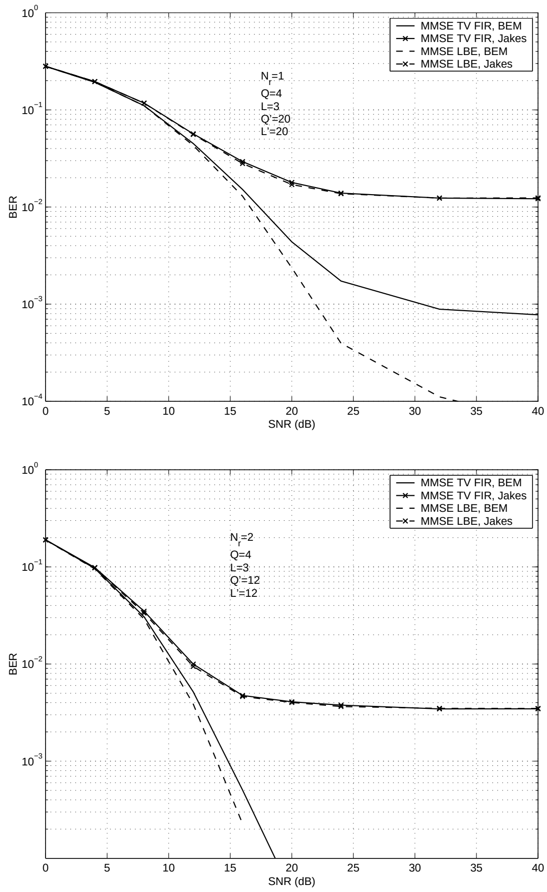

Fig. 2. BER vs. SNR for $N_r = 1$ (top) and $N_r = 2$ (bottom).

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