TIME-OF-ARRIVAL (TOA) ESTIMATION BASED STRUCTURED SPARSE CHANNEL ESTIMATION ALGORITHM, WITH APPLICATIONS TO DIGITAL TV RECEIVERS

*Serdar Özen**, *Michael D. Zoltowski*

Purdue University
School of Electrical & Computer Engineering
West Lafayette, IN 47907-1285
{ozen, mikedz}@ecn.purdue.edu
Phone: +1 765 494-3512

ABSTRACT

In this paper we introduce a new structured channel impulse response (CIR) estimation method for sparse multipath channels where we demonstrate a robust way of restoring the pulse shape into the composite CIR. We call this novel CIR estimation method *Time-Of-Arrival* based *Blended Least Squares* (TOA-BLS) which uses symbol rate sampled signals, and it is based on blending *correlation* processing followed by *TOA* estimation in the frequency domain by the *least squares* based channel estimation. TOA estimation in the frequency domain is accomplished by estimating the AR model parameters by solving the *forward* and *forward-backward linear prediction* equations in the least squares sense. Simulation examples are drawn from the ATSC digital TV 8-VSB system [1]. The delay spread for digital TV systems can be as long as several hundred times the symbol duration; however digital TV channels are *sparse* where there are only a few dominant multipaths.

1. OVERVIEW OF DATA TRANSMISSION MODEL

For the communications systems utilizing periodically transmitted training sequence, *least-squares* (LS) based channel estimation or the *correlation* based channel estimation algorithms have been the most widely used two alternatives. Both methods use a stored copy of the known transmitted training sequence at the receiver. The properties and the length of the training sequence are generally different depending on the particular communication system's standard specifications. In the sequel, although the examples following the derivations of the blended channel estimator will be drawn from the ATSC digital TV 8-VSB system [1], to the best of our knowledge it could be applied with minor modifications to any digital communication system with linear modulation which employs a training sequence. We should note that we are not using semi-blind techniques (subspace based [6] or IQML[5]). Since our main focus is applying this technique in real time on the ATSC Digital TV system which has 10.76 MHz symbol rate [1], we want as low complexity as possible. Computing eigenvectors, or implementing an iterative search scheme, may be prohibitive in real-time for the very long channel lengths that one has to deal with in Digital TV, which may span up to 500 symbols or so.

The baseband symbol rate sampled receiver pulse-matched filter output is given by

$$y[n] \equiv y(t)|_{t=nT} \;=\; \sum_k I_k h[n-k] + \nu[n], \qquad (1)$$

$$I_k = \left\{ \begin{array}{ll} a_k, & 0 \le k \le N-1 \\ d_k, & N \le n \le N'-1, \end{array} \right\} \in \mathcal{A} \equiv \{\alpha_1, \cdots, \alpha_M\} \qquad (2)$$

where I_k is the M-ary complex valued transmitted sequence, $\mathcal{A} \subset \mathbb{C}^1$, and $\{a_k\} \in \mathbb{C}^1$ denote the first N symbols within a *frame* of length N' to indicate that they are the known training symbols; $\nu(t) = \tilde{\nu}(t) * q^*(-t)$ denotes the (colored) noise process after the pulse matched filter, with $\tilde{\nu}(t)$ being a zero-mean white Gaussian noise process with spectral density N_o per real and imaginary part; $h(t)$ is the complex valued impulse response of the composite channel, including pulse shaping filter $q(t)$, the physical channel $c(t)$, and the receive filter $q^*(-t)$, and is given by

$$h(t) \;=\; p(t) * c(t) = \sum_{k=-K}^{L} c_k p(t-\tau_k), \qquad (3)$$

and $p(t) = q(t) * q^*(-t)$ is the convolution of the transmit and receive filters where $q(t)$ has a finite support of $[-T_q/2, T_q/2]$, and the span of the transmit and receive filters, T_q, is integer multiple of the symbol period, T; that is $T_q = N_q T$, $N_q \in \mathbb{Z}^+$. $\{c_k\} \subset \mathbb{C}^1$ denote complex valued physical channel gains, and $\{\tau_k\}$ denote the multipath delays, or the Time-Of-Arrivals (TOA). It is assumed that the time-variations of the channel is slow enough that $c(t)$ can be assumed to be a static inter-symbol interference (ISI) channel, at least throughout the training period with the impulse response

$$c(t) \;=\; \sum_{k=-K}^{L} c_k \delta(t-\tau_k) \qquad (4)$$

for $0 \le t \le NT$, where N is the number of training symbols. The summation limits K and L denote the number of maximum anti-causal and causal multi-path delays respectively. The multipath delays τ_k are *not* assumed to be at integer multiples of the sampling period T.

1.1. Review of Least-Squares Channel Estimation

Without loss of generality symbol rate sampled composite CIR $h[n]$ can be written as a finite dimensional vector $\mathbf{h} = [h[-N_a], \cdots, h[0], \cdots, h[N_c]]^T$ where N_a and N_c denote the number of anti-causal and the causal taps of the channel, respectively, and $N_a + N_c + 1$ is the total memory of the channel. Based on Equation (1) and assuming that $N \ge N_a + N_c + 1$, we can write the pulse matched filter output corresponding only to the known training symbols compactly as

$$\mathbf{y} \;=\; \mathbf{A}\mathbf{h} + \boldsymbol{\nu}, \qquad (5)$$

*This research was partially supported by NSF Grant no. CCR-0118842 and by Zenith Electronics Corporation

where

$$\mathbf{y} = [y[N_c], y[N_c+1], \cdots, y[N-1-N_a]]^T, \quad (6)$$

$$\mathbf{A} = \mathcal{T}\{[a_{N_c+N_a}, \cdots, a_{N-1}]^T, [a_{N_c+N_a}, \cdots, a_0]\} \quad (7)$$

where $\mathbf{A}$ is $(N - N_a - N_c) \times (N_a + N_c + 1)$ Toeplitz convolution matrix with first column $[a_{N_c+N_a}, \cdots, a_{N-1}]^T$ and first row $[a_{N_c+N_a}, \cdots, a_0]$, and $\boldsymbol{\nu} = [\nu[N_c], \nu[N_c+1], \cdots, \nu[N-1-N_a]]^T$. As long as the matrix $\mathbf{A}$ is a tall matrix and of full column rank, that is **(i)** $N \geq 2(N_a+N_c)+1$, **(ii)** rank$\{\mathbf{A}\} = N_a+N_c+1$ then the least squares solution which minimizes the objective function $J_{LS}(\mathbf{h}) = \|\mathbf{y} - \mathbf{A}\mathbf{h}\|^2$ exists and unique, and is given by $\widehat{\mathbf{h}}_{LS} = (\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H\mathbf{y}$. For a single antenna receiver the problems associated with the standard least squares based CIR estimation for digital TV systems is summarized by Özen, et al[7].

2. OVERVIEW OF THE PROPOSED CIR ESTIMATOR

We will first briefly overview the initial channel estimation which involves *correlation, cleaning* followed by TOA estimation in the frequency domain. Once the TOA's are properly obtained we will then present the Blended Least Squares (BLS) algorithm.

2.1. Initial Channel Estimation

Cross correlating the stored training sequence with the received sequence, which is primarily done for frame synchronization [3], yields a raw (uncleaned) channel estimate

$$\tilde{h}_u[n] = \frac{1}{r_a[0]}\sum_{k=0}^{N-1} a_k^* y[k+n], \ n = -N_a, \cdots, 0, \cdots, N_c \quad (8)$$

where $r_a[0] = \sum_{k=0}^{N-1} \|a_k\|^2$. Equation (8) can be written as

$$\tilde{\mathbf{h}}_u = \frac{1}{r_a[0]}\tilde{\mathbf{A}}^H\tilde{\mathbf{y}}, \quad (9)$$

where $\tilde{\mathbf{A}} = \mathcal{T}\{[a_0, \cdots, a_{N-1}, \underbrace{0, \cdots, 0}_{N_a+N_c}]^T, [a_0, \underbrace{0, \cdots, 0}_{N_a+N_c}]\}$ which means that $\tilde{\mathbf{A}}$ is a $(N + N_a + N_c) \times (N_a + N_c + 1)$ Toeplitz matrix with first column $[a_0, a_1, \cdots, a_{N-1}, 0, \cdots, 0]^T$, and first row $[a_0, 0, \cdots, 0]$, and $\tilde{\mathbf{y}} = [y[-N_a], \cdots, y[N + N_c - 1]]^T$. In order to get rid of the sidelobes of the aperiodic autocorrelation we can simply invert the normalized autocorrelation matrix $\boldsymbol{R}_{aa}$ of the training symbols, defined by

$$\boldsymbol{R}_{aa} = \frac{1}{r_a[0]}\tilde{\mathbf{A}}^H\tilde{\mathbf{A}}. \quad (10)$$

Then the *cleaned* channel estimate $\tilde{\boldsymbol{h}}_c$ is obtained from

$$\tilde{\mathbf{h}}_c = \boldsymbol{R}_{aa}^{-1}\tilde{\mathbf{h}}_u, \quad (11)$$

however the channel estimate $\tilde{\mathbf{h}}_c$ obtained by Equation (11) has the contributions due to unknown symbols prior to and after the training sequence, as well as the additive channel noise; only the sidelobes due to aperiodic auto-correlation is removed.

If all the symbols involved in the correlation of Equation (9) were perfectly known then the baseline noise in the estimation vector would have been due to finite correlation of known symbols only. Then the cleaning algorithm would have cleaned this deterministic noise perfectly and we wouldn't have needed any thresholding on the cleaned estimation vector. But we generally have unknown symbols involved in the correlation in most of the practical applications. Previously Özen et al[7] used thresholding in the time domain to obtain the locations of the TOA's as well as to decrease the contributions of the unknown symbols in the correlation output. The correlations of Equation (9) ideally yield *peaks* at $\{D_k^a \in \mathbb{Z}^+\}$, for $k = -K, \cdots, -1, 0$, and at $\{D_k^c \in \mathbb{Z}^+\}$, for $k = 1, \cdots, L$. These delays are the sampling instants closest to the locations of the actual physical channel multi-path TOAs τ_k, $k = -K, \cdots, -1, 0, 1, \cdots, L$, within a symbol interval. If we apply a uniform thresholding to the vector obtained by Equation (11) which is in the form of setting the estimated channel taps to zero if they are below a certain preselected threshold for $n = -N_a, \cdots, -1, 0, 1, \cdots, N_c$, then in general we can choose the tap location with largest magnitude and denote it as the *cursor* (reference) path, and the TOAs prior to and after this reference TOA are denoted as pre- and post-cursor channel impulse responses. However in this work we introduce the estimation of TOA's in the frequency domain via linear prediction.

2.2. TOA Estimation by Estimating the AR model parameters via Forward and Forward-Backward Linear Prediction

Consider the CIR, $h(t)$, given by Equation (3). If we take the Fourier transform of $h(t)$ we obtain

$$H(f) = C(f)P(f) = \left(\sum_{n=-K}^{L} c_n \exp\{-j2\pi f\tau_n\}\right) P(f). \quad (12)$$

Evaluating $H(f)$ at N_{fft} discrete frequency points, that is having $f = k/N_{\mathit{fft}}$, and defining $H[k] = H(k/N_{\mathit{fft}})$, $C[k] = C(k/N_{\mathit{fft}})$, and $P[k] = P(k/N_{\mathit{fft}})$ we obtain

$$H[k] = C[k]P[k] = \left(\sum_{n=-K}^{L} c_n \exp\left\{-\frac{j2\pi k\tau_n}{N_{\mathit{fft}}}\right\}\right) P[k]. \quad (13)$$

Since the pulse shape $p(t)$ and its discrete Fourier transform $P[k]$ is known, we can convert the CIR estimation problem into a *complex sinusoid estimation* problem by writing the CIR discrete frequency response, for the frequencies k where $P[k]$ is nonzero, by

$$C[k] = H[k]/P[k] = \sum_{n=-K}^{L} c_n \exp\left\{-\frac{j2\pi k\tau_n}{N_{\mathit{fft}}}\right\} \quad (14)$$

for all k such that $P[k] \neq 0$. Without loss of generality we assume that $P[k] \neq 0$ for $0 \leq k \leq N_f$ where $N_f = N_{\mathit{fft}}/2$. The rest of the sinusoid estimation problem can be accomplished by following a similar approach to the one outlined in [4, 8]: we can form the *forward linear predictor* (FLP) of order N_p of each sample $C[k]$ for $N_f - 1 \geq k \geq N_p$ based on N_p previous samples, and also *backward linear predictor* (BLP) of order N_p of each sample $C[k]$ for $N_f - N_p - 1 \geq k \geq 0$ based on N_p forward samples. For the forward-backward linear prediction (FBLP) we minimize the *sum* of the FLP and BLP errors in the least squares sense, denoted by $\mathcal{E}_p^{fb}$, and we write

$$\mathcal{E}_p^{fb} = \mathcal{E}_p^f + \mathcal{E}_p^b \quad (15)$$

$$\mathcal{E}_p^f = \sum_{k=N_p}^{N_f-1}\left|C[k] - \sum_{n=1}^{N_p}\rho_n^f C[k-n]\right|^2, \quad (16)$$

$$\mathcal{E}_p^b = \sum_{k=N_p}^{N_f-1}\left|C[k-N_p] - \sum_{n=0}^{N_p-1}\rho_n^b C[k-n]\right|^2, \quad (17)$$

where $\mathcal{E}_p^f$ and $\mathcal{E}_p^f$ denote the forward and backward LP errors respectively; $\rho_n^f, n = 1, \cdots, N_p$, and $\rho_n^b, n = 0, \cdots, N_p - 1$, denote the *forward* and *backward* LP AR-model parameters respectively[8]. It is well known that $\rho_n^b = \rho_{N_p-n}^{f*}$ for $n = 0, 1, \cdots, N_p$, and $\rho_{N_p}^b = \rho_0^f = 1$, which enables us to rewrite the FBLP error equation (15) in terms of the forward LP parameters $\{\rho_n^f\}$ only[8]. Then Equation (15) can be written as (and can be solved via any stable and numerically efficient algorithm such as Conjugate Gradients [2])

$$\boldsymbol{R}_{fb}^H \boldsymbol{R}_{fb} \boldsymbol{\rho} = \boldsymbol{R}_{fb}^H \boldsymbol{c}_{fb} \tag{18}$$

where $\boldsymbol{\rho} = [\rho_1^f, \cdots, \rho_{N_p}^f]^T$,

$$\boldsymbol{R}_{fb} = \left[\boldsymbol{R}_f^T, \boldsymbol{R}_b^T\right]^T, \tag{19}$$

$$\boldsymbol{R}_f = \mathcal{T}\{[C[N_p-1], \cdots, C[N_f-2]]^T, [C[N_p-1], \cdots, C[0]]\} \tag{20}$$

$$\boldsymbol{R}_b = \mathcal{H}\{[C^*[1], \cdots, C^*[N_f-N_p]]^T, [C^*[N_f-N_p], \cdots, C^*[N_f-1]]\}, \tag{21}$$

$$\boldsymbol{c}_{fb} = [\boldsymbol{c}_f^T, \boldsymbol{c}_b^T]^T, \tag{22}$$

where $\boldsymbol{c}_f = [C[N_p], \cdots, C[N_f-1]]^T$, $\boldsymbol{c}_b = [C^*[0], \cdots, C^*[N_f - N_p - 1]]^T$, $\mathcal{H}\{\boldsymbol{v}_{col}, \boldsymbol{v}_{row}\}$ denotes *Hankel* matrix with first column $\boldsymbol{v}_{col}$ and last row $\boldsymbol{v}_{row}$. FLP error of Equation (16) only can be minimized by solving the set of equations

$$\boldsymbol{R}_f^H \boldsymbol{R}_f \boldsymbol{\rho} = \boldsymbol{R}_f^H \boldsymbol{c}_f. \tag{23}$$

Once the unknown AR model parameter vector, $\boldsymbol{\rho}$, is obtained then the complex sinusoids $\{\exp\{-\frac{j2\pi k\tau_n}{N_f}\}\}$ which is also equivalent to estimating the TOAs $\{\tau_n\}$, is accomplished via thresholding the discrete power spectrum

$$\Psi[m] = \frac{\mathcal{E}_p^{fb}}{\left|1 + \sum_{k=1}^{N_p} \rho_k^* e^{-j2\pi mk/N_{ft}}\right|^2}, \tag{24}$$

for $0 \le m \le N_{ft} - 1$, with $N_{ft} > N_f$, and the thresholding is accomplished by

$$\text{set} \quad \hat{\tau}_k = kTN_{fft}/N_{ft} \quad \text{if } \Psi[k] > \varepsilon' \tag{25}$$

where ε' can be set experimentally to a value which is a few standard deviations above the average value of $\Psi[m]$.

We denote the estimated TOAs of the (estimated) channel by $\{D_k^a\}$ and $\{D_k^c\}$ corresponding to the *pre-cursor* (anti-causal) TOA's, and the *post-cursor* (causal) TOA's respectively. It is assumed that $1 \le D_1^c < \cdots < D_L^c$, and similarly $1 \le D_1^a < \cdots < D_K^a$. The relationship between the actual TOA's and the estimated TOAs is given by $D_k^a = -\text{round}\left(\frac{\tau_k}{T}\right)$, for $-K \le k \le 0$, $D_k^c = \text{round}\left(\frac{\tau_k}{T}\right)$ for $1 \le k \le L$. It is also important to note that all the preceding steps for the TOA estimator can only start with the cleaned channel estimate $\tilde{\mathbf{h}}_c$ of Equation (11).

3. OVERVIEW OF BLS ALGORITHM

The channel estimation is performed in two steps using symbol-spaced received samples after the receiver pulse matched filter. In the first step, the received samples are *correlated* with the stored training sequence, and *cleaning* is applied, summarized by Equations (9,11) respectively; and the TOAs are determined as in Section 2.2. The purpose of the second step is to incorporate the transmitted pulse shape $p(t)$ into the channel impulse response. To do this, we locate three copies of $p(t)$ shifted by one-half of a symbol period around each multipath location and estimate complex scaling factors using a modified least squares approach.

In order to recover the pulse shape $p(t)$ into the CIR estimate, for every multi-path we would like to approximate the shifted and scaled copies of the pulse shape $p(t)$ (shifted by τ_k and scaled by c_k) by a linear combination of three pulse shape functions shifted by half a symbol interval. More precisely

$$c_k p(nT - \tau_k) \approx \begin{cases} \sum_{l=-1}^{1} \gamma_l^{(k)} p((n + D_k^a - \frac{l}{2})T), & -K \le k \le 0 \\ \sum_{l=-1}^{1} \gamma_l^{(k)} p((n - D_k^c - \frac{l}{2})T), & 1 \le k \le L \end{cases} \tag{26}$$

where $\{\gamma_l^{(k)}, -K \le k \le L\}_{l=-1}^{1} \subset \mathbb{C}^1$. By making this approximation we claim to efficiently recover the tails of the complex pulse shape $p(t)$ into the CIR estimate. To accomplish this approximation we introduce three vectors $\boldsymbol{p}_k$, for $k = -1, 0, 1$, each containing T spaced samples of the complex pulse shape $p(t)$ shifted by $kT/2$, such that

$$\mathbf{p}_k = [p(-N_q T - \frac{kT}{2}), \cdots, p(-\frac{kT}{2}), \cdots, p(N_q T - \frac{kT}{2})]^T, \tag{27}$$

for $k = -1, 0, 1$, and by concatenating these vectors side by side we define a $(2N_q + 1) \times 3$ matrix $\mathbf{P}$ by $\mathbf{P} = [\mathbf{p}_{-1},\ \mathbf{p}_0,\ \mathbf{p}_1]$.

Then we form the matrix denoted by Γ whose columns are composed of the shifted vectors $\mathbf{p}_k$, where the shifts represent the relative delays of the multi-paths; that is

$$\Gamma = \begin{bmatrix} \mathbf{P} & & \\ \mathbf{0}_{(D_K^a + D_L^c)\times 3} & \ddots & \mathbf{0}_{D_K^a \times 3} \\ & & \mathbf{P} & & \\ & & \mathbf{0}_{D_L^c \times 3} & \ddots & \mathbf{0}_{(D_K^a + D_L^c)\times 3} \\ & & & & \mathbf{P} \end{bmatrix} \tag{28}$$

where Γ is of dimension $(D_K^a + D_L^c + 2N_q + 1) \times 3(K + L + 1)$, and $\mathbf{0}_{m \times n}$ denotes an m by n zero matrix. Then the observation vector $\mathbf{y}$, and the convolution matrix, $\mathbf{A}$, composed only of the known training symbols are defined as in Equations (6, 7) respectively. Since it was assumed that $q(t)$ spans N_q symbol durations, it implies that $q[n]$ has $N_q + 1$ sample points, which in turn implies $p[n]$ has $2N_q + 1$ samples. Hence $N_a = D_K^a + N_q$, and

Table 1. Simulated channel delays in symbol periods, relative gains ($K = 2$ *pre-cursor* ghosts, $L = 6$ *post* ghosts)

Channel taps	Delay $\{\tau_k\}$	Gain $\{\lvert c_k \rvert\}$
$k = -2$	-60.277	0.55
$k = -1$	-0.957	0.7263
Main $k = 0$	0	1
$k = 1$	3.551	0.6457
$k = 2$	15.250	0.9848
$k = 3$	24.032	0.7456
$k = 4$	29.165	0.8616
$k = 5$	221.2345	0.6150
$k = 6$	332.9810	0.4900

$N_c = D_L^c + N_q$. Defining $\boldsymbol{\gamma}^{(k)} = [\gamma_{-1}^{(k)}, \gamma_0^{(k)}, \gamma_1^{(k)}]$ for $-K \leq k \leq L$, we define $\boldsymbol{\gamma} = [\boldsymbol{\gamma}^{(-K)}, \cdots, \boldsymbol{\gamma}^{(0)}, \cdots, \boldsymbol{\gamma}^{(L)}]^T$, as the unknown vector of the coefficients with $\{\gamma_n^{(k)}, n = -1, 0, 1; k = -K, \cdots, 0, \cdots, L\}$, of length $3(K + L + 1)$. Then the observation vector is given by

$$\mathbf{y} = \mathbf{A}\Gamma\boldsymbol{\gamma} + \boldsymbol{\nu} \tag{29}$$

where $\boldsymbol{\nu}$ is the noise vector. We can estimate the unknown coefficient vector by $\widehat{\gamma}_{BLS} = \left(\Gamma^H \mathbf{A}^H \mathbf{A}\Gamma\right)^{-1} \Gamma^H \mathbf{A}^H \mathbf{y}$. Once the vector $\widehat{\gamma}_{BLS}$ is obtained, the new channel estimate $\widehat{\mathbf{h}}_{BLS}$, can simply be obtained by

$$\widehat{\mathbf{h}}_{BLS} = \Gamma\widehat{\gamma}_{BLS}. \tag{30}$$

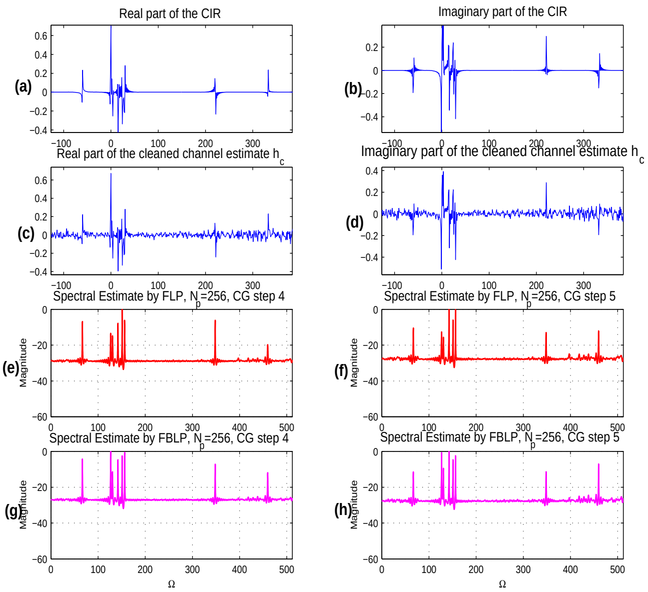

Fig. 1. Parts (a-b) show the real and imaginary parts of the actual CIR; parts (c-d) show the cleaned CIR estimate, $\tilde{\boldsymbol{h}}_c$; parts (e-f) show the TOA estimates with FLP of order $N_p = 256$ at CG iterations 4 and 5 respectively; parts (g-h) show the TOA estimates with FBLP of order $N_p = 256$ at CG iterations 4 and 5 respectively.

4. SIMULATIONS

We considered an 8-VSB [1] receiver with a single antenna. 8-VSB system has a complex raised cosine pulse shape [1]. The CIR we considered is given in Table 1. The phase angles of individual paths for all the channels are taken to be $\arg\{c_k\} = \exp(-j2\pi f_c \tau_k)$, for $k = -K, \cdots, L$ where $f_c = \frac{50}{T_{sym}}$ and $T_{sym} = 92.9$nsec. The simulations were run at 28dB Signal-to-Noise-Ratio (SNR) measured at the input to the receive pulse matched filter, and it is calculated by

$$\text{SNR} = \frac{\mathcal{E}_s(\sum_{\forall k} \|\{c(t) * q(t)\}_{t=kT}\|^2)}{N_0}, \tag{31}$$

where $E_s = 21$ is the symbol energy for 8-VSB system, and N_0 is the variance of channel noise $nu(\tilde{k}T)$. We set $N_{fft} = 2^{10}$, $N_{ft} = 2^{13}$, and $\varepsilon' = \overline{\Psi[k]} + 4\sigma_{\{\Psi[k]\}}$, that is we set the ε' to 4 standard deviations above the average of the power spectrum $\Psi[k]$. Figure 2 shows the simulation results for the test channel provided in Table 1. Part (a) shows the actual CIR; part (b) shows the CIR estimate, $\widehat{h}_{BLS}$, based on BLS with estimated TOAs; part (c) shows the CIR estimate, $\widehat{h}_{BLS}$, based on BLS with perfectly known TOAs. As can be seen in Figure 2 either the 4th or the 5th iterations of the CG algorithm can be taken as the solutions to the FBLP and FLP equations (18,23) to compute the spectrum $\Psi[k]$. Finally we have observed a very promising performance by using estimated TOA parameters for the BLS algorithm to compute the CIR estimate, where the normalized least-squares error is very close to that of the BLS when the true TOAs are used.

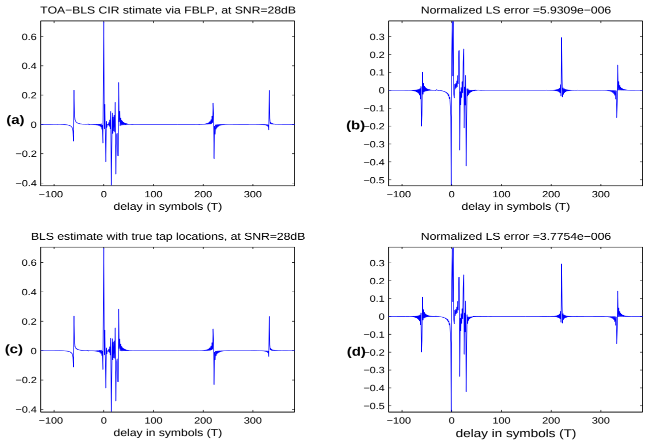

Fig. 2. Parts (a-b) show the real and imaginary parts of the CIR estimate, $\widehat{h}_{BLS}$, based on BLS with estimated TOAs via FBLP; parts (c-d) show the CIR estimate based on BLS with perfectly known TOAs.

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