MATRIX GRADUATED NONCONVEXITY ANNEALED NEURAL NETWORK FOR DS-CDMA MULTIUSER DETECTOR

Bijaya Nepal, M.H. Tan, T.T. Tjhung and Y.H. Chew
Institute for Communications Research, 20 Science Park Road,
Tele Tech Park, Singapore Science Park II, Singapore 117674
(e-mail:engp1710@nus.edu.sg)

ABSTRACT

In this paper, we propose an efficient two-stage DS-CDMA multiuser detector (MUD). The first stage is a reduced linear detector and the second stage is a matrix graduated nonconvexity annealed neural network (MGN-ANN). By using a first stage linear detector, the computational complexity in the second stage can be significantly reduced. We carried out extensive simulations in order to compare the error probability performance of our proposed detector with other competing multiuser detectors, which are based on conventional matched filter detector (CD), Hopfield Neural Network (HNN), multistage detector (MS-10) and annealed neural network (ANNMD). We also use the optimum multiuser detector (OMD) as comparison benchmark for all the MUDs. We show that the error probability performance of our proposed detector is significantly better than the other suboptimum MUD's and approaches the performance of the OMD.

1. INTRODUCTION

Verdu's maximum likelihood sequence estimation (MLSE) based optimum multiuser detector (OMD) [1], which can be implemented by a dynamic programming algorithm, has a computation complexity that grows exponentially with the number of users and becomes infeasible in real time demodulation under multiple access user environment. This has motivated researchers around the globe to propose other feasible sub-optimal scheme whose complexity is only proportional to the number of users. Multistage detector, decorrelating detector and neural network based detectors are some of the extensively studied suboptimum detectors.

In [2], Kechriotis et al. proposed a hybrid type of multiuser detector that uses a linear detector (reduced detector) in the first stage and a Hopfield neural network multiuser detector in the second stage to solve the remaining optimization problem. In this paper, as in [2], we also propose a hybrid type of multiuser detector that consists of a reduced detector in the first stage but in the second stage, we use a matrix graduated nonconvexity annealed neural network (MGN-ANN) to get a more superior error probability performance.

2. DS-CDMA COMMUNICATION SYSTEM

We start with a mathematical description of a DS-CDMA communication system. In a synchronous CDMA channel, there is no relative delay among different user signals when arriving at the base station receiver. But in practical CDMA applications, the channel is asynchronous i.e. there exist relative time delays among different user signals at the receiver. Assuming there are K active users sharing the same radio channel and the k^{th} active user is assigned a signature waveform $S_k(t)$, $t=(0,T)$, where T is the bit duration. The signal at a base station receiver is the superposition of K transmitted signals and channel noise as follows:

$$r(t)=\sum_{l=-P}^{P}\sum_{k=1}^{K}\sqrt{2W_k}\,b_k^{(l)}S_k(t-lT-\tau_k)+n(t) \tag{1}$$

Here $\tau_k \in [0,T]$ denotes the relative time delays between the users, $2P+1$ is the packet size, $b_k^{(l)} \in \{+1,-1\}$ is the lth information bit of the kth user, W_k is the power of the kth user and $n(t)$ is the additive zero mean Gaussian receiver noise (AWGN). For the synchronous case where τ_k =0, k=1…..K, after the received signal $r(t)$ passes through a bank of matched filters which matches to each users' signature waveform $S_k(t)$, its sampled output $y_k^{(l)}$ at $t=lT$ are as follows:

$$y_k^{(l)}=\int_{lT}^{(l+1)T} r(t)S_k(t-lT)\,dt,\ k=1,2,\ldots,K \tag{2}$$

$\mathbf{y}^{(l)}=[\,y_1^{(l)}\ y_2^{(l)}\ \ldots\ldots\ y_k^{(l)}\,]^T$ is the sufficient statistics for demodulating $\mathbf{b}^{(l)}=[b_1^{(l)}\ b_2^{(l)}\ \ldots\ldots\ b_k^{(l)}]^T$.

Substituting (1) into (2) for all the K users, the K equations expressed in matrix form , we have

$$\mathbf{y}(l) = \mathbf{H}\mathbf{b}(l) + \mathbf{n} \tag{3}$$

where $\mathbf{H} \in R^{K\times K}$ is the symmetric nonnegative definite matrix of signature cross correlations with its ijth element

$hij = \int_{lT}^{(l+1)T} S_i(t).Sj(t)dt$ and $\mathbf{n}$ is the K-th dimensional Gaussian noise vector.

The optimum multiuser detector (OMD) obtains its detection output by selecting the most likely hypothesis $\hat{\mathbf{b}} = [\hat{b}_1 \;\; \hat{b}_2 \ldots\ldots \hat{b}_k]^T$ given the observation $\mathbf{y}$ which corresponds to selecting the noise realization with minimum energy

$$\hat{\mathbf{b}} = \underset{b\in\{+1,-1\}^K}{\arg\min}\left\{\mathbf{b}^T\mathbf{H}\mathbf{b} - 2\mathbf{y}^T\mathbf{b}\right\} \tag{4}$$

or equivalently, $\hat{\mathbf{b}} = \underset{b\in\{+1,-1\}^K}{\arg\max}\left\{2\mathbf{y}^T\mathbf{b} - \mathbf{b}^T\mathbf{H}\mathbf{b}\right\}$

The above expressions can be generalized for the asynchronous case as follows:

$$\hat{\mathbf{b}} = \underset{b\in\{+1,-1\}^{(2P+1)K}}{\arg\max}\left\{2\mathbf{y}^T\mathbf{b} - \mathbf{b}^T\widetilde{\mathbf{H}}\mathbf{b}\right\} \tag{5}$$

where

$\mathbf{y} = [y_1^{(-p)}\, y_2^{(-p)} \ldots\, y_k^{(-p)}\, y_1^{(-p+1)}\, y_2^{(-p+1)} \ldots\, y_k^{(-p+1)} \ldots y_1^{(p)}\, y_2^{(p)} \ldots\, y_k^{(p)}]^T \in \mathrm{R}^{(2p+1)K\times 1}$ and $\widetilde{\mathrm{H}} \in \mathrm{R}^{(2p+1)K\times(2p+1)K}$ is the symmetric matrix.

3. MUD BASED ON HOPFIELD AND ANNEALED NEURAL NETWORKS

HNN is a feedback recurrent neural network with many nonlinear processing units that are interconnected by feedback lines. The dynamics of the hopfield networks are described by a system of nonlinear ordinary differential equations and by an associated computation energy (Lyapunov) function, which is minimized during the computation process [3]. The Hopfield networks are particularly suitable for optimization of difficult combinatorial problems.

The energy function and the equation of motion describing the time evolution of Hopfield network can be written respectively as follows:

$$E = -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} T_{ij}V_iV_j - \sum_{i=1}^{N} V_iI_i \tag{6}$$

$$\Delta U_i = -\frac{U_i}{\tau_i} + \sum_{i\neq j} T_{ij}V_j + I_i \tag{7}$$

Here U_i and V_i are the state and output of the i^{th} unit, T_{ij} is the conductance between neuron i and neuron j, I_i is the externally supplied bias to the neuron i, N is the number of neurons and τ is the time constant of the RC circuit. Hopfield [4] showed that if the conductances are symmetric ($T_{ij} = T_{ji}$), this type of network will always converge to a stable state.

The input-output relation in the Hopfield network can be given using the sigmoid function

$$V_i = \mathrm{f}(U_i) = \frac{1-\exp(-\alpha U_i)}{1+\exp(-\alpha U_i)} , \tag{8}$$

Here α is the gain of the sigmoid function.

It is due to this similarity in (4) and (6) that makes Hopfield network suitable for the multiuser detection. Therefore, the OMD objective function (4) can be directly transformed into the HNN energy function (6) with slightly modified weight matrix $T_{ij} = (h_{ij} - e_{ij})$, where e_{ij} is the ij^{th} element of a diagonal matrix E with $e_{ii} = \int_0^T S_i^2 dt$ and $I_i = y_i$.

Even with the correct choice of parameters for the HNN, the performance will be degraded if the OMD objective function has a lot of local minima. The number of local minima that OMD objective function exhibits depend on the additive channel noise, the relative powers and delays of the users and the spreading codes assigned to them [5]. The HNN is only guaranteed to converge to local minima. So in order to avoid the network converging into a local minima instead of global minima, simulated annealing technique is used which perturb the energy function and seeks the global minimum by slowly decreasing the temperature from a high value or slowly increasing the neural gain of the sigmoid function from a low value. The ANNMD is based on simulated annealing and does not require good initial states [6].

4. PROPOSED HYBRID DETECTOR

This detector is a combination of two stages as shown in the Figure 1. The first stage is a digital signal-processing device (also known as reduced detector [2]) that will reduce the size of the optimization process. The second stage involves a matrix graduated nonconvexity annealed neural network. The outputs from the conventional matched filters (MF) form the inputs to the reduced detector, which in turn reduces the size of the OMD objective function to a manageable size before passing these to the second stage MGN-ANN detector.

Kechriotis et al. in [2] has proved that if for some element i of the observation vector y in (4), the inequality

$$\sum_{j=1, j\neq i}^{K} \left|\mathrm{h}_{ij}\right| < \left|\mathrm{y}_i\right| \qquad (9)$$

holds, then the OMD's estimate for the corresponding transmitted information bit will be $b_{opt,i} = sign(y_i)$. This means that if for some user i, (9) holds, then the OMD's estimate for the corresponding transmitted information bit will coincide with the conventional detector's estimate. And once the condition (9) is checked for all the users, we can partition the observation vector y as $\mathbf{y}^T = [\mathbf{y}_r^T \mid \mathbf{y}_w^T]$ where $\mathbf{y}_r^T$ denotes all the users that satisfy (9), and $\mathbf{y}_w^T$ denotes those users which do not satisfy (9). In the same way we can also partition the matrix $\mathbf{H}$ and vector $\mathbf{b}$ as follows:

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{rr} & \mathbf{H}_{rw} \\ \mathbf{H}_{wr} & \mathbf{H}_{ww} \end{bmatrix} \quad \text{and } \mathbf{b} = \begin{bmatrix} \mathbf{b}_r \\ \mathbf{b}_w \end{bmatrix}$$

By partitioning $\mathbf{y}$, $\mathbf{b}$ vectors and $\mathbf{H}$ matrix, the OMD optimization problem now can be reduced to a smaller equivalent problem of the same structure

$$\mathbf{b}_r = \underset{b\in\{+1,-1\}^K}{\arg\min} \{-2\mathbf{y}_{new}^T \mathbf{b}_w + \mathbf{b}_w^T \widetilde{\mathbf{H}}_{ww} \mathbf{b}_w\} \qquad (10)$$

where $$\mathbf{y}_{new} = \mathbf{y}_w - \mathbf{H}_{rw}^T \mathbf{b}_r \qquad (11)$$

After iterating the reduced detector algorithm and each time checking for the compliance of (9), we finally arrive at a point where each remaining element does not satisfy the above condition in an iteration and we need to employ annealed neural network for the remaining optimization problem.

Besides using the annealing technique, we also use the matrix graduated nonconvexity technique to get the better quality solution. In this technique, we add one penalty term E_p, $E_p = k \sum_{i=1}^{N}(1 - V_i^2)$, k is penalty parameter, in the annealed energy function so that the final output converges to a valid solution on the corner of an N-dimensional hypercube and minimizes the probability of being trapped in a bad local minima.

The penalty parameter k is tuned in such a way that the energy function E is initially kept as convex as possible so that the optimization problem has a single global minimum and then it is increased gradually ending with a positive value [3]. Using both the annealing and matrix graduated nonconvexity methods; we are able to get better solution for our multiuser detection problem.

5. SIMULATION RESULTS AND DISCUSSIONS

In our first example, we consider K=7 synchronous users employing poorly designed spreading codes of length L=4, sharing a common radio channel. The first user's energy is 10 times larger than the rest of the users. We append a +1 to the binary representation of decimal number from 0 to 6 to form the seven spreading codes. Thus we have the following spreading codes: $a^{(1)}$=(1,-1,-1,-1), $a^{(2)}$=(1,-1,-1,1), $a^{(3)}$=(1,-1,1,-1), $a^{(4)}$=(1,-1,1,1), $a^{(5)}$=(1,1,-1,-1), $a^{(6)}$=(1,1,-1,1), $a^{(7)}$=(1,1,1,-1). In Figure 2 we show the logarithm of the cumulative bit error rate (BER). It is clearly seen from the figure that our proposed hybrid detector performs better than other sub-optimal detectors such as multistage interference cancellation detector (MS-10), Hopfield neural network detector (HNN) and Annealed neural network detector (ANNMD) and almost as well as the OMD.

In our second example, we consider K=5 synchronous users employing poorly designed spreading codes of length L=4. The spreading codes are: $a^{(1)}$=(1,-1,-1,-1), $a^{(2)}$=(1,-1,-1,1), $a^{(3)}$=(1,-1,1,-1), $a^{(4)}$=(1,-1,1,1), $a^{(5)}$=(1,1,-1,-1) and the power ratio is such that the energies of the first 4 users are 6 dB higher than the last user. In this simulation we are interested in the BER of the weakest user in a very severe near far condition. From Figure 3 we can see that the proposed hybrid detector performs better than the other sub-optimal detectors.

In the third example, we consider 4 asynchronous users employing spreading codes of length L=7 (m-sequence). Each user sends $2P+1$=31 bits long packets. The near far ratio is such that the energy of the other three users is 4, 8 and 10 dB higher than the energy of the first user. We show the cumulative BER of the whole asynchronous system. Figure 4 shows that the hybrid detector performs best amongst the sub-optimal detectors.

6. CONCLUSIONS

We have proposed a sub-optimum hybrid annealed multiuser detector for DS-CDMA system and have investigated its BER performance in AWGN channel. We show from simulations that the proposed MUD outperforms other sub-optimal detectors and approaches the performance of the OMD but with much lesser computational complexity than the OMD.

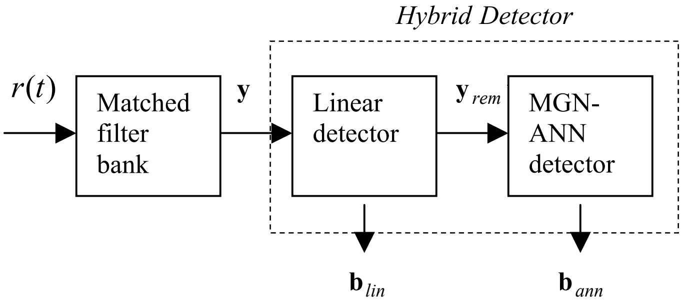

Fig. 1. *Hybrid MUD*

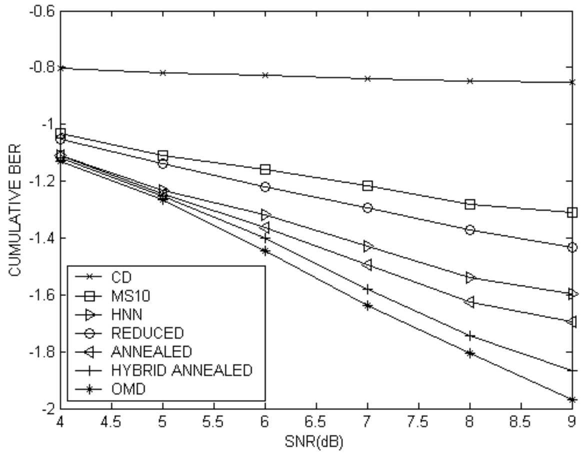

Fig. 2. BER performance of 7 synchronous users

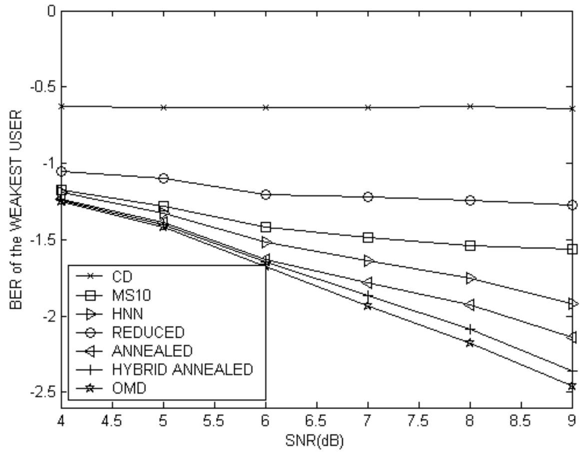

Fig. 3. BER performance of the weakest user

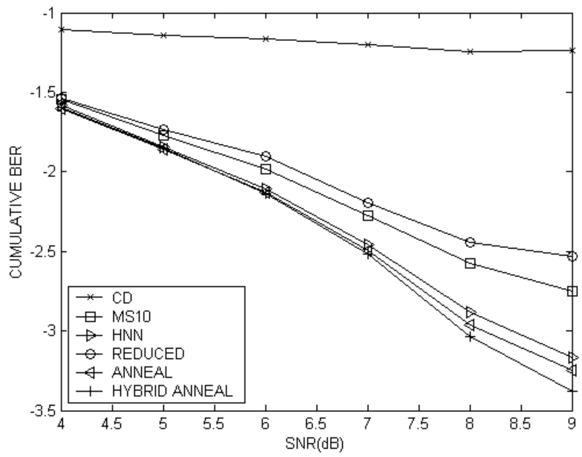

Fig. 4. BER Performance for 4 asynchronous users

REFERENCES

[1] S. Verdu, “Minimum Probability of Error for Asynchronous Gaussian Multiple access Channels”, IEEE Trans. on Info. Theory, vol. 32, no. 1, pp. 85-96, January 1986

[2] George I. Kechriotis and E.S.Manolakos,”A Hybrid Digital Signal Processing – Neural CDMA Multiuser Detection Scheme”, IEEE Trans. on Circuits and Systems – II: Analog and Digital Signal Processing, vol. 43, no. 2, pp. 96-104, February 1996

[3] A.Cichocki and R. Unbehauen, “Neural Networks for Optimization and Signal Processing”, John Wiley & Sons Ltd., 1993

[4] J.J. Hopfield,”Neurons with graded response have collective computational properties like those of two state neurons”, in Proc. Nat. Acad. Sci.USA, vol. 81, pp. 3088-3092,1984

[5] George I. Kechriotis and E.S.Manolakos,”Hopfield Neural Network Implementation of the Optimal CDMA Multiuser Detector”, IEEE Trans. on Neural Network, vol 7, no. 1, pp. 131-141, January 1996

[6] S.H.Yoon and S.S. Rao, “Multiuser Detector in Code Division Multiple Access Communications Using Annealed Neural Networks”, Report, University of Villanova, Dept. of Electrical Engineering, Villanova, PA, USA, 1997.