IMPROVED PARALLEL INTERFERENCE CANCELLATION BASED ON CONDITIONAL LIKELIHOOD FUNCTION

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ABSTRACT

An improved parallel interference cancellation (PIC) detector based on conditional likelihood function is presented. The improved PIC benefits from an additional correcting module after the last stage, where the maximum likelihood algorithm using only the conditional likelihood function, which is calculated from the information of the tentative decisions, is employed. Analysis and simulation show that the new algorithm can improve the bit error rate (BER) performance of PIC with much lower computation complexity than optimum detector.

1. INTRODUCTION

In DS-CDMA systems, multi-user detection is the key technique for alleviating or even eliminating multiple access interference (MAI) [1]. The optimum detector was first proposed by Verdú, which employs the maximum likelihood (ML) algorithm and suffers from the fact that its complexity grows exponentially with the number of the users [2]. In 1990, Varanasi and Aszhang suggests a parallel interference cancellation (PIC) detector, which has modular structure and is easy to realize [3]. But its BER is much higher than the optimum detector.

In PIC, the information of the tentative decisions can be used to produce the conditional likelihood function, whose solution space has much smaller dimensions than that of ML function, so maximum likelihood detection on the conditional function behind PIC can provide performance improvement without much higher complexity.

2. SYSTEM MODEL

A K-user synchronous DS-CDMA system is considered. Sampled at the chip rate $1/T_c$, the base band received signal in a symbol period T is given by

$$\mathbf{r}(i) = \sum_{k=1}^{K} A_k \mathbf{s}_k b_k(i) + \mathbf{n}(i) \quad (1)$$

where A_k, $\mathbf{s}_k = \left[s_{k,1}, \cdots s_{k,N}\right]^T$ and $b_k(i)$ are the signal amplitude, spreading sequence and data bits of the kth user; $N = T/T_c$ is processing gain; $\mathbf{n}(i)$ is white gaussian noise with $E[\mathbf{n}(i)\mathbf{n}^T(i)] = \sigma^2\mathbf{I}$.

The spreading sequences of all users are known in the base station, so the signal is passed through a group of matched filters

$$\mathbf{y} = \mathbf{RAb} + \mathbf{z} \quad (2)$$

where $R = [\rho_{ij}]_{j=1:K}^{i=1:K}$ is the correlation matrix .

3. PIC BASED ON CONDITIONAL LIKELIHOOD FUNCTION

Suppose y_k^m is the input signal of the kth user in the mth stage of PIC, obviously $y_k^1 = y_k$; and a following sign function will give the tentative decision d_k^m, so

$$y_k^{m+1} = y_k - \sum_{j\neq k}^{K} \rho_{jk} d_k^m = y_k - \sum_{j \neq k}^{K} \rho_{jk}\,\mathrm{sgn}(y_k^m) \quad (3)$$

Rewrite PIC in iterative form:

$$\mathbf{d}^{m+1} = \mathrm{sgn}(\mathbf{y} - (\mathbf{R} - \mathbf{I})\,\mathbf{A}\mathbf{d}^m) \quad (4)$$

From the preceding study, some useful facts are shown:

1. After many enough stages, the tentative decisions will convergence to a fixed point or come into a cycle [4]; the output of optimum detectors is one of the fixed points.
2. In high enough SNR, BER of PIC will be so low that only a few bits are different between the tentative decisions of two neighboring stages [5].
3. In the study of differential PIC [6], if the tentative decisions of two neighboring stages are equal, the output of PIC can be given by the decisions and it makes little difference in BER performance.

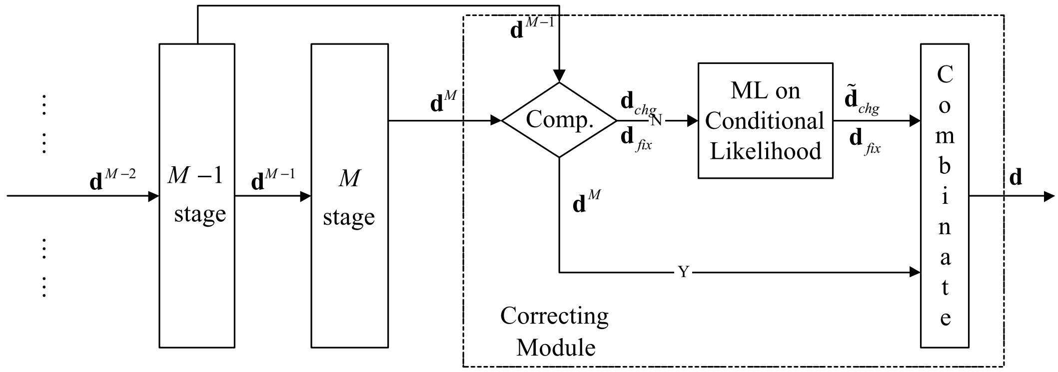

Fig.1 Detector Structure of CLPIC

By comparing tentative decisions of the last two stages, users are divided into two groups: in the first group, tentative decisions are equal and so considered right; in the second group, tentative decisions are unequal and considered wrong. It is easy to conclude that the users' number in the second group is relatively small. In traditional PIC, the output is given directly by the tentative decisions of the last stage; while in this paper, group information is used to produce conditional likelihood function, then ML algorithm based on the function will be adopted after the last stage.

3.1. Conditional Likelihood Function

Optimum detectors tend to solve maximum likelihood problem with K users:

$$\tilde{\mathbf{d}} = \arg\max_{\mathbf{d}\in\{-1,+1\}^K} 2\mathbf{d}^T\mathbf{Ay} - \mathbf{d}^T\mathbf{ARAd} \tag{5}$$

Its solution space comprises 2^K vectors. But a lot of know bits can decrease the dimension of the space. Suppose $\mathbf{d}_f$ and $\mathbf{d}_c$ denote separately known bits and unknown bits. Accordingly, $\mathbf{x}=\mathbf{Ay}$ is divided to $\mathbf{x}_f$ and $\mathbf{x}_c$; $\mathbf{H}=\mathbf{ARA}$ is divided to $\mathbf{H}_f$, $\mathbf{H}_c$ and $\mathbf{H}_I$, which denotes interaction between the two groups. Rewrite ML function as

$$\begin{aligned} L(\mathbf{d}) &= 2\mathbf{d}_c^T(\mathbf{x}_c - \mathbf{H}_I\mathbf{d}_f) - \mathbf{d}_c^T\mathbf{H}_c\mathbf{d}_c \\ &\quad + 2\mathbf{d}_f^T\mathbf{x}_f - \mathbf{d}_f^T\mathbf{H}_f\mathbf{d}_f \end{aligned} \tag{6}$$

So conditional likelihood function, which has the same form as ML function, is given by

$$L(\mathbf{d}_c \mid \mathbf{d}_f) = 2\mathbf{d}_c^T(\mathbf{x}_c - \mathbf{H}_I\mathbf{d}_f) - \mathbf{d}_c^T\mathbf{H}_c\mathbf{d}_c \tag{7}$$

3.2. Detector Structure

Fig.1 shows the detector structure of improved PIC called CLPIC. Compared to traditional PIC, a correcting module is added to the last stage of PIC. In the module, tentative decisions of the last two stages are compared first. If they are equal, $\mathbf{d}=\mathbf{d}^M$, otherwise, solve the following optimization problem:

$$\tilde{\mathbf{d}}_c = \arg\max_{\mathbf{d}_{chg}\in\{-1,+1\}^K} L(\mathbf{d}_c \mid \mathbf{d}_f) \tag{8}$$

then $\mathbf{d}$ is the combination of $\mathbf{d}_f$ and $\tilde{\mathbf{d}}_c$.

4. ANALYSIS

Suppose X^m is the bit error of one user, who is average of all users and an imaginary user. $X^m=0$ denotes the bit decision is right while $X^m=1$ denotes it's wrong. Obviously

$$\begin{aligned} P(X^m=0) &= 1-P_e^m \\ P(X^m=1) &= P_e^m \end{aligned} \tag{9}$$

where P_e^m is average BER in m th stage. So $\{X^m\}$ has no after effect and is a Markov chain with transient probability matrix

$$\mathbf{P}(1) = \begin{bmatrix} p^m & 1-p^m \\ 1-q^m & q^m \end{bmatrix} \tag{10}$$

When the number of the stages approaches infinite, BER of PIC will convergence to a fixed value [7]; and it is also true for transient probability. So omit all the superscripts in (10), and we will get transient probability in the case of infinite stages. Suppose ultimate distribution of PIC's BER is P_e^{PIC}. It is obvious

$$\begin{aligned} P_1 &= (1-P_e^{PIC})p \\ P_2 &= P_e^{PIC}q \\ P_3 &= (1-P_e^{PIC})(1-p) = P_e^{PIC}(1-q) \end{aligned} \tag{11}$$

where P_1, P_2, P_3 are the joint probability of the cases when both tentative decisions of two neighboring stages are right, when both decisions are wrong, when the former is right and the latter is wrong or when the former is wrong and the latter is right. So

$$P_e^{PIC} = P_2 + P_3$$
$$P_1 + P_2 + 2P_3 = 1 \quad (12)$$

CLPIC compares tentative decisions of the last two stages and employs ML detection on conditional likelihood function. It falls into two occasions:

1. No user's tentative decisions are both wrong. In this case, conditional likelihood function involves all the wrong bits and a solution better than that of optimum detectors is possible. BER is so low compared to PIC that bit errors are omitted to simplify our discussion.
2. Some users' decisions are both wrong. It may bring more bit errors. The worst case that all bits involved in conditional likelihood function is considered. So BER of CLPIC is shown below:

$$\begin{aligned} P_e^{CL} &\le \frac{\sum_{j=1}^{K} C_k^j P_2^j \sum_{i=0}^{K-j} C_{K-j}^i (i+j) P_1^{K-j-i} (2P_3)^i}{K} \\ &= P_2 + (1-P_1-P_2)[1-(1-P_2)^{K-1}] \\ &= P_2 + 2P_3[1-(1-P_2)^{K-1}] \end{aligned} \quad (13)$$

Comparing (12) and (13), if only $(1-P_2)^{K-1} > 1/2$, we have $P_e^{CL} < P_e^{PIC}$. In high SNR, the condition is easy to satisfy. Further, it can be concluded from (11) that $P_2 = o(P_e)$, so $P_e^{CL} = o(P_e)$.

The average size of $\mathbf{d}_c$ is

$$\begin{aligned} \bar{l} &= \sum_{i=1}^{K} i C_K^i (\mathrm{P}_1 + \mathrm{P}_2)^{K-i} (1 - \mathrm{P}_1 - \mathrm{P}_2)^i \\ &= K(1-P_1-P_2) = 2KP_3 \end{aligned} \quad (14)$$

The computation complexity is $O(2^{2KP_3})$. When the users' number is very large, the computation will be costly.

5. COMPUTATION COMPLEXITY

For simplification, suppose that the complexity of matched filters and sign function are omitted; multiplication of known matrixes is calculated beforehand and the complexity is not considered; compare is equal to minus; computation complexity is measured by computation units, one of which is consisted of one multiplication and one plus-minus.

5.1. Maximum Likelihood

For each vector $\mathbf{d}$ in (5), calculating $2\mathbf{d}^T\mathbf{x}$ needs K multiplication and $K-1$ plus minus, and calculating $\mathbf{d}^T\mathbf{H}\mathbf{d}$ needs K^2 multiplication and K^2-1. In the solution space there are 2^K vectors, so 2^K-1 compare is needed. Hence, the computation complexity of ML is $2^K(K^2+K)$ units.

5.2. PIC

In (4), calculating $(\mathbf{R}-\mathbf{I})\mathbf{A}\mathbf{d}^m$ needs K^2 multiplication and K^2-K plus-minus, and interference cancellation needs K minus. Hence, if the number of stages is M, the computation complexity of PIC is MK^2 units.

5.3. CLPIC

The complexity of CLPIC is the sum of those of the correcting module and PIC. In the module, compare of tentative decisions needs K minus, and calculating conditional likelihood function needs $\bar{l}(K-\bar{l})$ units. Hence, the computation complexity of CLPIC is about $MK^2 + K + \bar{l}(K-\bar{l}) + 2^{\bar{l}}(\bar{l}^2 + \bar{l})$ units.

6. SIMULATION

This section presents various simulation results to demonstrate the performance of the improved PIC. We consider a synchronous DS-CDMA system over an AWGN channel. The processing gain is 31 and random sequence (long codes) is used as the spreading sequence. Finally it is assumed that the system is with perfect power control. MF denotes conventional detectors; PIC-2 and PIC-3 represent traditional PIC with two stages and three stages; CLPIC-2 and CLPIC-3 represent the improved PIC with two stages and three stages; ML denotes optimum detectors.

Fig.2, Fig.3 shows the performance in different SNR environments with $K=12$. CLPIC performs much better than traditional PIC and MF. When SNR>8dB, BER of CLPIC is lower by more than one order of magnitude. However, computation complexity of CLPIC increases relatively slight. When SNR>4dB, it is shown that CLPIC-2 outperforms PIC-3 in BER with lower complexity.

Fig.4, Fig.5 compares the performance in SNR=10dB with different user numbers. When the system load is low ($K/N<1/3$), CLPIC has the same performance in BER as PIC which contains 4~6 users less than CLPIC; when the system load is very high, CLPIC still enhances system capacity greatly, but the computation complexity increase becomes significant.

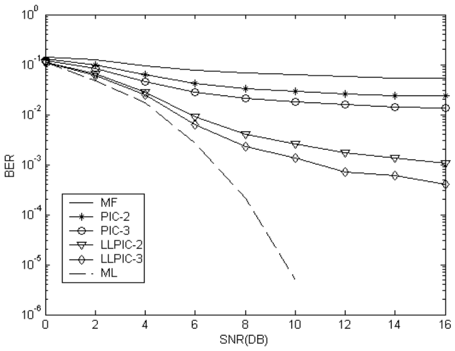

Fig.2 BER vs. SNR with K=12

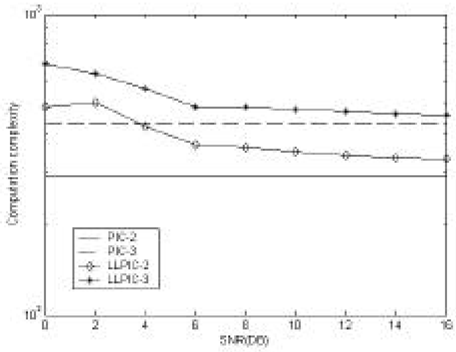

Fig.3 Complexity vs. SNR with K=12

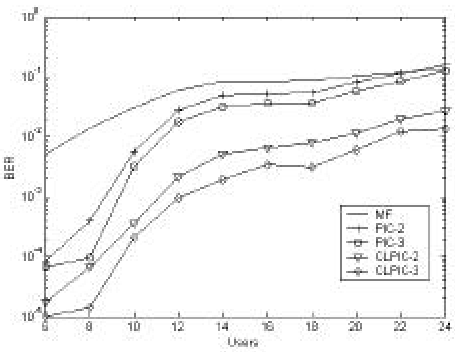

Fig.4 BER vs. Users with SNR=10dB

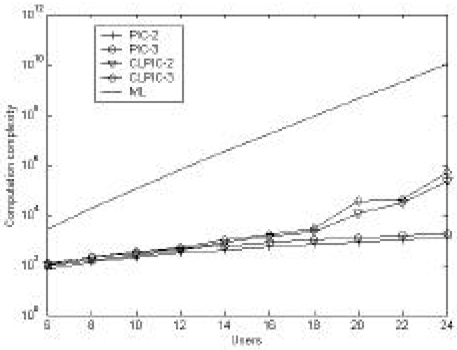

Fig.5 Complexity vs. Users with SNR=10dB

7. CONCLUSIONS

In this paper, an improved PIC called CLPIC is studied in details. The properties of tentative decisions in traditional PIC are considered, and compare between the last two stages will bring group information, which is used to produce conditional likelihood function. So in CLPIC, a correcting module employing ML algorithm improves the BER performance with relatively low increase of complexity. From the simulation, it is inferred that

1. CLPIC has much better BER performance than traditional PIC, while its computation complexity is quite lower than ML.
2. When system load is not very high ($K/N<1/2$), computation complexity approaches traditional PIC.
3. The computation complexity of CLPIC increases fast with high system load.
4. CLPIC preserves modular structure of traditional PIC and is well suited for hardware realization.

Further study will focus on the case of high system load. Two methods may be employed: a pre-filter will reduce BER of the last stage, then reduce the dimensions of solution space; some simplified ML algorithms such as OFMUD will be used to replace ML and will directly reduce the computation complexity [8].

8. REFERENCES

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