BLIND MULTIUSER DETECTION BASED ON LINEAR CONSTRAINED MMSE

Shangkun Xiong, Fangjiong Chen, Gang Wei

School of Electronic & Information Engineering
South China University of Technology, Guangzhou, P.R.China, 510640
Email:xiongshangkun@263.net, cfjiong@21cn.com, ecgwei@scut.edu.cn

ABSTRACT

Based on the minimum mean-square error (MMSE) criterion, a new blind multiuser detector is proposed in this paper. The searching space for the optimal detector is constrained according to the spreading code and channel properties of the desired user. The proposed constraint technique can guarantee that the optima associated with the desired user will locate in the searching space, whereas the existing methods consider only the spreading code and consequently cannot guarantee that the target optimal would locate in the constrained searching space. Theoretical analysis and numerical results are presented.

1. INTRODUCTION

Multiple access interference (MAI) caused by code non-orthogonality and inter-symbol interference (ISI) due to the time dispersion of wireless channels are the main difficulties in wide-band code division multiple access (CDMA) systems. Multiuser detectors that suppress the MAI as well as the ISI remain interesting topics in the past decades [1]. In this paper we focus on constrained linear inverse filtering. Existing studies are based on criteria including Minimum-Mean-Square-Error (MMSE) [2], Minimum-Output-Energy(MOE) [2][3] and Maximum Likelihood(ML) [4][5] etc.. The constraint technique proposed in [2] limits the searching space according to the spreading code of the desired user, however it cannot guarantee that the optimal MMSE solution associated with the desired user would locate in the constrained searching space. Therefore the solution is not guaranteed to coincide with the global MMSE solution and performance loss should be anticipated. In [4], a similar constraint has been applied and a ML based detector has been proposed. Since it is also not guaranteed that the solution would coincide with the optimal MMSE solution, a rake receiver has been proposed to compensate the performance loss caused by the constraint. However, the rake receiver needs to estimate a filter for each path, and therefore the computation complexity significantly increases especially when the channel impulse response has long time duration. Furthermore, it is not guaranteed that the rake receiver would totally compensate the performance loss.

In this paper, we introduce a new constraint technique that takes into account the spreading code and channel properties. The new constraint is based on the classical MMSE equation and therefore it is guaranteed that the solution will coincide with the optimal MMSE solution. Based on this constraint technique, an iterative MMSE detector is proposed and termed as linear-constrained MMSE(LC-MMSE) detector. Simulation results show that it has better performance than the linear-constrained maximum likelihood linear(LCMLL) detector in [4].

2. SIGNAL MODEL

We consider a baseband direct-sequence(DS) CDMA system with K users. The discrete-time equivalent model is shown in Fig.1. The k th user transmits a symbol-stream $b_k(n)$ at rate $1/T_s$ where T_s is the symbol duration. The symbols are assumed to be independent and identically distributed (i.i.d.) and have zero mean. To each user is associated a spreading code sequence $\{c_k(l)\}_{l=0}^{L-1}$ with chip rate $1/T_c$, where $T_c = T_s/L$ and L is the spreading gain. The received code is [4]

$$d_k(i) = c_k(i) * g_k(i) = \sum_{p=0}^{P-1} g_k(p) c_k(i-p) \qquad i = 0, \cdots, L+P-1 \tag{1}$$

where $g_k(i)$ is the discrete-time equivalent channel response which accounts for matching filter response, propagation channel response and relative time delays of different users. We assume $g_k(i)$ $(k = 1, \cdots, K)$ has the same length P. The j th received sample during n th symbol period is given by [4]

$$x(n, j) = x(nL + j) = \sum_{k=1}^{K} \sum_{r=0}^{m-1} d_k(r, j) b_k(n-r) + e(n, j) \tag{2}$$

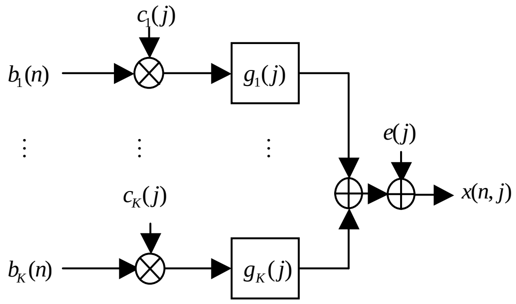

Fig.1. Baseband discrete-time equivalent model of a DS-CDMA system

where $d_i(r,j) = d_i(rL+j)$, $m = \lceil (L+P-1)/L \rceil$ and $e(n,j) = e(nL+j)$ is the j th component of the additive white Gaussian noise (AWGN) sequence.

Using vector notation, the $L \times 1$ vector given by the observations in (2) can be written as

$$\mathbf{x}(n) = \sum_{k=1}^{K} \sum_{r=0}^{m-1} \mathbf{d}_k(r) b_k(n-r) + \boldsymbol{\varepsilon}(n)$$

$$= \sum_{k=1}^{K} \mathbf{D}_k \mathbf{b}_k(n) + \boldsymbol{\varepsilon}(n) \tag{3}$$

where

$\mathbf{x}(n) = [x(n,0),\cdots,x(n,L-1)]^T$,

$\mathbf{d}_k(r) = [d_k(r,0),\cdots,d_k(r,L-1)]^T$,

$\mathbf{D}_k = [\mathbf{d}_i(0),\cdots,\mathbf{d}_i(m-1)]$,

$\mathbf{b}_k(n) = [b_k(n),\cdots b_k(n-m+1)]^T$,

$\boldsymbol{\varepsilon}(n) = [e(n,0),\cdots,e(n,L-1)]^T$,

$E[\boldsymbol{\varepsilon}(n)\boldsymbol{\varepsilon}(n)^H] = \sigma_e^2 \mathbf{I}_L$.

The linear multiuser detector consists of a finite impulse response (FIR) filter $\mathbf{w} = [\omega_{1,\cdots,}\omega_L]^T$,the soft estimate corresponding to the n th symbol period can be written as

$$y(n) = \mathbf{w}^H \mathbf{x}(n) \tag{4}$$

3. A MMSE DETECTOR

In this section, we derive a new blind adaptive detector based on MMSE criterion. We assume user 1 is the desired user. The MMSE solution can be written as

$$[\mathbf{w}_*, A_*] = \arg\min_{\mathbf{w},A}\{E_{\mathbf{x}(n)}[|\,\mathbf{w}^H \mathbf{x}(n) - Ab_1(n)\,|^2]\} \tag{5}$$

where $\mathbf{w}_*$ is the optimal MMSE detector, A_* is the amplitude related to the channel parameters and the scale of $\mathbf{w}_*$, and $b_1(n)$ is the data sequence of the desired user.

If only a block of samples is available the time average can be used to take the place of statistical average.

$$[\hat{\mathbf{w}}, \hat{A}] = \arg\min_{\mathbf{w},A}\{\zeta(\mathbf{w},A)\} \tag{6}$$

where $\zeta(\mathbf{w},A) = \sum_{n=0}^{N-1}\left(\left|\mathbf{w}^H \mathbf{x}(n) - Ab_1(n)\right|^2\right)$ is the cost function. N is length of data block.

From (6) we have

$$\hat{\mathbf{w}} = [\sum_{n=0}^{N-1} \mathbf{x}(n)\mathbf{x}^H(n)]^{-1} A [\sum_{n=0}^{N-1} b_1(n)\mathbf{x}(n)] \tag{7}$$

$$\hat{A} = [\sum_{n=0}^{N-1} b_1^2(n)]^{-1} [\sum_{n=0}^{N-1} \mathbf{w}^H \mathbf{x}(n) b_1(n)] \tag{8}$$

However, the cost function $\zeta(\mathbf{w},A)$ does not have unique global optimal solution. In fact it has an optimal for each user besides zero. Therefore the minimization of $\zeta(\mathbf{w},A)$ may capture an interfering user instead of the desired user. A common technique to avoid this is to constrain the searching space for $\mathbf{w}$. A general formulation of the constraint is (e.g see [2][4])

$$\mathbf{w}^H \mathbf{F} = \mathbf{u}^T \tag{9}$$

where $\mathbf{F}$ is a matrix related to the desired user's spreading code and channel parameters, and $\mathbf{u}$ is a vector. The linear constraint in (9) can be converted into an unconstrained form

$$\mathbf{w} = \mathbf{w}_q - \mathbf{B}\mathbf{w}_u \tag{10}$$

where $\mathbf{w}_q$ is one solution of the overdetermined linear system $\mathbf{w}^H \mathbf{F} = \mathbf{u}^T$, and $\mathbf{B}$ spans the null column subspace of $\mathbf{F}$, i.e., $\mathbf{B}^H \mathbf{F} = 0$. $\mathbf{w}_u$ is the unconstrained part of $\mathbf{w}$.

Since it is impossible to find a close-form solution to problem (6) under the constraint (9), variant iterative approaches have been proposed(e.g. [2][4]). In this paper we propose a new iterative technique showed in Fig.2 and detailed as follows

Step 1) *Initialization*: initialize $\hat{\mathbf{w}}_{u(0)}$, $\hat{A}_0$ and set $i = 0$

Step 2) *recursion*:

a) Compute $\hat{\mathbf{w}}_i = \mathbf{w}_q - \mathbf{B}\hat{\mathbf{w}}_{u(i)}$

b) Symbol estimation

$$\hat{b}_1(n) = sign\{(\hat{\mathbf{w}}_i)^H \mathbf{x}(n)\} \quad n = 0,\cdots,N-1$$

c) Detector estimation

$$\hat{A}_{i+1} = \{\sum_{n=0}^{N-1}[\hat{b}_1(n)]^2\}^{-1}[\sum_{n=0}^{N-1} \hat{\mathbf{w}}_i^H \mathbf{x}(n)\hat{b}_1(n)]$$

$$\hat{\mathbf{w}}_{u(i+1)} = (\mathbf{B}^H\mathbf{B})^{-1}\mathbf{B}^H\{\mathbf{w}_q - [\sum_{n=0}^{N-1}\mathbf{x}(n)\mathbf{x}^H(n)]^{-1}$$

$$\hat{A}_{i+1}[\sum_{n=0}^{N-1} b_1(n)\mathbf{x}(n)]\}$$

Step 3) if converged, stop, else set $i = i+1$ and go to step 2.

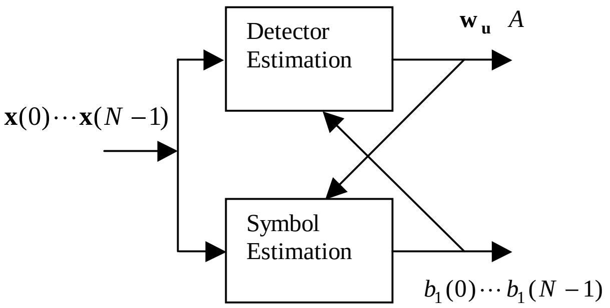

Fig.2. A new iterative algorithm

4. LINEAR MMSE CONSTRAINT

An efficient constraint technique should guarantee that the global optimal associated with the desired user will locate in the constrained searching space. At the same time the interference solutions should be excluded from the constrained searching space. The constraint applied in [4] (similar formulation in [2]) is

$$\mathbf{w}^H\mathbf{C}_1(0)=\mathbf{u}_d^T \tag{11}$$

where $\mathbf{C}_1(0)$ is constructed from the desired user's spreading code, and $\mathbf{u}_d$ is restricted to $\mathbf{u}_d=[0\cdots0\ \underset{d}{1}\ 0\cdots0]^T$. Note that in (11) $\mathbf{u}_d$ is *forced* to $\mathbf{u}_d=[0\cdots0\ \underset{d}{1}\ 0\cdots0]^T$, but in fact it does not have such formulation, and therefore the constrained searching space by (11) may not contain the real optimal $\mathbf{w}_*$ [2]. Furthermore, the effect of channel variation has not been considered in (11).

In the following part we propose a new constraint taking into account the desire user's spreading code as well as the channel properties. It is straightforward that the optimal MMSE detector $\mathbf{w}_*$ satisfies the following

$$\mathbf{R}\mathbf{w}_*=A_*\mathbf{p} \tag{12}$$

where $\mathbf{R}=E[\mathbf{x}(n)\mathbf{x}^H(n)]$ denotes the autocorrelation matrix, and $\mathbf{p}=E[\mathbf{x}(n)b_1(n)]$ denotes the inter-correlation vector. When the user symbols are i.i.d. we have

$$\begin{aligned}\mathbf{p}&=E[\mathbf{x}(n)b_1(n)]\\&=E\{\sum_{k=1}^{K}[\mathbf{D}_k\mathbf{b}_k(n)]b_1(n)+\boldsymbol{\varepsilon}(n)b_1(n)\}\\&=\mathbf{d}_1(0)=\mathbf{C}_1(0)\mathbf{g}_1 \qquad (13)\end{aligned}$$

where

$$\mathbf{d}_k(r)=\begin{bmatrix}d_k(r,0)\\ \vdots\\ d_k(r,L-1)\end{bmatrix}=\begin{bmatrix}d_k(rL)\\ \vdots\\ d_k(rL+L-1)\end{bmatrix}$$

$$=\mathbf{C}_k(r)\mathbf{g}_k \tag{14}$$

$$\mathbf{g}_k=[g_k(0),\ldots,g_k(P-1)]^T \tag{15}$$

$$\mathbf{C}_k(r)=\begin{bmatrix}c_k(rL) & \cdots & c_k(rL-P+1)\\ \cdots & \ddots & \cdots\\ c_k((r+1)L-1) & \cdots & c_k((r+1)L-P)\end{bmatrix} \tag{16}$$

Then we have

$$\mathbf{R}\mathbf{w}_*=A_*\mathbf{C}_1(0)\mathbf{g}_1 \tag{17}$$

Let $\mathbf{D}$ be an $L\times(L-P)$ matrix that spans null column subspace of $\mathbf{C}_1(0)$, i.e., $\mathbf{D}^H\mathbf{C}_1(0)=\mathbf{0}$. Construct a matrix $\mathbf{E}^H=\begin{bmatrix}1\ 0\ \cdots\ 0\\ \mathbf{D}^H\end{bmatrix}$. Left multiply $\mathbf{E}^H$ to both sides of eq. (13), then we have

$$\begin{aligned}\mathbf{E}^H\mathbf{R}\mathbf{w}_*&=\mathbf{E}^H A_*\mathbf{C}_1(0)\mathbf{g}_1\\&=A_*[c_1(0)g_1(0)\quad 0\quad\cdots\quad 0]^T \qquad (18)\end{aligned}$$

Let $A_*=(c_1(0)g_1(0))^{-1}$(note that $|g_1(0)|>0$), we have

$$\mathbf{E}^H\mathbf{R}\mathbf{w}_*=[1\quad 0\quad\cdots\quad 0]^T \tag{19}$$

Let $\mathbf{F}^H=\mathbf{E}^H\mathbf{R}$, we have a new constraint having same formulation as (9)

$$\mathbf{w}_*{}^H\mathbf{F}=[1\quad 0\quad\cdots\quad 0] \tag{20}$$

It is straightforward that the optimal solution of (12) must satisfy the proposed new linear constraint of (20), which means the constrained space by (20) must contain the global MMSE solution. Therefore the proposed algorithm can achieve global optimality in theory.

Eq. (20) differs from (11) in that the spreading code as well as the channel properties are included in (20), whereas (11) cannot reflect the effect of channel parameters and consequently the constraint of (11) does not hold in practice. One may argue that interference optimal may also locate in the constrained searching space by (20). We shall show that it depends on the spreading codes. Optimal selection of spreading codes is beyond the scope of this paper and we leave it to future work.

5. COMPUTATION SIMULATIONS

Computer simulations are presented to illustrate the efficiency of our approach. We consider an asynchronous DS-CDMA communication system with K users transmitting BPSK symbols. The processing gain is $L=16$. The spreading sequences of all users are generated randomly. The length of the discrete-time equivalent channel response for each user is $P=4$. We consider a Rayleigh channel model where the channel coefficients $h_k(p)$ $(k=1,\cdots,K, p=0,\cdots,P-1)$ are complex random variables with statistically independent real and imaginary parts. Both the real part and imaginary part are Gaussian with zero mean and standard deviation

$\sigma_h = 0.3$. In order to estimate the symbol error rate (SER), the simulation results are averaged over 150 randomly generated channels. For each channel a block of data with length $N = 300$ is transmitted and 500 independent trials are done. Symbol error rate (SER) is adopted to measure the performance. In our iterative approach, $\hat{\mathbf{w}}_{u\,(0)}$, $\hat{A}_0$ are initialized to $\hat{\mathbf{w}}_{u\,(0)} = [1/P, \cdots, 1/P]$ and $\hat{A}_0 = 1$.

We compare the performance of the proposed LC-MMSE detector with that of the LCMLL detector [4]. Fig.3 and Fig.4 show the results. Fig.3 depicts the SER comparison for several values of the input SNR where user number $K = 7$. Fig.4 depicts the SER comparison for several values of the user number where the input SNR=12dB. It can be seen that the proposed LC-MMSE detector has better performance than the LCMLL detector. The SER curve of LC-MMSE detector approaches the theoretical limit more closely. The reason is that the constraint in [4] cannot guarantee that the optimal solution locates in the constrained searching space whereas the proposed constraint in section 4 ensures that the optimal solution locates in the searching space and hence can achieve the optimal solution in theory.

Fig.5 illustrates the convergence property of the LC-MMSE detector when user number $K = 7$ and SNR=12dB. We show the mean square error (MSE) of detector output as a function of the number of iterations. It can be seen that the proposed algorithm has high convergence rate and therefore is suitable for situation when the channel parameters vary rapidly.

6. CONCLUSION

We have proposed a new linear constrained multiuser detector for asynchronous CDMA system in unknown multipath channels. This blind multiuser detector is based on MMSE criterion and the proposed new linear constraint coincides with MMSE criterion. Simulation results demonstrate the effectiveness of the proposed techniques.

7.ACKNOWLEDGEMENTS

This work was supported by the National Natural Science Fund of China(60172048) and Doctoral Program Fund(20010561007) of China Education Ministry.

8. REFERENCES

[1] S. Verdú, *Multiuser Detection*, Cambridge, U.K.: Cambridge Univ. Press, 1998.

[2] M. K. Tsatsanis, "Inverse filtering criteria for CDMA systems," *IEEE Trans. Signal Processing*, vol. 45, no. 1, pp. 102-122, Jan.1997.

[3] M. K. Tsatsanis and Z. Xu, "Performance analysis of minimum variance CDMA receivers," *IEEE Trans. Signal Processing*, vol. 46, no. 11, pp. 3014-3022, Nov. 1998.

[4] Móica F. Bugallo, Joaquín Míguez and Luis Castedo, "A maximum likelihood approach to blind multiuser interference cancellation," *IEEE Trans. Signal processing*, vol. 49, no. 6, pp. 1228-1239, June 2001.

[5] M. F. Bagallo, J. Miguez, and L. Castedo, "Semiblind linear muliuser interference cancellation: a maximum likelihood approach," *IEEE Trans. Signal Processing*, vol. 81, no. 4, pp. 2041-2057, April 2001

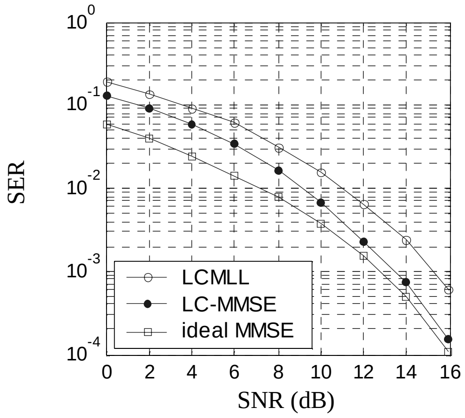

Fig.3. SER versus SNR in a DS-CDMA system with $K = 7$ users

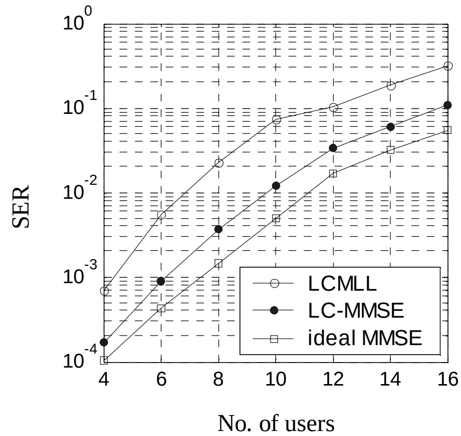

Fig.4. SER versus the number of users in a DS-CDMA system with SNR=12dB

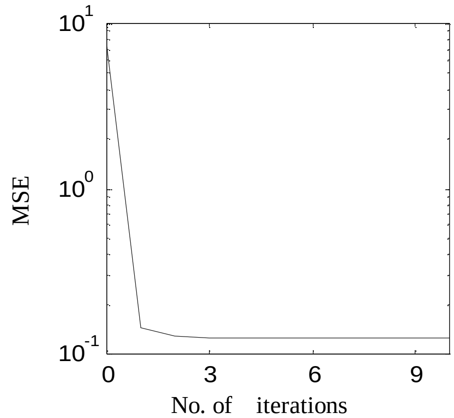

Fig.5. MSE versus the number of iteration in a DS-CDMA system with $K = 7$ users and SNR=12dB