PARTICLE FILTERING FOR JOINT SYMBOL AND PARAMETER ESTIMATION IN DS SPREAD SPECTRUM SYSTEMS

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ABSTRACT

In this paper, we develop a new receiver for joint symbol, channel characteristics and code delay estimation for DS spread spectrum systems under conditions of multipath fading. This nonlinear estimation problem is extremely complex. An efficient simulation-based algorithm based on particle filtering is proposed to solve it. The method combines sequential importance sampling, a selection scheme and a variance reduction technique. An extensive simulation study is carried out and demonstrates good performance of the suggested approach.

1. INTRODUCTION

Direct sequence (DS) spread spectrum systems are robust to many channel impairments, allow multiuser (CDMA) and low-detectability signal transmission, and, therefore, are widely used in different areas of digital communications. Unlike with many other communication systems, however, spread spectrum receivers require additional code synchronization, which might be a rather challenging task under conditions of multipath fading, when severe amplitude and phase variations take place.

The problem of joint delay and multipath estimation has been addressed in the literature before (see [6, 7, 9], for example), and proved to be a difficult one due to its inherited nonlinearity. The previously proposed approaches were mainly based on the use of the Extended Kalman Filter (EKF). However, many of them concentrated on the channel parameters and delay estimation only; moreover, in a number of cases, when EKF methods were applied, the estimated parameters were divergent [6].

In this paper, we propose to estimate the channel parameters, code delays and symbols jointly using particle filtering techniques, a set of powerful and versatile simulation-based methods recently appeared in the literature (see [5] for a survey). These methods have already been successfully applied in digital communications for demodulation in fading channels [2, 5, 11] and detection in synchronous CDMA [10, 12]. In this work, however, only the symbols needed to be imputed since the unknown fading channel characteristics were integrated out. In DS spread spectrum systems, one faces a more complex task involving both discrete (symbols) and continuous-valued (delays) unknown parameters. We show in this paper that particle filtering allows to solve efficiently this problem.

The key idea of particle filtering methods is to approximate the posterior distribution of interest by swarms of N ($N \gg 1$) weighted points in the sample space, called particles, which evolve randomly in time in correlation with each other and either give birth to offspring particles or die according to their ability to represent the different zones of interest of the state space. These methods are very flexible and converge asymptotically ($N \to \infty$) towards the posterior of interest. The algorithm developed in this paper is designed so as to make use of the structure of the model, and incorporates efficient variance reduction strategies based on Kalman filtering techniques [4]. At each iteration the algorithm has a computational complexity that is linear in the number of particles, and can easily be implemented on parallel processors.

The rest of the paper is organized as follows. The model specification and estimation objectives are stated in Section 2. In Section 3, a particle filtering method is developed for joint symbol/channel coefficients/code delay estimation with simulation results presented in Section 4. Finally, a conclusion is drawn in Section 5.

2. PROBLEM STATEMENT AND ESTIMATION OBJECTIVES

Transmitted waveform. Let us denote for any generic sequence κ_t, $\kappa_{i:j} \triangleq (\kappa_i, \kappa_{i+1}, \ldots, \kappa_j)^{\mathrm{T}}$, and let d_n be the nth information symbol, and $s(\tau)$ be the corresponding analog bandpass spread-spectrum signal waveform transmitted in the symbol interval of duration T_d:

$$s_{\text{trans}}(\tau) = \mathrm{Re}[r_n(d_n) PN(\tau) \exp(j2\pi f_c \tau)],$$
$$\text{for } (n-1)T_d < \tau \leq nT_d,$$

where $r_n(.)$ performs the mapping from the digital sequence to waveforms and corresponds to the modulation technique employed, f_c denotes the carrier frequency and $PN(\tau)$ is a wide-band pseudo-noise (PN) waveform defined by $PN(\tau) = \sum_{h=1}^{H} a_h \eta(\tau - hT_c)$. Here, $a_{1:H}$ is a spreading code sequence[1] consisting of H chips (with values $\{\pm 1\}$) per symbol, $\eta(\tau - hT_c)$ is a rectangular pulse of unit height and duration T_c, and T_c is the chip interval satisfying the relation $T_c = T_d/H$.

Channel model. The signal is passed through a noisy multipath fading channel which causes random amplitude and phase variations on the signal. The channel can be represented by a time-varying tapped-delayed line with taps spaced T_s seconds apart, where T_s is the Nyquist sampling rate for the transmitted waveform; $T_s = T_c/2$ due to the PN bandwidth being approximately $1/T_c$. The equivalent discrete-time impulse response of the channel is given by

$$h_{c,t} = \sum_{n_f=0}^{N_f-1} f_t^{(n_f)} \delta_{t,n_f},$$

where t is a discrete time index, N_f is the number of paths of the channel, $f_t^{(n_f)}$ are the complex-valued time-varying multipath coefficients arranged into the vector $\mathbf{f}_t$, and δ_{t,n_f} denotes the Kronecker delta.

We assume here that the channel coefficients $\mathbf{f}_t$ and code delay θ_t propagate according to the first-order autoregressive (AR) model:

$$\mathbf{f}_t = \mathbf{A}_f \mathbf{f}_{t-1} + \mathbf{B}_f \mathbf{v}_t, \mathbf{v}_t \overset{i.i.d.}{\sim} \mathcal{N}_c\left(\mathbf{0}, \mathbf{I}_{N_f}\right), \quad (1)$$

$$\theta_t = \gamma\theta_{t-1} + \sigma_\theta \epsilon_t,\ \epsilon_t \overset{i.i.d.}{\sim} \mathcal{N}(0,1), \quad (2)$$

which corresponds to a Rayleigh uncorrelated scattering channel model; here $\mathbf{A}_f \triangleq diag(\alpha_0, \ldots, \alpha_{N_f-1})$, $\mathbf{B}_f \triangleq diag(\sigma_{f,0}, \ldots, \sigma_{f,N_f-1})$, with σ_{f,n_f}^2 being the noise variance, and α_{n_f} accounting for the Doppler spread (see [7] for details and discussion on the use of the higher order AR).

Received signal. The complex output of the channel sampled at the Nyquist rate, (in which case $t = 2H(n-1)+1, \ldots, 2Hn$ samples correspond to the nth symbol transmitted, i.e. $d_n \leftrightarrow y_{2H(n-1)+1:2Hn}$) can, thus, be expressed as

$$y_t = \mathbf{C}(d_{1:n}, \theta_{1:t}) + \sigma\varepsilon_t,\ \varepsilon_t \overset{i.i.d.}{\sim} \mathcal{N}_c(0,1), \quad (3)$$

where $\mathbf{C}(d_{1:n}, \theta_{1:t}) = \sum_{n_f=0}^{N_f-1} f_t^{(n_f)} s\left((t-n_f)T_s - \theta_t\right)$ and σ^2 being the noise variance[2]. The noise sequences ε_t, ϵ_t and $v_t^{(n_f)}$, $n = 0, \ldots, N_f - 1$ are assumed mutually independent and independent of the initial states $\mathbf{f}_0 \sim \mathcal{N}_c\left(\hat{\mathbf{f}}_0, \mathbf{P}_{\mathbf{f},0}\right)$, $\theta_0 \sim \mathcal{N}\left(\hat{\theta}_0, P_{\theta,0}\right)$.

[1] The extension to a multiuser DS CDMA transmission is straightforward, see [10, 12], for example.

[2] The case of non-Gaussian noise can be easily treated using the techniques presented in [11].

Estimation objectives. The symbols d_n, which are assumed i.i.d., the channel characteristics $\mathbf{f}_t$ and the code delay θ_t are unknown for $n, t > 0$. Our aim is to obtain sequentially in time an estimate of the joint posterior probability density of these parameters $p\left(d_{1:n}, \mathbf{f}_{0:2Hn}, \theta_{0:2Hn} \middle| y_{1:2Hn}\right)$, and some of its characteristics, such as $\mathbb{E}\left(d_{1:n} \middle| y_{1:2Hn}\right), \mathbb{E}\left(\mathbf{f}_{0:2Hn} \middle| y_{1:2Hn}\right)$ and $\mathbb{E}\left(\theta_{0:2Hn} \middle| y_{1:2Hn}\right)$ in particular. This problem, unfortunately, does not admit any analytical solution and, thus, approximate methods must be employed. One of the methods that has proved to be useful in practice is particle filtering, and in the next chapter we propose a receiver based on the use of these techniques for joint estimation of symbols, channel coefficients and code delay in DS spread-spectrum systems.

3. PARTICLE FILTERING ALGORITHM

A straightforward application of the particle filtering methods focuses on the estimation of the joint posterior distribution $p\left(d_{1:n}, d\mathbf{f}_{0:2Hn}, d\theta_{0:2Hn} \middle| y_{1:2Hn}\right) = p\left(d_{1:n}, \mathbf{f}_{0:2Hn}, \theta_{0:2Hn} \middle| y_{1:2Hn}\right) d\mathbf{f}_{0:2Hn} d\theta_{0:2Hn}$. In our case, however, this problem can be reduced to one of sampling from a lower-dimensional posterior $p\left(d_{1:n}, d\theta_{0:2Hn} \middle| y_{1:2Hn}\right)$. This is based on the fact that, conditional upon the sequences $d_{1:n}, d\theta_{0:2Hn}$, the probability density $p\left(\mathbf{f}_{0:2Hn} \middle| y_{1:2Hn}, d_{1:n}, d\theta_{0:2Hn}\right)$ can be computed using the Kalman filter associated with the Eq.(1, 3) (see [1, 3, 4]), and, therefore, $p\left(\mathbf{f}_{0:2Hn} \middle| y_{1:2Hn}\right)$ can be approximated by a random mixture of Gaussian distributions, thus leading to an increased algorithm efficiency:

$$p\left(\mathbf{f}_{0:2Hn} \middle| y_{1:2Hn}\right) = \sum_{i=1}^{N} p\left(\mathbf{f}_{0:2Hn} \middle| y_{1:2Hn}, d_{1:n}^{(i)}, \theta_{0:2Hn}^{(i)}\right)$$

Strictly speaking, we are interested in estimating the information symbols only and the tracking of the channel is naturally incorporated into the proposed algorithm. However, following this approach, the MMSE (conditional mean) estimates of fading coefficients can, of course, be obtained if necessary.

We can now proceed with the estimation of $p\left(d_{1:n}, d\theta_{0:2Hn} \middle| y_{1:2Hn}\right)$ using particle filtering techniques. The method is based on the following remark. Suppose N particles $\left\{d_{1:n}^{(i)}, \theta_{0:2Hn}^{(i)}\right\}_{i=1}^{N}$ can be easily simulated according to an arbitrary convenient importance distribution $\pi\left(d_{1:n}, d\theta_{0:2Hn} \middle| y_{1:n}\right)$ (such that $p\left(d_{1:n}, d\theta_{0:2Hn} \middle| y_{1:n}\right) > 0$ implies $\pi\left(d_{1:n}, d\theta_{0:2Hn} \middle| y_{1:n}\right) > 0$). Then, using the importance sampling identity, an estimate of $p\left(d_{1:n}, d\theta_{0:2Hn} \middle| y_{1:2Hn}\right)$ is given by the following point mass approximation:

$$\hat{p}\left(d_{1:n}, d\theta_{0:2Hn} \middle| y_{1:2Hn} \right) = \sum_{i=1}^{N} \tilde{w}_{1:n}^{(i)} \delta_{\left(d_{1:n}^{(i)}, \theta_{0:2nH}^{(i)}\right)} \left(d_{1:n}, d\theta_{0:2Hn} \right),$$

where $\tilde{w}_n^{(i)}$ are the so-called *importance weights*

$$\tilde{w}_{1:n}^{(i)} = \frac{w_{1:n}^{(i)}}{\sum_{j=1}^{N} w_{1:n}^{(j)}},\; w_{1:n}^{(i)} \propto \frac{p\left(d_{1:n}^{(i)}, \theta_{0:2nH}^{(i)} \middle| y_{1:2Hn} \right)}{\pi\left(d_{1:n}^{(i)}, \theta_{0:2nH}^{(i)} \middle| y_{1:2Hn} \right)}.$$

An additional condition of $\pi\left(d_{1:n}^{(i)}, \theta_{0:2nH}^{(i)} \middle| y_{1:2Hn} \right)$ having to admit $\pi\left(d_{1:n-1}^{(i)}, \theta_{0:2H(n-1)}^{(i)} \middle| y_{1:2H(n-1)} \right)$ as a marginal distribution allows to propagate this estimate sequentially in time without subsequently modifying the past simulated trajectories; $w_{1:n} = w_{1:n-1} w_n$ in this case. Also a selection procedure introduced at each time step helps to avoid the degeneracy of the algorithm by discarding particles with low normalized importance weights and multiply those with high ones (see [3, 4, 10] for the details of the algorithm).

Given for the $(n-1)$th symbol N particles $\left\{ d_{1:n-1}^{(i)}, \theta_{0:2H(n-1)}^{(i)} \right\}_{i=1}^{N}$ distributed approximately according to $p\left(d_{1:n-1}, d\theta_{0:2H(n-1)} \middle| y_{1:2H(n-1)} \right)$, the general particle filtering receiver, proceeds as follows:

Particle Filtering Algorithm

Sequential Importance Sampling Step

- For $i = 1, \ldots, N$, sample $(\widetilde{d}_n^{(i)}, \widetilde{\theta}_{2H(n-1)+1:2Hn}^{(i)}) \sim \pi(d_n, \theta_{2H(n-1)+1:2Hn} | \, d_{1:n-1}^{(i)}, \theta_{0:2H(n-1)}^{(i)}, y_{1:2Hn})$.
- For $i = 1, \ldots, N$, evaluate the importance weights $w_n^{(i)}$ up to a normalizing constant.
- For $i = 1, \ldots, N$, normalize $w_n^{(i)}$ to obtain $\tilde{w}_n^{(i)}$.

Selection Step

- Multiply/discard particles with respect to high/low $\tilde{w}_n^{(i)}$ to obtain N unweighted particles $(d_{1:n}^{(i)}, \theta_{1:2Hn}^{(i)})$.

The choice of the importance distribution and a selection scheme is discussed in [4]; depending on the one chosen, the computational complexity of the algorithm varies. If, say, the prior is taken to be the importance distribution, as in this paper, i.e.

$$\pi(d_n, \theta_{2H(n-1)+1:2Hn} | \, d_{1:n-1}, \theta_{0:2H(n-1)}, y_{1:2Hn}) = p(d_n) \prod_{t=2H(n-1)+1}^{2Hn} p(\theta_t | \, \theta_{t-1}),$$

then w_n becomes

$$w_n \propto p\left(y_{2H(n-1)+1:2Hn} \middle| y_{1:2H(n-1)}, d_{1:n}, \theta_{0:2Hn} \right) = \prod_{t=2H(n-1)+1}^{2Hn} p\left(y_t \middle| d_{1:n}, \theta_{0:t}, y_{1:t-1} \right),$$

and requires evaluation of $2H$ one-step Kalman filter updates for each symbol. As far as the selection step is concerned, a stratified sampling [8] employed here can be implemented in $O(N)$ operations.

Sequential Importance Sampling
(prior as an importance distribution)

- For $i = 1, \ldots, N$,
  sample $\widetilde{d}_n^{(i)} \sim p(d_n)$,
  $w_n^{(i)} = 1$,
  For $t = 2H(n-1)+1, \ldots, 2Hn$,
  sample $(\widetilde{\theta}_t^{(i)}) \sim p(\theta_t | \, \theta_{t-1})$,
  perform one-step Kalman filter update ($w_n^{(i)} = w_n^{(i)} p(y_t | \, d_{1:n}, \theta_{0:t}, y_{1:t-1})$).
- For $i = 1, \ldots, N$, normalize $w_n^{(i)}$ to obtain $\tilde{w}_n^{(i)}$.

If H is long, it is useful to resample the particles at intermediate steps between $t = 2H(n-1)+1$ and $t = 2Hn$. One can also use Markov chain Monte Carlo (MCMC) steps so as to rejuvenate the particle and in particular d_n.

We would also like to note that, of course, using the prior distribution in our case can be inefficient, as no information carried by observations is used to explore the state space. Further research should concentrate on development suboptimal importance distributions, perhaps, on a case by case basis, so as to reduce the variance of the importance weights and thus increase the efficiency of the procedure.

4. SIMULATION RESULTS

The algorithm presented above was applied to perform joint symbols/channel coefficients/code delay estimation for DS spread spectrum systems with $H = 15$, $N_f = 4$ (modulation scheme $2PSK$) and the multipath channel response as in [7, channel B]. The AR parameters were also set as in [7], i.e. $\alpha_{n_f} = 0.999$, $\gamma = 0.999$, $\sigma_{f,n_f}^2 = 0.001$, $\sigma_\theta^2 = 0.001$, and corresponded to the case of nearly constant coefficients and constant delay.

As it is shown in Fig. 1, the algorithm exhibits good bit-error-rate (BER) performance even for just $N = 100$ particles (pilot symbol rate is $1:30$). A tracking error trajectory $\theta_{2Hn} - \hat{\theta}_{2Hn}$ for 100 information symbols (corresponding to 1500 chips and 3000 channel samples) and an average signal to noise ratio (SNR) equal to 15 dB is presented in Fig. 2. Fig. 3 also illustrates the mean-square delay error MSE as a function of SNR:

$$MSE = \frac{1}{N_n}\sum_{n=1}^{N_n}\left(\theta_{2Hn} - \hat{\theta}_{2Hn}\right)^2,$$

where N_n is a length of the symbol sequence, $N_n = 1000$.

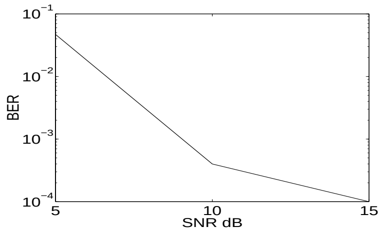

Fig. 1. Bit error rate.

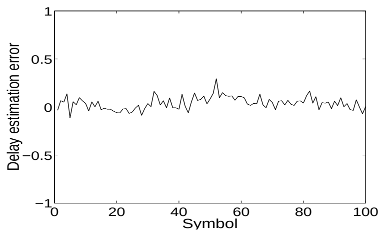

Fig. 2. The error in delay estimation. SNR=15 dB.

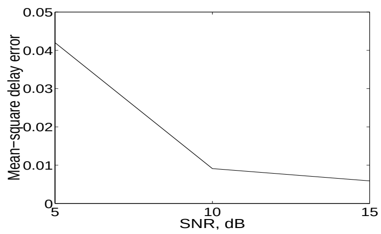

Fig. 3. Mean-square delay error via SNR.

5. CONCLUSION

In this paper, we propose the application of particle filtering techniques to a challenging problem of joint symbols, channel coefficients and code delay estimation for DS spread spectrum systems in multipath fading. The algorithm is designed to make use of the structure of the model, and incorporates efficient variance reduction techniques. An extensive simulation study demonstrates the good performance of the method. In addition, the tracking of the channel is naturally incorporated in the proposed algorithm, and the estimates of the channel coefficients can be obtained if required. The algorithm also can be easily simplified to consider just channel coefficients and code delay estimation. The extension to a multiuser DS CDMA transmission is straightforward. Future research should concentrate on the development of suboptimal importance distributions capable of increasing the algorithm efficiency.

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