BLIND MULTIUSER CHANNEL ESTIMATION IN TIME-VARYING DIRECT SEQUENCE CODE DIVISION MULTIPLE ACCESS SYSTEMS

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ABSTRACT

A multistep linear prediction (MSLP) approach is presented for blind channel estimation for short-code DS-CDMA (direct sequence code division multiple access) signals in time-varying multipath channels. The time-varying channel is assumed to be described by a complex exponential basis expansion model (CE-BEM). We first extend a recently proposed MSLP approach to blind channel estimation for time-varying SIMO (single-input multiple-output) systems, to time-varying MIMO systems in order to define a "signal" subspace. Then the knowledge of the spreading code of a desired user is exploited in conjunction with the signal subspace to estimate the time-varying channel of the desired user. Sufficient conditions for channel identifiability are investigated. An illustrative simulation example is provided.

1. INTRODUCTION

Consider a short-code DS-CDMA system with M users and N chips per symbol with the j-th user's spreading code denoted by $\mathbf{c}_j = [c_j(0), \cdots, c_j(N-1)]^T$. Then the j-th user's transmitted signal at the chip rate in a baseband discrete-time model representation is given by [5]-[7]

$$x_j(k) = \sum_{n=-\infty}^{\infty} s_j(n) c_j(k - nN), \quad j = 1, 2, \cdots, M, \tag{1}$$

where $s_j(n)$ is the j-th user's n-th symbol. In the presence of a linear dispersive channel where the receiver collects one sample per chip, the received discrete-time (sampled) signal $\tilde{x}_j(k)$ due to user j is

$$\tilde{x}_j(k) = \sum_{l=0}^{L_g-1} x_j(k-l) g_j(k;l) \tag{2}$$

where $g_j(k;l)$, $0 \le l \le L_g - 1$, (response at time k to a unit input at time $k-l$) is the effective time-varying channel impulse response (IR) sampled at the chip interval T_c. The total received signal at chip-rate is the superposition of contributions of all users observed in additive white Gaussian noise $w(k)$ as

$$y(k) = \sum_{j=1}^{M} \tilde{x}_j(k) + w(k). \tag{3}$$

The problem is to estimate the time-varying channel $g_{j_0}(k;l)$ of a desired user j_0 given the noisy data $\{y(k)\}$ and the knowledge of the desired user's spreading code $\mathbf{c}_{j_0}$. There is no training (pilot) signal present and the codes of other users' are not known.

This problem has various solutions [5]-[7] in the time-invariant case when $g_j(k;l) = g_j(l)\ \forall k$. In this paper we consider time-varying multipath channels described by a complex exponential basis expansion model (CE-BEM) [1]-[4]

$$g_j(k;l) = \sum_{q=1}^{Q} g_{q,j}(l) e^{j\omega_q k} \tag{4}$$

where $g_{q,j}(l)$ (for $q = 1, 2, \cdots, Q$) are time-invariant. Eqn. (4) is a basis expansion of $g_j(k;l)$ in the time variable k onto complex exponentials with frequencies $\{\omega_q\}$. We assume the knowledge of the basis frequencies. The basis frequencies are assumed to be the same for all users which is certainly true in the downlink case.

The paper is organized as follows. The basic baseband system model is derived in Sec. 2 to yield an MIMO formulation. The proposed solution together with some analysis are presented in Sec. 3. In Sec. 3.1 the MSLP approach of [4] to blind channel estimation for time-varying SIMO systems, is extended to time-varying MIMO systems to define a signal subspace. In Sec. 3.2 the spreading code of the desired user is exploited in conjunction with the signal subspace to estimate the time-varying channel of the desired user. Sufficient conditions for channel identifiability are investigated in Sec. 3.3. An illustrative simulation example is in Sec. 4.

2. SYSTEM MODEL

From (1)-(4) we have

$$\tilde{x}_j(k) = \sum_{q=1}^{Q} e^{j\omega_q k} \left\{ \sum_{n=-\infty}^{\infty} s_j(n) h_{q,j}(k - nN) \right\} \tag{5}$$

where

$$h_{q,j}(k) := \sum_{m=0}^{N-1} c_j(m) g_{q,j}(k-m). \tag{6}$$

Define

$$\tilde{\mathbf{x}}_j(n) := [\tilde{x}_j(nN),\ \tilde{x}_j(nN+1),\ \cdots,\ \tilde{x}_j(nN+N-1)]^T. \tag{7}$$

Then, at the symbol rate, we have

$$\tilde{\mathbf{x}}_j(n) = \sum_{l=0}^{L} \mathbf{h}_j(n;l) s_j(n-l) \tag{8}$$

where

$$\mathbf{h}_j(n;l) := \sum_{q=1}^{Q} e^{j\overline{\omega}_q n} \mathbf{h}_{q,j}(l), \ \ \overline{\omega}_q := \omega_q N, \tag{9}$$

$$\mathbf{h}_{q,j}(l) := [h_{q,j}(lN),\ h_{q,j}(lN+1)e^{j\omega_q},\ \cdots,\ h_{q,j}(lN+N-1)e^{j\omega_q(N-1)}]^T. \tag{10}$$

If we collect N chip-rate measurements of received signal (from all users) into N-vector $\mathbf{y}(n)$, then we obtain, **at**

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the symbol rate, the MIMO model (additive white Gaussian noise $\mathbf{w}(n)$ is defined in a manner similar to $\mathbf{y}(n)$) and $\tilde{\mathbf{x}}_j(n)$:

$$\mathbf{y}(n) = \sum_{j=1}^{M}\sum_{l=0}^{L} \mathbf{h}_j(n;l)s_j(n-l) + \mathbf{w}(n). \tag{11}$$

Thus, at the symbol rate (in the vector stationary formulation), the frequencies in vector CE-BEM are $N\omega_q$ $(q = 1, 2, \cdots, Q)$.

Assume that $g_j(k;l) = 0$ for $l > L_g - 1$ (in addition to $g_j(k;l) = 0$ for $l < 0$) $\forall k$ (i.e. multipath spread is L_g chips). Let $L_g \le mN$. It then follows that

$$\tilde{\mathbf{h}}_j(n) := \left[\begin{array}{cccc} \mathbf{h}_j^T(n;0) & \mathbf{h}_j^T(n+1;1) & \cdots & \mathbf{h}_j^T(n+m;m) \end{array}\right]^T$$

$$= \sum_{q=1}^{Q} \Gamma_{q,n}\mathbf{C}_j\mathbf{g}_{q,j} \tag{12}$$

where

$$\Gamma_{q,n} := \text{diag}\{e^{j\omega_q nN}, e^{j\omega_q(nN+1)}, \cdots, e^{j\omega_q(nN+N+mN-1)}\}, \tag{13}$$

($\mathbf{C}_j$ is $((m+1)N) \times (mN)$, $\mathbf{g}_{q,j}$ is $(mN) \times 1$)

$$\mathbf{C}_j := \left[\begin{array}{cccc} c_j(0) & 0 & \cdots & 0 \\ c_j(1) & c_j(0) & \ddots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ c_j(N-1) & \ddots & \ddots & c_j(0) \\ 0 & c_j(N-1) & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & c_j(N-1) \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & 0 \end{array}\right], \tag{14}$$

$$\mathbf{g}_{q,j} := \left[\begin{array}{c} g_{q,j}(0) \\ g_{q,j}(1) \\ \vdots \\ g_{q,j}(mN-1) \end{array}\right]. \tag{15}$$

It follows that $\mathbf{h}_j(n;l) = 0$ for $l < 0$ or $l > (m+1)$. Finally, we may write (11) as

$$\mathbf{y}(n) = \underbrace{\sum_{l=0}^{L} \mathbf{H}(n;l)\mathbf{s}(n-l)}_{=:\mathbf{x}(n)} + \mathbf{w}(n), \tag{16}$$

$$\mathbf{H}(n;l) := \left[\begin{array}{cccc} \mathbf{h}_1(n;l) & \mathbf{h}_2(n;l) & \cdots & \mathbf{h}_M(n;l) \end{array}\right], \tag{17}$$

$$\mathbf{s}(n) := \left[\begin{array}{cccc} s_1(n) & s_2(n) & \cdots & s_M(n) \end{array}\right]^T. \tag{18}$$

3. MULTI-STEP LINEAR PREDICTORS (MSLP) BASED SOLUTION

3.1. Multi-Step Linear Predictors [4]

Assume the following:

(**H1**) $N > M$.

(**H2**) There exists a time-varying left inverse $\{\mathbf{G}(n;l)\}_{l=0}^{\overline{K}}$ such that $\forall n$ and $\forall k$ $\sum_{l=0}^{\overline{K}} \mathbf{G}(n;l)\mathbf{H}(n-l;k-l) = I_M\delta(k)$ where $\mathbf{G}(n;l)$ is $M \times N$ and $\overline{K} < \infty$.

(**H3**) The information sequences $\{s_j(k)\}_{j=1}^{M}$ are zero-mean, mutually independent and temporally white. Take $E\{|s_j(k)|^2\} = 1$ by absorbing any non-identity correlation of $s_j(k)$ into the channel.

(**H4**) $\{\mathbf{w}(k)\}$ is zero-mean with $E\{\mathbf{w}(k+\tau)\mathbf{w}^H(k)\} = \sigma_w^2 I_N\delta(\tau)$ where I_N is the $N \times N$ identity matrix and the superscript H is the Hermitian operator (complex conjugate transpose).

It is not too hard to establish (see [3],[8] for the SIMO case) (**H2′**) implies (**H2**) with $\overline{K} = K$.

(**H2′**) Consider the $[N(K+1)] \times [M(K+L+1)]$ "time-varying Sylvester" matrix $\mathcal{T}_{K;n}(\mathbf{H})$ associated with time-varying MIMO impulse response $\{\mathbf{H}(n;l)\}$, defined as

$$\mathcal{T}_{K;n}(\mathbf{H}) :=$$

$$\left[\begin{array}{cccccc} \mathbf{H}(\text{n;0}) & \mathbf{H}(\text{n;1}) & \cdots & \mathbf{H}(\text{n;L}) & \cdots & 0 \\ 0 & \mathbf{H}(\text{n-1;0}) & \cdots & \mathbf{H}(\text{n-1;L-1}) & \cdots & 0 \\ \vdots & \ddots & \ddots & & \ddots & \\ 0 & 0 & \cdots & \mathbf{H}(\text{n-K;0}) & \cdots & \mathbf{H}(\text{n-K;L}) \end{array}\right]. \tag{19}$$

Assume that $N > M$, and $\mathcal{T}_{K;n}(\mathbf{H})$ is of full column rank $\forall K \ge ML - 1$ and $\forall n$.

Rewrite noise-free part of (16) as $(d \ge 1)$

$$\mathbf{x}(n) = \mathbf{e}(n|n-d) + \widehat{\mathbf{x}}(n|n-d) \tag{20}$$

where

$$\mathbf{e}(n|n-d) := \sum_{l=0}^{d-1} \mathbf{H}(n;l)\mathbf{s}(n-l), \tag{21}$$

$$\widehat{\mathbf{x}}(n|n-d) := \sum_{l=d}^{L} \mathbf{H}(n;l)\mathbf{s}(n-l). \tag{22}$$

As in [4], it then follows that $\{\mathbf{x}(n)\}$ can be decomposed as in (20)-(22) such that

$$E\{\mathbf{e}(n|n-d)\mathbf{x}^H(n-m)\} = 0 \quad \forall m \ge d, \tag{23}$$

and

$$\widehat{\mathbf{x}}(n|n-d) = \sum_{i=1}^{K} \mathbf{A}_{i;n}^{(d)}\mathbf{x}(n-i-d+1) \quad \text{for some } K \le \overline{K}+L. \tag{24}$$

Mimicking [4] and using (20)-(24), we have, for $m = d, d+1, \cdots, K+d-1$,

$$\mathbf{R}_{xx}(m;n) = \sum_{i=1}^{K} \mathbf{A}_{i;n}^{(d)}\mathbf{R}_{xx}(m-i-d+1; n-i-d+1), \tag{25}$$

where

$$\mathbf{R}_{xx}(m;n) := E\{\mathbf{x}(n)\mathbf{x}^H(n-m)\}. \tag{26}$$

In a matrix formulation, we may write $(K_d := K + d - 1)$

$$[\mathbf{A}_{1;n}^{(d)}, \; \cdots, \; \mathbf{A}_{M;n}^{(d)}]\mathcal{R}_{xxK}^{(n-d+1)} = [\mathbf{R}_{xx}(d;n), \; \cdots, \; \mathbf{R}_{xx}(K_d;n)] \tag{27}$$

where $\mathcal{R}_{xxK}^{(n)}$ denotes an $[NK] \times [NK]$ matrix with its ij-th block element as $\mathbf{R}_{xx}(j-i; n-i)$. Note that $\mathcal{R}_{xxK}^{(n)}$ is not necessarily full rank, therefore, the coefficients $\mathbf{A}_{i;n}^{(d)}$s are not necessarily unique. A minimum norm solution to (27) may be obtained as

$$[\mathbf{A}_{1;n}^{(d)},\ \cdots,\ \mathbf{A}_{K;n}^{(d)}]$$

$$= [\mathbf{R}_{xx}(d;n),\ \cdots,\ \mathbf{R}_{xx}(K+d-1;n)][\mathcal{R}_{xxK}^{(n-d+1)}]^{\#} \quad (28)$$

where the superscript # denotes the pseudoinverse.

Consider multistep linear predictors for steps $d = 1, 2, \cdots, \overline{d}+1$. For $d \geq 2$, define

$$\overline{\mathbf{e}}_d(n) := \mathbf{e}(n|n-d) - \mathbf{e}(n|n-d+1) = \mathbf{H}(n;d-1)\mathbf{s}(n-d+1) \quad (29)$$

where we have used (21). Consider the $[N(\overline{d}+1)]-$vector

$$\mathbf{E}(n) := \left[\ \mathbf{e}^T(n|n-1) \vdots\ \overline{\mathbf{e}}_2^T(n+1) \vdots \cdots \vdots\ \overline{\mathbf{e}}_{\overline{d}+1}^T(n+\overline{d})\ \right]^T. \quad (30)$$

Using (21) and (30), we have

$$\mathbf{E}(n) = \begin{bmatrix} \mathbf{H}(n;0) \\ \mathbf{H}(n+1;1) \\ \vdots \\ \mathbf{H}(n+\overline{d}-1;\overline{d}-1) \\ \mathbf{H}(n+\overline{d};\overline{d}) \end{bmatrix} \mathbf{s}(n) =: \tilde{\mathbf{H}}_{\overline{d}}(n)\mathbf{s}(n). \quad (31)$$

By (20) and (29) it follows that

$$\overline{\mathbf{e}}_d(n+d-1) = \sum_{i=1}^{K} \mathbf{A}_{i;n+d-1}^{(d-1)}\mathbf{x}(n-i+1) - \sum_{i=1}^{K} \mathbf{A}_{i;n+d-1}^{(d)}\mathbf{x}(n-i)$$

$$= \mathbf{A}_{1;n+d-1}^{(d-1)}\mathbf{x}(n) + \sum_{i=1}^{K-1} \mathbf{D}_{i;n+d-1}^{(d,d-1)}\mathbf{x}(n-i) - \mathbf{A}_{K;n+d-1}^{(d)}\mathbf{x}(n-K) \quad (32)$$

where

$$\mathbf{D}_{i;n+d-1}^{(d,d-1)} := \mathbf{A}_{i+1;n+d-1}^{(d-1)} - \mathbf{A}_{i;n+d-1}^{(d)},\ \ i = 1, 2, \cdots, K-1. \quad (33)$$

Using (29)-(33) we have

$$\mathbf{E}(n) = \mathcal{D}_n X(n) \quad (34)$$

where

$$X(n) := \left[\ \mathbf{x}^T(n) \vdots\ \mathbf{x}^T(n-1) \vdots \cdots \vdots\ \mathbf{x}^T(n-K)\ \right]^T \quad (35)$$

is a $[N(K+1)]-$column vector and $\mathcal{D}_n$ is a $[N(\overline{d}+1)] \times [N(K+1)]$ matrix composed of $\mathbf{D}_{i;n}^{(d,d-1)}$s and $\mathbf{A}_{i;n}^{(d)}$s for $1 \leq d \leq \overline{d}+1$. By (31), (34) and (**H3**), it follows that

$$\mathbf{R}_{EE}(0;n) = E\{\mathbf{E}(n)\mathbf{E}^H(n)\} = \tilde{\mathbf{H}}_{\overline{d}}(n)\tilde{\mathbf{H}}_{\overline{d}}^H(n) \quad (36)$$

$$= \mathcal{D}_n \mathcal{R}_{xx(K+1)}^{(n+1)} \mathcal{D}_n^H. \quad (37)$$

It has been shown in [4] that given noisy measurements $\mathbf{y}(n)$, one can consistently estimate noise variance σ_w^2 under (**H4**), using the correlation function of $\mathbf{y}(n)$. Therefore, one can estimate the correlation function of the noise-free data $\mathbf{x}(n)$ from that of the noisy data. In the following discussion it is assumed that such is the case. Calculate $\mathbf{R}_{EE}(0;n)$ as

$$\mathbf{R}_{EE}(0;n) = \mathcal{D}_n \left[\mathcal{R}_{yy(K+1)}^{(n+1)} - \sigma_w^2 I_{N(K+1)}\right] \mathcal{D}_n^H. \quad (38)$$

3.2. Code-Constrained Solution

Pick $\overline{d} = m$. Then by (12) and (31), we have

$$\tilde{\mathbf{H}}_m(n) = [\tilde{\mathbf{h}}_1(n),\ \tilde{\mathbf{h}}_2(n),\ \cdots\ \tilde{\mathbf{h}}_M(n)]. \quad (39)$$

Define $(j = 1, 2, \cdots, M)$

$$\tilde{\mathbf{g}}_j := [\mathbf{g}_{1,j}^T,\ \mathbf{g}_{2,j}^T,\ \cdots\ \mathbf{g}_{Q,j}^T]^T, \quad (40)$$

$$\tilde{\Gamma}_{nj} := [\Gamma_{1,n,j},\ \Gamma_{2,n,j},\ \cdots\ \Gamma_{Q,n,j}], \quad (41)$$

$$\Gamma_{q,n,j} := \Gamma_{q,n}\mathbf{C}_j,\ \ q = 1, 2, \cdots, Q. \quad (42)$$

Then $\tilde{\mathbf{h}}_j(n) = \tilde{\Gamma}_{nj}\tilde{\mathbf{g}}_j$ where $\tilde{\Gamma}_{nj}$ is known for $j = 1$, the desired user.

Let

$$\mathcal{D}_n \left[\mathcal{R}_{yy(K+1)}^{(n+1)} - \sigma_w^2 I_{N(K+1)}\right] \mathcal{D}_n^H$$

$$= [\mathcal{U}_{sn}\ \mathcal{U}_{nn}] \begin{bmatrix} \Lambda_{sn} & \\ & \Lambda_{nn} \end{bmatrix} [\mathcal{U}_{sn}\ \mathcal{U}_{nn}]^H \quad (43)$$

where $\Lambda_{sn} = \mathrm{diag}\,(\lambda_{1n}, \cdots, \lambda_{Mn})$ contains the M largest eigenvalues in descending order, $\mathcal{U}_{sn} = [\mathbf{u}_{1n}\ \cdots\ \mathbf{u}_{Mn}]$ contains the M corresponding orthonormal eigenvectors defining the signal subspace, $\Lambda_{nn} = \alpha_n I_{N(m+1)-M}$ $(\alpha_n \to 0)$, and $\mathcal{U}_{nn} = \left[\mathbf{u}_{(M+1)n}\ \cdots\ \mathbf{u}_{(Nm+N)n}\right]$ contains the $Nm + N - M$ orthonormal eigenvectors corresponding to the noise subspace (ideally eigenvalues equaling zero).

By orthogonality of $\tilde{\mathbf{H}}_m(n)$ to $\mathrm{sp}\{\mathcal{U}_{nn}\}$, it follows that

$$\tilde{\mathbf{h}}_j^H(n)\mathbf{u}_{ln} = \tilde{\mathbf{g}}_j^H \tilde{\Gamma}_{nj}^H \mathbf{u}_{ln} = 0,\ \ l = M+1, \cdots, Nm+N,\ \forall j. \quad (44)$$

Pick P time points $n \in \mathcal{N} := \{n_0, n_0+l, \cdots, n_0+(P-1)l\}$ for some $l \geq 1$. Then for the desired user ($j = 1$), (44) is satisfied for $n \in \mathcal{N}$ where $\tilde{\Gamma}_{n1}$ is known. Therefore, an estimate of the desired user's multipath channel coefficients $\tilde{\mathbf{g}}_1$ can be obtained (up to a time-invariant scale factor) by minimizing the cost

$$\tilde{\mathbf{g}}_1^H \left(\sum_{n\in\mathcal{N}} \tilde{\Gamma}_{n1}^H \left[\sum_{l=M+1}^{Nm+N} \mathbf{u}_{ln}\mathbf{u}_{ln}^H\right] \tilde{\Gamma}_{n1}\right) \tilde{\mathbf{g}}_1 =: \tilde{\mathbf{g}}_1^H \mathcal{A} \tilde{\mathbf{g}}_1 \quad (45)$$

subject to the constraint $\tilde{\mathbf{g}}_1^H\tilde{\mathbf{g}}_1 = 1$. The solution (up to a scale factor) is given by the eigenvector corresponding to the smallest eigenvalue of the matrix $\mathcal{A}$. Once $\tilde{\mathbf{g}}_1$ is estimated, we can obtain $g_1(n;l)$ via (4), (15) and (40).

3.3. Identifiability

Now we investigate the conditions (in addition to (**H1**)-(**H4**)) under which the solution of Sec. 3.2 will yield the desired solution. With no loss of generality, assume that the desired user is $j = 1$. Consider

(**H5**) The $[N(m+1)] \times [mNQ + M - 1]$ matrix $[\tilde{\Gamma}_{n1}\ \vdots\ \tilde{\mathbf{h}}_2(n)\ \vdots\ \cdots\ \vdots\ \tilde{\mathbf{h}}_M(n)]$ has rank equal to $\mathrm{rank}(\tilde{\Gamma}_{n1}) + M - 1\ \forall\ n \in \mathcal{N}$. Moreover, $\mathrm{rank}([\mathbf{g}_{1,1} \vdots\ \mathbf{g}_{2,1} \vdots \cdots \vdots\ \mathbf{g}_{Q,1}])$=Q.

Lemma: Suppose that mNQ-vector $\tilde{\mathbf{g}}'$ minimizes $\tilde{\mathbf{g}}'^H \mathcal{A} \tilde{\mathbf{g}}'$ (see (45)) subject to $\tilde{\mathbf{g}}'^H \tilde{\mathbf{g}}' = 1$. If $P \geq 2Q$ and (**H5**) holds true, then $\tilde{\mathbf{g}}' = \beta\tilde{\mathbf{g}}_1$ for some $\beta \neq 0$ where $\tilde{\mathbf{g}}_1$ satisfies (15) for $j = 1$.

Proof: By construction $\tilde{\Gamma}_{n1}\tilde{\mathbf{g}}' = \sum_{j=1}^{M} \alpha_{jn}\tilde{\mathbf{h}}_j(n)$ which implies $\tilde{\Gamma}_{n1}(\tilde{\mathbf{g}}' - \alpha_{1n}\tilde{\mathbf{g}}_1) = \sum_{j=2}^{M} \alpha_{jn}\tilde{\mathbf{h}}_j(n)\ \forall n$. Hence under (**H5**), $\tilde{\Gamma}_{n1}(\tilde{\mathbf{g}}' - \alpha_{1n}\tilde{\mathbf{g}}_1) = 0 \Rightarrow \tilde{\Gamma}_{n1}\tilde{\mathbf{g}}' = \alpha_{1n}\tilde{\mathbf{h}}_1(n)$ for $n \in$

$\mathcal{N}$. It follows from (12)-(15) and (40)-(42) that $\tilde{\Gamma}_{n1}\tilde{\mathbf{g}}' = \sum_{q=1}^{Q} e^{j\overline{\omega}_q n}\mathbf{g}'_{cq}$ where for some $(mN)\times 1$ $\mathbf{g}'_q$s,

$$\mathbf{g}'_{cq} = \text{diag}\{1, e^{j\omega_q}, \cdots, e^{j\omega_q(mN+N-1)}\}\mathbf{C}_1\mathbf{g}'_q. \quad (46)$$

Thus we have the system of equations

$$\sum_{q=1}^{Q} e^{j\overline{\omega}_q n}\mathbf{g}'_{cq} = \alpha_{1n}\tilde{\mathbf{h}}_1(n), \quad n \in \mathcal{N}, \quad (47)$$

where the unknowns are $\mathbf{g}'_{cq}$s and α_{1n}s. By [4, Lemma 1] we have a unique solution (up to a scale factor) to (47) if $P \geq 2Q$ and rank($[\mathbf{g}'_{c1} \vdots \mathbf{g}'_{c2} \vdots \cdots \vdots \mathbf{g}'_{cQ}]$)=Q. Note that (47) is satisfied for $\mathbf{g}'_{cq} = \text{diag}\{1, e^{j\omega_q}, \cdots, e^{j\omega_q(mN+N-1)}\}\mathbf{C}_1\mathbf{g}_{q,1}$. Hence, the rank condition on $\mathbf{g}_{q,1}$s in (**H5**) leads to the desired solution. □

Remark: It is seen from the rank condition on $\mathbf{g}_{q,1}$s in (**H5**) that if Q is "large" and the multipath spread is "small," the rank condition may not be fulfilled. A possible way to alleviate this is to use multiple sensors (receive antenna array) in addition to spreading – this is an interesting topic for future research □.

4. SIMULATION EXAMPLE

We consider the case of M (=2 or 5) users, each transmitting 4-QAM signals, and short-codes with N (=8 or 16) chips per symbol ($M = 2$ when $N = 8$ and $M = 5$ when $N = 16$). The spreading codes were randomly generated binary (±1, with equal probability) sequences. For multipath channels we took $L_g = 4$ (multipath spread of 4 chip intervals, assuming a synchronous system) and $Q = 2$ with

$$\omega_1 = 0, \quad \omega_2 = \frac{2\pi}{50}. \quad (48)$$

Using the model (4), the coefficients $g_{jq}(l)$ were randomly generated for each l (mutually independent, complex Gaussian, independent real and imaginary parts each with zero-mean and unit variance – Rayleigh fading). The channels were the same for each user (downlink) and were randomly generated in each run (i.e. were different in different Monte Carlo runs).

Normalized mean-square error in estimating the channel coefficients $g_{q,j_0}(l)$ of the desired user $j_0 = 1$, averaged over 100 Monte Carlo runs, was taken as the performance measure for channel identification. It is defined as (before Monte Carlo averaging)

$$\text{NCMSE} := \frac{\min_\beta \left\{\sum_{q=1}^{Q}\sum_{m=0}^{L-1} \|g_{q,j_0}(m) - \beta\widehat{g}_{q,j_0}(m)\|^2\right\}}{\sum_{q=1}^{Q}\sum_{m=0}^{L-1} \|g_{q,j_0}(m)\|^2} \quad (49)$$

Complex white zero-mean Gaussian noise was added to the received signal from the M users. The SNR refers to the symbol SNR of the desired user, which was user $j_0 = 1$, and it equals the energy per symbol divided by N_0 (= one-sided power spectral density of noise = $2\sigma_w^2$}). In the equal-power case (0dB MUIs), all users have the same power; in the near-far case (10dB MUIs), the desired user power is 10 dB below that of other users. The results of averaging over 100 Monte Carlo runs are shown in Fig. 1 for various SNR's for a record length of 1000 symbols. The proposed approach works well.

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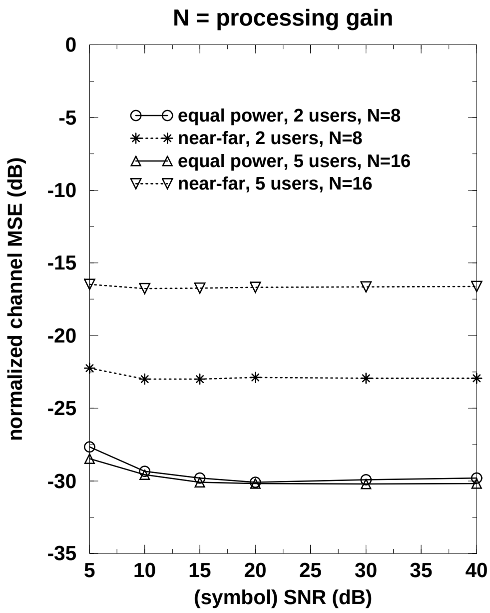

Fig. 1. Normalized channel MSE (49) for desired user j_0 =1: based on 1000 symbols per run, 100 Monte Carlo runs.