



NOVEL CHANNEL ESTIMATION ALGORITHM USING KALMAN FILTER FOR DS-CDMA RAYLEIGH FADING CHANNEL

Kuk-Jin Song and Sug-Ky Hong

CDMA Handsets Laboratory
LG Electronics
E-mail) ksong@lge.com

Sung-Yoon Jung and Dong-Jo Park

Department of EECS
Korea Advanced Institute of Science and Technology
Email) jsunguni@mail.kaist.ac.kr

ABSTRACT

We estimate DS-CDMA channel described by Rayleigh fading using a Kalman filter. A Kalman filter is proposed for the time varying channel estimation. The proposed scheme is constructed without bit estimation, that is, neither training sequence nor estimated bit symbol is used for channel estimation. We verify the performance of the channel estimation scheme with simulation results.

1. INTRODUCTION

Multiuser detectors for direct-sequence (DS) code-division-multiple-access (CDMA) communications have been studied over the past years. Most of the proposed multiuser detection schemes were focused on suppressing of multiple-access interference (MAI) from other users. Among the multiuser detectors, the minimum mean squared error (MMSE) detector offers the advantage of the implementation including adaptive and blind detection schemes.

To apply the MMSE criterion into the CDMA systems over the multipath channels, we must estimate the channel parameters. Many methods based on second order statistics [1] and constrained optimization approaches [2] have been proposed. These methods have used second order information of the received signal and proved reliable performance. The correlation matching technique has been applied to the CDMA channel estimation problem [3], as well. The decorrelating RAKE receiver proposed in [4] estimates the channel parameter after suppressing interference in the each delay finger.

In the time varying channel such as Rayleigh fading channel, it is hard to estimate the autocorrelation matrix of the received signal for adaptive receivers. The previously proposed channel estimation methods are difficult to apply the time varying channel. In this paper, we propose an estimation scheme of Rayleigh fading channels using the Kalman filter which needs no estimated bit symbol so that the method is independent of the bit detection scheme. Therefore, we can say that the channel estimation is inde-

pendent of the bit estimation algorithm. The constant minimum variance method (CMVM) is used to suppress interference.

2. PROBLEM FORMULATION

Assume K -user asynchronous DS-CDMA systems using binary phase shift keying (BPSK) modulation where each user has J resolvable paths. The k th user's bit symbol, $b_k(m)$ to be transmitted at the m th time is spread by a spreading waveform, $s_k(t)$ uniquely assigned to the specific user as following

$$x_k(t) = \sum_m b_k(m) s_k(t - mT_s) \quad (1)$$

where T_s is the bit duration. The spreading waveform $s_k(t)$ is given by

$$s_k(t) = \sum_{n=1}^N c_k(n) p(t - nT_c) \quad (2)$$

where $T_c = T_s/N$ is the chip duration, $c_k(n)$ is the spreading sequence of the k th user, $p(t)$ is the chip waveform and N is the processing gain.

$x_k(t)$ is transmitted through the multipath of which each channel has different delay and path gain. If the maximum delay is D , then we can set the multipath channel as a vector $\mathbf{h}_k(m)$ of order $q = D + 1$. Now, we define the *effective spreading code* of user one as

$$\bar{\mathbf{c}}_k(m) = \mathbf{C}_k \mathbf{h}_k(m) \quad (3)$$

where \mathbf{C}_k is $(N + D) \times D$ matrix as

$$\mathbf{C}_k = \begin{bmatrix} c_k(0) & 0 & \cdots & 0 \\ \vdots & c_k(0) & \cdots & \vdots \\ c_k(N-1) & \vdots & \ddots & 0 \\ 0 & c_1(N-1) & \cdots & c_k(0) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & c_k(N-1) \end{bmatrix}. \quad (4)$$

From the above deduction, the received baseband signal is

$$\begin{aligned}\mathbf{r}(m) &= [r(0) \quad r(1) \quad \cdots \quad r(N+D-1)]^T \\ &= \sum_{k=1}^K \mathbf{C}_k \mathbf{h}_k(m) b_k(m) + \mathbf{u}(m) \\ &= \sum_{k=1}^K \bar{\mathbf{c}}_k(m) b_k(m) + \mathbf{u}(m)\end{aligned}\quad (5)$$

where \mathbf{u} includes the multiple access interference (MAI) signals and inter-symbol interference (ISI) signals and we stack $N+D$ samples considering the maximum delay.

As it can be seen in (3), the multipath channel is time varying. In this paper, the first-order Gauss-Markov model [6] is used to describe the fading channel and given by

$$\mathbf{h}_k(m+1) = a \mathbf{h}_k(m) + \boldsymbol{\nu}(m). \quad (6)$$

where $a = \exp(-2\pi f_d T_s)$ and $\boldsymbol{\nu}$ is a zero-mean white Gaussian variable with variance σ_ν^2 .

The autocorrelation matrix of the received signal can be

$$\begin{aligned}\mathbf{R}_r(m) &\equiv E\{\mathbf{r}(m)\mathbf{r}^T(m)\} \\ &= \sum_{k=1}^K \mathbf{C}_k \mathbf{h}_k(m) \mathbf{h}_k^T(m) \mathbf{C}_k + \mathbf{R}_i(m)\end{aligned}\quad (7)$$

where $\mathbf{R}_i(m)$ is the autocorrelation matrix of the interference signals, $E\{\mathbf{u}(m)\mathbf{u}^T(m)\}$, and assumed to be stationary.

Without loss of generality, we consider the signal from the first user as the desired signal and the signals from all other users as interfering signals throughout the paper.

3. CHANNEL ESTIMATION WITHOUT BIT ESTIMATION

In general, the time-varying channel estimation requires the training bit sequences or the estimated bits. However, if the reliable bit estimation is not guaranteed, the Kalman filter would estimate wrong channel parameters. We propose a channel estimation scheme only using a Kalman filter without the bit estimation. Therefore, the proposed channel estimation scheme is independent of bit estimation. The block diagram is represented in Fig. 1.

In the front end of Fig. 1, CMVM filter is used to suppress interference existing in the received baseband signals.

The new state vector at the time m is defined not to use the estimated bit symbol using vector operation [5] as following:

$$\mathbf{d}_1(m) = \text{vec}(\mathbf{h}_1(m)\mathbf{h}_1^T(m)). \quad (8)$$

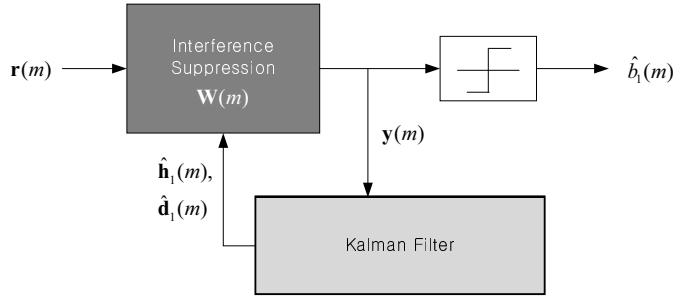


Fig. 1. System block diagram of channel estimation using Kalman filter without bit estimation.

Therefore, the processing equation and measurement equation must be redefined with the new state vector. The new processing equation matrix is obtained by (6) as

$$\mathbf{d}_1(m+1) = \text{vec}(\mathbf{h}_1(m+1)\mathbf{h}_1^T(m+1)) \quad (9)$$

$$= a^2 \mathbf{d}_1(m) + \boldsymbol{\nu}_2(m) \quad (10)$$

where $\boldsymbol{\nu}_2(m) = \text{avec}(\mathbf{h}_1(m)\boldsymbol{\nu}^T(m) + \boldsymbol{\nu}(m)\mathbf{h}_1^T(m)) + \text{vec}(\boldsymbol{\nu}(m)\boldsymbol{\nu}^T(m))$. Define \mathbf{R}_p as the correlation matrix of $\boldsymbol{\nu}_2(m)$ and express

$$\mathbf{R}_p(m) = E\{\boldsymbol{\nu}_2(m)\boldsymbol{\nu}_2^T(m)\}. \quad (11)$$

To complete the Kalman filter we must find out the $\mathbf{R}_p(m)$. The inside equation in the expectation (11) can be written as

$$\begin{aligned}\boldsymbol{\nu}_2(m)\boldsymbol{\nu}_2^T(m) &= \\ &a^2 \text{vec}(\mathbf{h}_1(m)\boldsymbol{\nu}^T(m)) \{\text{vec}(\mathbf{h}_1(m)\boldsymbol{\nu}^T(m))\}^T \\ &+ a^2 \text{vec}(\boldsymbol{\nu}(m)\mathbf{h}_1^T(m)) \{\text{vec}(\boldsymbol{\nu}(m)\mathbf{h}_1^T(m))\}^T \\ &+ \text{vec}(\boldsymbol{\nu}(m)\boldsymbol{\nu}^T(m)) \{\text{vec}(\boldsymbol{\nu}(m)\boldsymbol{\nu}^T(m))\}^T \\ &+ a^2 \text{vec}(\mathbf{h}_1(m)\boldsymbol{\nu}^T(m)) \{\text{vec}(\boldsymbol{\nu}(m)\mathbf{h}_1^T(m))\}^T \\ &+ \text{avec}(\mathbf{h}_1(m)\boldsymbol{\nu}^T(m)) \{\text{vec}(\boldsymbol{\nu}(m)\boldsymbol{\nu}^T(m))\}^T \\ &+ a^2 \text{vec}(\boldsymbol{\nu}(m)\mathbf{h}_1^T(m)) \{\text{vec}(\mathbf{h}_1(m)\boldsymbol{\nu}^T(m))\}^T \\ &+ \text{avec}(\boldsymbol{\nu}(m)\mathbf{h}_1^T(m)) \{\text{vec}(\boldsymbol{\nu}(m)\boldsymbol{\nu}^T(m))\}^T \\ &+ \text{avec}(\boldsymbol{\nu}(m)\boldsymbol{\nu}^T(m)) \{\text{vec}(\mathbf{h}_1(m)\boldsymbol{\nu}^T(m))\}^T \\ &+ \text{avec}(\boldsymbol{\nu}(m)\boldsymbol{\nu}^T(m)) \{\text{vec}(\boldsymbol{\nu}(m)\mathbf{h}_1^T(m))\}^T.\end{aligned}\quad (12)$$

The expectation of each term in the right side of (12) is solved separately.

$$\begin{aligned}E\{a^2 \text{vec}(\mathbf{h}_1(m)\boldsymbol{\nu}^T(m)) \{\text{vec}(\mathbf{h}_1(m)\boldsymbol{\nu}^T(m))\}^T\} \\ = a^2 \sigma_\nu^2 \mathbf{I}_J \otimes (\mathbf{h}_1(m)\mathbf{h}_1^T(m))\end{aligned}\quad (13)$$

$$\begin{aligned}E\{a^2 \text{vec}(\boldsymbol{\nu}(m)\mathbf{h}_1^T(m)) \{\text{vec}(\boldsymbol{\nu}(m)\mathbf{h}_1^T(m))\}^T\} \\ = a^2 \sigma_\nu^2 (\mathbf{h}_1(m)\mathbf{h}_1^T(m)) \otimes \mathbf{I}_J\end{aligned}\quad (14)$$

←
→

$$E \left\{ \text{vec} (\boldsymbol{\nu}(m) \boldsymbol{\nu}^T(m)) \{ \text{vec} (\boldsymbol{\nu}(m) \boldsymbol{\nu}^T(m)) \}^T \right\} = \sigma_{\nu}^4 \mathbf{I}_{J \times J} \quad (15)$$

$$E \left\{ a^2 \text{vec} (\mathbf{h}_1(m) \boldsymbol{\nu}^T(m)) \{ \text{vec} (\boldsymbol{\nu}(m) \mathbf{h}_1^T(m)) \}^T \right\} = a^2 \sigma_{\nu}^2 \mathbf{h}_1^T(m) \otimes (\mathbf{I}_J \otimes \mathbf{h}_1(m)) \quad (16)$$

$$E \left\{ \text{avec} (\mathbf{h}_1(m) \boldsymbol{\nu}^T(m)) \{ \text{vec} (\boldsymbol{\nu}(m) \boldsymbol{\nu}^T(m)) \}^T \right\} = \mathbf{0}_{J \times J} \quad (17)$$

$$E \left\{ a^2 \text{vec} (\boldsymbol{\nu}(m) \mathbf{h}_1^T(m)) \{ \text{vec} (\mathbf{h}_1(m) \boldsymbol{\nu}^T(m)) \}^T \right\} = a^2 \sigma_{\nu}^2 \mathbf{h}_1(m) \otimes (\mathbf{I}_J \otimes \mathbf{h}_1^T(m)) \quad (18)$$

$$E \left\{ \text{avec} (\boldsymbol{\nu}(m) \mathbf{h}_1^T(m)) \{ \text{vec} (\boldsymbol{\nu}(m) \boldsymbol{\nu}^T(m)) \}^T \right\} = \mathbf{0}_{J \times J} \quad (19)$$

$$E \left\{ \text{avec} (\boldsymbol{\nu}(m) \boldsymbol{\nu}^T(m)) \{ \text{vec} (\boldsymbol{\nu}(m) \boldsymbol{\nu}^T(m)) \}^T \right\} = \mathbf{0}_{J \times J} \quad (20)$$

$$E \left\{ \text{avec} (\boldsymbol{\nu}(m) \boldsymbol{\nu}^T(m)) \{ \text{vec} (\boldsymbol{\nu}(m) \mathbf{h}_1^T(m)) \}^T \right\} = \mathbf{0}_{J \times J} \quad (21)$$

The measurement vector $\mathbf{y}(m)$ is defined as

$$\mathbf{y}(m) = \text{vec} \{ (\mathbf{W}^T(m) \mathbf{r}(m)) (\mathbf{W}^T(m) \mathbf{r}(m))^T \} \quad (22)$$

where $\mathbf{W}(m)$ is the decorrelating filter to suppress interference as follows

$$\begin{aligned} \mathbf{W}(m) &= \min_{\mathbf{W}(m)} E \left\{ (\mathbf{W}^T(m) \mathbf{r}(m))^T (\mathbf{W}^T(m) \mathbf{r}(m)) \right\} \\ &\text{subject to} \quad \mathbf{C}_1^T \mathbf{W}(m) = \mathbf{I}_{N+D} \\ &= \mathbf{R}_r^{-1}(m) \mathbf{C}_1 (\mathbf{C}_1^T \mathbf{R}_r^{-1}(m) \mathbf{C}_1)^{-1}. \end{aligned} \quad (23)$$

where \mathbf{I}_{N+D} is the $(N+D) \times (N+D)$ identity matrix and the autocorrelation matrix must be obtained.

Since the autocorrelation matrix of the received signal (7) is time varying, the estimated autocorrelation matrix $\hat{\mathbf{R}}_r(m)$ can not be obtained by time averaging method. Therefore, the autocorrelation matrix is estimated using the estimated channel parameters of interested user and the autocorrelation matrix of the interference signal, by

$$\begin{aligned} \hat{\mathbf{R}}_r(m) &\equiv E \{ \mathbf{r}(m) \mathbf{r}^T(m) \} \\ &= \mathbf{C}_1 \hat{\mathbf{h}}_1(m) \hat{\mathbf{h}}_1^T(m) \mathbf{C}_1 + \hat{\mathbf{R}}_{in}(m) \end{aligned} \quad (24)$$

where $\hat{\mathbf{R}}_{in}(m)$ is the estimated autocorrelation matrix of interference signals including MAI and ISI and estimated by time averaging as

$$\begin{aligned} \hat{\mathbf{R}}_{in}(m) &= \frac{m-1}{m} \hat{\mathbf{R}}_{in}(m-1) \\ &+ \frac{1}{m} \left\{ \mathbf{r}(m) \mathbf{r}^T(m) - \mathbf{C}_1 \hat{\mathbf{h}}_1(m) \hat{\mathbf{h}}_1^T(m) \mathbf{C}_1 \right\}, \end{aligned} \quad (25)$$

because it is assumed that the interference signals are stationary. $\hat{\mathbf{h}}_1(m)$ can be found from kalman filter.

Assuming that the decorrelating filter suppress interference signals sufficiently, the measurement equation is constructed using the output of the CMVM filter as following:

$$\begin{aligned} \mathbf{y}(m) &= \text{vec} \{ (\mathbf{W}^T(m) \mathbf{C}_1 \mathbf{h}_1(m) \mathbf{b}_1(m) + \mathbf{W}^T(m) \mathbf{v}(m)) \\ &\quad \cdot (\mathbf{W}^T(m) \mathbf{C}_1 \mathbf{h}_1(m) \mathbf{b}_1(m) + \mathbf{W}^T(m) \mathbf{v}(m))^T \} \\ &\equiv \mathbf{Q}_1(m) \mathbf{d}_1(m) + \mathbf{v}_w(m) \end{aligned} \quad (26)$$

where

$$\mathbf{Q}_1(m) = (\mathbf{W}^T(m) \mathbf{C}_1) \otimes (\mathbf{W}^T(m) \mathbf{C}_1) \quad (27)$$

$$\begin{aligned} \mathbf{v}_w(m) &= \\ &b_1(m) (\mathbf{W}^T(m) \otimes \mathbf{W}^T(m)) \text{vec} (\mathbf{C}_1 \mathbf{h}_1(m) \mathbf{v}(m)) \\ &+ b_1(m) (\mathbf{W}^T(m) \otimes \mathbf{W}^T(m)) \text{vec} (\mathbf{v}(m) \mathbf{h}_1^T(m) \mathbf{C}_1^T) \\ &+ (\mathbf{W}^T(m) \otimes \mathbf{W}^T(m)) \text{vec} (\mathbf{v}(m) \mathbf{v}^T(m)) \end{aligned} \quad (28)$$

and we employed the following property [5] (\mathbf{A} , \mathbf{B} and \mathbf{C} are matrices)

$$\text{vec}(\mathbf{ABC}) = (\mathbf{C}^T \otimes \mathbf{A}) \text{vec}(\mathbf{B}). \quad (29)$$

where \otimes is the Kronecker product [5]. The correlation matrix of the measurement noise vector $\mathbf{v}_w(m)$ is defined as $\mathbf{R}_m(m)$:

$$\begin{aligned} E \{ \mathbf{v}_w(m) \mathbf{v}_w^T(m) \} &= \\ \mathbf{W}_2(m) E \{ \text{vec} (\mathbf{v}(m) \mathbf{v}^T(m)) \text{vec} (\mathbf{v}(m) \mathbf{v}(m)) \} \mathbf{W}_2^T(m) & \end{aligned} \quad (30)$$

$$= \mathbf{W}_2(m) \mathbf{W}_2^T(m) \sigma_n^4 \mathbf{I}_{(N+D) \times (N+D)} \quad (31)$$

From (10) and (26), the Kalman filter can be constructed without the estimated bit symbol as followings:

- Kalman gain computation

$$\begin{aligned} \mathbf{\Gamma}(m) &= a^2 \mathbf{\Lambda}(m, m-1) \mathbf{Q}_1^T(m) \\ &\cdot [\mathbf{Q}_1 \mathbf{\Lambda}(m, m-1) \mathbf{Q}_1^T(m) + \mathbf{R}_m(m)]^{-1} \end{aligned} \quad (32)$$

- One-step prediction

$$\mathbf{\alpha}(m) = \mathbf{y}(m) - \mathbf{Q}_1(m) \hat{\mathbf{d}}_1(m) \quad (33)$$

$$\hat{\mathbf{d}}_1(m+1) = a^2 \hat{\mathbf{d}}_1(m) + \mathbf{\Gamma}(m) \mathbf{\alpha}(m) \quad (34)$$

- Riccati equation solving

$$\begin{aligned} \mathbf{K}(m) &= \mathbf{\Lambda}(m, m-1) - \\ &a^2 \mathbf{\Gamma}(m) \mathbf{Q}_1(m) \mathbf{\Lambda}(m, m-1) \end{aligned} \quad (35)$$

$$\mathbf{\Lambda}(m+1, m) = a^4 \mathbf{K}(m) + \mathbf{R}_p(m) \quad (36)$$

where $\mathbf{\Gamma}(m)$ is Kalman gain at time m , $\mathbf{a}(m)$ is the innovations vector at time m , $\mathbf{\Lambda}(m)$ is the correlation matrix of the error in $\hat{\mathbf{d}}_1(m+1)$, and $\mathbf{K}(m)$ is the correlation matrix of the error in $\hat{\mathbf{d}}_1(m)$. Finally, we can obtain the channel parameter $\hat{\mathbf{h}}_1(m)$ from the estimated $\hat{\mathbf{d}}_1(m)$.

In the next section, the performance of the two Kalman filters is verified.

4. SIMULATION RESULTS

The proposed channel estimation algorithm is evaluated by numerical examples. The input SNR is defined to be $E\{|b_1(m)|^2/\sigma_n^2\}$ dB and the near-far-ratio (NFR) is the ratio of the signal power to the MAI power before despread-ing. The path gains are assumed to be independent, identically distributed unit variance Gaussian random variables, the path delays are assumed to be uniform over $[0, 3T_c]$, then the number of resolvable paths is $J = 4$ for all users ($q = 4$).

All simulations involved 5 CDMA signals spread by the randomly generated spreading codes of length $N = 20$ and BPSK modulation schemes are used. The number of fingers is $L = 4$ and the input SNR is 10 dB, that is, high SNR. A total 50 Monte-Carlo runs are executed to obtain the resulting statistics. The Rayleigh fading parameters are set to be $-2\pi f_d T_s = 0.004$.

For the comparison purpose, the Decorrelating RAKE receiver [4] without Kalman filter and Kalman Filter with bit estimation using the CMVM are simulated simultaneously. The performance measure is the mean-squared error of the channel estimation as

$$E\{\|\mathbf{h}_1(m) - \hat{\mathbf{h}}_1(m)\|^2\} \quad (37)$$

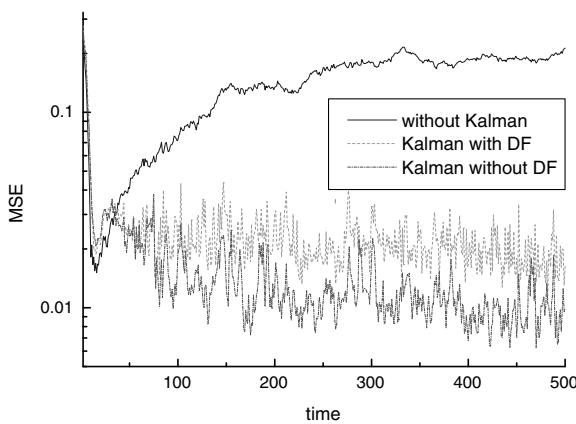


Fig. 2. MSEs of the channel estimation schemes when the number of user $K = 5$.

In Fig. 2, ‘without Kalman’ represents the decorrelat-ing RAKE receiver without Kalman filter, ‘Kalman with DF’

represents the channel estimation scheme using a Kalman filter with bit estimation (Decision feedback) and ‘Kalman without DF’ represents the proposed channel estimation scheme using a Kalman filter without estimated bits. The MSE of the scheme without decision feedback is better than that of the scheme with decision feedback since it is independent of the estimated bit symbol.

Above results show that the Kalman filter without decision feedback can track the time varying channel, efficiently. The proposed method has the advantage of the independence of the estimated bit symbols. Therefore, the bit error do not effect on the performance of the channel estimation.

5. CONCLUSIONS AND FURTHER WORKS

In this paper, the channel estimation scheme has been proposed for time varying channel. The Rayleigh fading model is applied and the solution to estimate the channel is suggested. We focused on estimation of the channel without training sequence or estimated bit symbols. This advantage makes the channel estimation be independent to the bit estimation. To improve the performance of the channel estimation must be studied as further works. In addition, non-stationary interference signal must be considered.

6. REFERENCES

- [1] X. Wang and H. V. Poor, “Blind equalization and multiuser detection in dispersive CDMA channels,” *IEEE Trans. Commun.*, vol.46, pp. 91–103, 1998.
- [2] M. K. Tsatsanis and Z. Xu, “Performance analysis of minimum variance cdma receivers,” *IEEE Trans. Signal Processing.*, vol. 46, no. 11, pp. 3014–3022 Nov. 1998.
- [3] Z. Xu, “On correlation matching approaches to multipath parameter estimation in multiuser CDMA systems,” *Proc. 2nd IEEE Workshop on Signal Processing Advances in Wireless Communications*, pp. 38–41, 1999.
- [4] H. Liu and K. Li, “A decorrelating RAKE receiver for CDMA communications over frequency-selective fading channels,” *IEEE Trans. Commun.*, Vol. 47, pp.1036–1046, 1999.
- [5] P. Lancaster and M. Tismenetsky, “The Theory of Matrices (2nd edition),” *Academic Press, San Diego, CA*, 1985.
- [6] Z. Zvonar and M. Stojanovic, “Performance of antenna diversity multiuser receivers in CDMA channels with imperfect fading estimation,” *Wireless Personal Commun.*, vol. 3, pp.91–110, 1996.