TURBO RECEIVERS FOR NARROW-BAND MIMO SYSTEMS

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ABSTRACT

The MIMO systems promise high spectral efficiency attainable if appropriate signal processing algorithms are used at the receiver. This paper compares several turbo-processing algorithms which may be used in the narrow-band systems. The algorithms are studied in the common simulations setup for two transmitter classes defined as horizontal and vertical encoding. The coded BER and BLER are compared to their respective lower-bounds using numerical simulations.

1. INTRODUCTION

Future wireless networks will have to provide high data rates with great spectral efficiency. In this context, so-called MIMO systems using multiple antennas both at the receiver and at the transmitter has raised considerable interest, as they exploit spatial diversity of the propagation environment. The first proposition and practical implementation of such a system was BLAST presented by Lucent [3, 10]. In the MIMO systems, each antenna transmitting different data streams use identical time-frequency resources. Therefore, each of the sub-streams is treated as interference to the others and has to be eliminated by some signal processing algorithm at the receiver. The algorithm originally implemented in V-BLAST [10] was based on interference nulling (Zero-Forcing) and successive cancellation. Since then, different algorithms have been considered based on MMSE, MAP and iterative processing [5, 7, 8, 10] . The literature also considers wide-band cases requiring space-time processing eg. [1] . In this paper we classify and evaluate the performance of various narrow-band MIMO receivers based on so-called turbo principle. These algorithms which gained recently considerable importance improve the performance measured in Bit- or Block Error Rates (BER, BLER) exchanging information between the channel decoder and the signal-processing part of the receiver. Application of turbo algorithms has been already proposed in the framework of MIMO systems [7, 8]. Here we show new extensions of some known algorithms to the iterative processing framework. In this sense, the present paper contribution consists in a unified description of known algorithms, proposition of new solutions and a comprehensive comparison of their performance in a common simulation setup. In this paper we compare the advantage of using two different transmitter setups resulting from so-called horizontal and vertical encodings. In order to gain the insight into the quality of the studied algorithms we use a lower bound criterion called Perfect Diversity Combining (PDC). In the following we define the system under study, describe the algorithms and presents the simulation results and conclusions.

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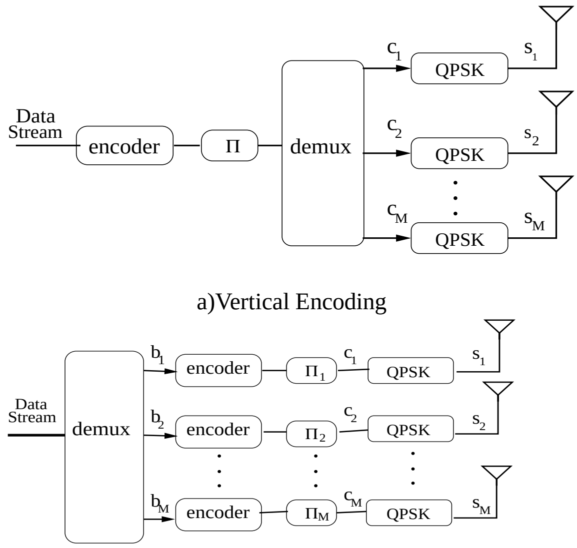
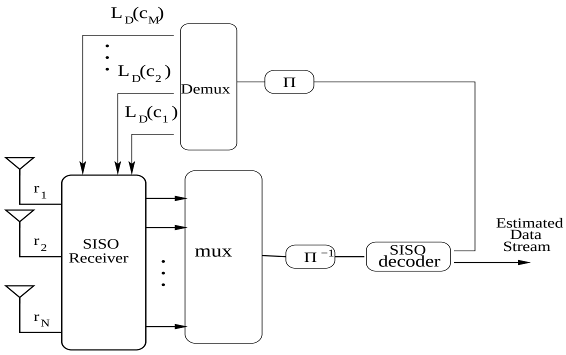


b)Horizontal Encoding

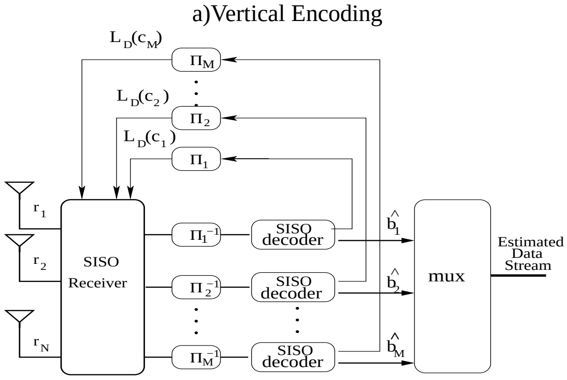
Fig. 1. Transmitter structures

2. SYSTEM DESCRIPTION

Consider two structures of multiantenna transmitter. The first one uses so-called vertical encoding (VE) where information bits are encoded and interleaved before being split into sub-stream as shown in Fig1.a; Π and Π^{-1} denote interleaver and deinterleaver respectively. The second system is based on so-called horizontal encoding (HE) where information bits are separated into sub-streams - then encoded and interleaved independently as shown in Fig1.b. The HE system may be seen as a particular case of multi-users SDMA system, since the data sub-streams are created independently. The choice of a transmitter structure will have an impact on the system performance and on the design of the receiver algorithms. Here we consider the particular system with $N = M$ antennas but it may straightforwardly extended to $N > M$ and $N < M$ cases. The narrow-band assumption results in the follow-



a)Vertical Encoding



b)Horizontal Encoding

Fig. 2. Receiver structures

ing model of the data transmission [3]:

$$\mathbf{r}(n) = \mathbf{H}\mathbf{s}(n) + \eta(n) \tag{1}$$

where $\mathbf{s} = [s_1(n), s_2(n), ..., s_M(n)]^T$ and the elements of $\mathbf{H}_{MxN}$ are unitary norm Rayleigh variables, η is the vector of random noise modeled as Gaussian variables with variance N_0. This define the average Signal-To-Noise ratio as $SNR = M/N_0$. The quasi static assumption is adopted i.e. channels remain constant during transmission of one data block. Transmissions being synchronized, the time index n may be dropped for convenience. We will assume that $\mathbf{H}$ is perfectly known.

3. RECEIVER ALGORITHMS

We analyze the algorithms apt to accept/produce soft input/outputs necessary for an iterative exchange of information with the decoder under the form of Logarithmic Likelihood Ratio (LLR). The literature proposes for this purpose the optimal MAP receiver [5, 9] and simplified linear receivers with hard or soft interference canceling [1, 7, 8, 10].

To avoid the exponential complexity of the algorithm MAP, we will analyze the algorithms linearly combining the received vector $\mathbf{r}$ and the statistics of the symbols $\mathbf{s}$ available from the decoders

$$y_k = \mathbf{w}_k^H \mathbf{r} + b_k \tag{2}$$

The variable b_k will convey a priori information about all the symbols $\mathbf{s}$. The methods using this linear filtering are Turbo Maximum Ratio Combining (T-MRC) and Turbo Minimum Mean Square Error (T-MMSE), the LLRs are further obtained from y_k as explained in Section 3.5.

3.1. T-MAP

The solution minimizing the raw bit error probability is obtained delivering to the decoders the *a posteriori extrinsic* LLR given by [5, 9] :

$$L_e(c_{k,j}|\mathbf{r}) = \ln \frac{P(c_{k,j} = 1|\mathbf{r})}{P(c_{k,j} = 0|\mathbf{r})} - L(c_{k,j}) =$$

$$\ln \frac{\sum\limits_{\forall \mathbf{s}:c_{k,j}=1} e^{-\frac{\|\mathbf{r}-\mathbf{Hs}\|^2}{N_0}} \prod\limits_{\forall k' \neq k, \forall j' \neq j} P(c_{k',j'})}{\sum\limits_{\forall \mathbf{s}:c_{k,j}=0} e^{-\frac{\|\mathbf{r}-\mathbf{Hs}\|^2}{N_0}} \prod\limits_{\forall k' \neq k, \forall j' \neq j} P(c_{k',j'})} \tag{3}$$

where $c_k = [c_{k,1} \quad c_{k,2} \ldots c_{k,P}]$ is the bit sequence corresponding to s_k and 2^P is the constellation size ($P = 2$ for the QPSK). To reduce the complexity of the algorithm T-MAP, we use the max-log approximation of the sum in (3) (i.e.: $ln(e^x + e^y) = max\{x, y\}$).

3.2. T-MRC

This algorithm proposed in [7] for MIMO case and in [4] for turbo-equalization, is composed of a matched filter followed by the parallel soft interference canceler

$$\mathbf{w}_k = \mathbf{h}_k, \quad b_k = \mathbf{h}_k^H(\mathbf{h}_k E[s_k] - \mathbf{H}E[\mathbf{s}]) \tag{4}$$

where $\mathbf{h}_k$ is the k-th column of matrix $\mathbf{H}$.

3.3. MMSE algorithms

In this algorithm $\mathbf{w_k}$ and b_k are found minimizing

$$J_{MMSE} = E[|s_k - y_k|^2] \tag{5}$$

which results in [9] [8]

$$\mathbf{w}_k = ((\mathbf{H}\bar{\mathbf{V}}\mathbf{H}^H + (1 - \bar{v}_{k,k})\mathbf{h}_k\mathbf{h}_k^H + \mathbf{I}N_0)^{-1}\mathbf{h}_k \tag{6}$$

$$b_k = \mathbf{w}_k^H(\mathbf{h}_k E[s_k] - \mathbf{H}E[\mathbf{s}]) \tag{7}$$

where $\bar{\mathbf{V}} = \text{diag}(\bar{v}_{1,1}, \ldots, \bar{v}_{M,M})$ is time averaged covariance matrix $\mathbf{V} = \text{diag}(Var[s_1], \ldots, Var[s_M])$. The time averaging lowers the complexity of the algorithm [9] since the same receiver is used for all the indexes of n (in the exact implementation $\mathbf{w}_k$ and b_k would be re-calculated for each n).

If $\mathbf{w}_k$ are calculated independently of each other for $k = 1 \ldots M$, the algorithm is called here T-MMSE. Additionally, it may be combined with Successive Interference Cancellation as proposed for V-BLAST [3]. Then, within each (turbo) iteration, the sub-streams are detected successively. Once the sub-stream is detected, the preliminary estimates of its symbols are formed and their effect is subtracted from the vector $\mathbf{r}$ (cf.(1)) potentially improving the quality of the estimates in the subsequent sub-streams. Depending on how these preliminary estimates are obtained we consider two versions of the MMSE algorithm:

- T-MMSE-HARD-SIC: This is the extension V-BLAST algorithm to the iterative processing. It performs Successive Interference Cancellation using the hard decisions $\hat{s}_j = T[y_j]$ assumed to be error-free. Therefore, $E[s_k] = \hat{s}_k$ and $Var[s_k] = 0$ for $j = 1 \ldots k-1$ should be used in (6) and (7); $T[]$ is the threshold device mapping y_k into the closest modulation symbol.

  If HE is used, the reliable hard estimates might be taken from the decoder output; however in such a case the performance offered by T-MMSE-HARD-SIC is practically the same as that one obtained by means of T-MMSE-SOFT-SIC.
- T-MMSE-SOFT-SIC: This algorithm extends the algorithm proposed in [1] to the turbo processing case. It obtains $E[s_k]$ and $Var[s_k]$ for $j = 1 \ldots k-1$ from the decoders at the current iteration so it may be applied only in the case of HE system (once the sub-stream is detected it may be immediately decoded without knowledge of other sub-streams).

3.4. Order detection for SIC

The order in which the sub-streams are detected affects strongly the performance of the algorithms bases on SIC [1, 2, 10]. The ordering strategy proposed in [10] (called here MW) consists in choosing the sub-stream whose corresponding vector $\mathbf{w}_k$ has the smallest norm. It maximizes SINR if Zero-Forcing receivers are used but is not optimal in the case of the MMSE. For this algorithm we propose the ordering (called MJ) consisting in choosing the sub-stream that leads to the minimum mean square error J_{MMSE} (cf. (5)) which after simple transformations yields

$$J_{MMSE} = \mathbf{w}_k^H \mathbf{h}_k \bar{v}_{k,k} + |E[s_k]|^2 \quad (8)$$

This ordering results in the performance similar to the one attained with the algorithm proposed in [2].

3.5. LLR calculation

The linear receivers need to transform y_k into a soft output. For QPSK modulation with gray mapping LLRs transmitted to the decoders are computed as [9]

$$L_e(c_{k,1}) = 2\sqrt{2}\frac{\Re[y_k]\mathbf{w}_k^H \mathbf{h}_k}{\sigma_k^2}$$
$$L_e(c_{k,2}) = 2\sqrt{2}\frac{\Im[y_k]\mathbf{w}_k^H \mathbf{h}_k}{\sigma_k^2} \quad (9)$$

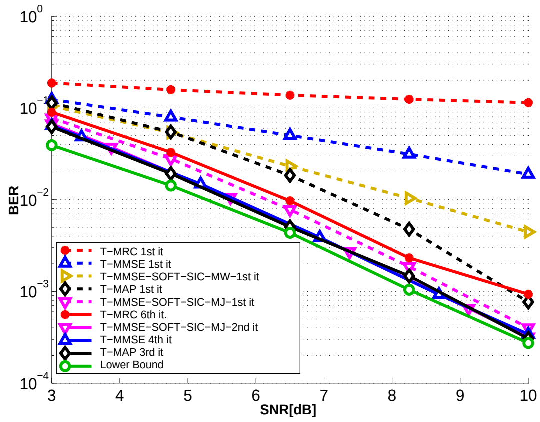
where $\Re$ and $\Im$ denote real and imaginary parts respectively, σ_k^2 is the variance of interference plus noise corrupting the signal y_k

$$\sigma_k^2 = \mathbf{w}_k^H(\mathbf{HVH}^H + \mathbf{I}N_0 - \text{Var}[s_k]\mathbf{h}_k\mathbf{h}_k^H)\mathbf{w}_k \quad (10)$$

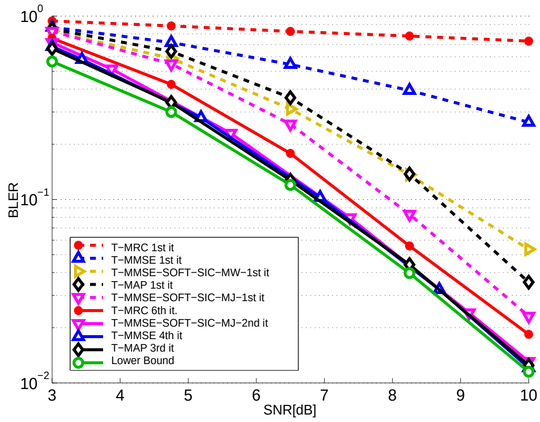
On the other hand, the symbols expectations (required by the algorithms T-MRC and T-MMSE) are obtained from the extrinsic LLRs available at the decoder output [9]:

$$E[s_k] = \frac{1}{\sqrt{2}}(\tanh(L_D(c_{k,1})/2) + i\tanh(L_D(c_{k,2})/2)) \quad (11)$$

The decoder used here is the max-log simplification of BCJR MAP algorithm [6].



a)Bit Error Rate v/s SNR



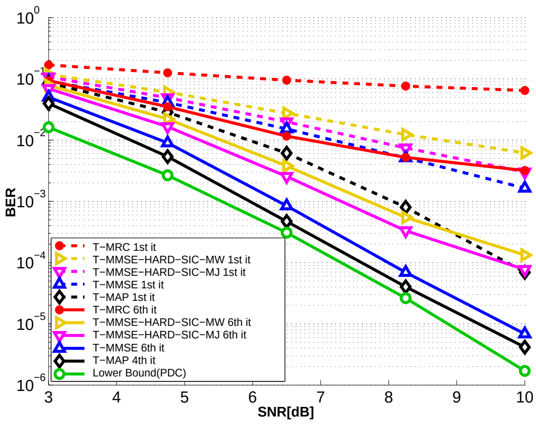
b) Block Error Rate v/s SNR

Fig. 3. Simulation results for horizontally encoded systems

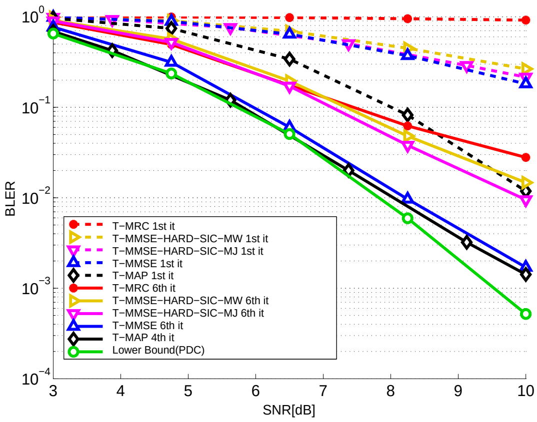
4. SIMULATION RESULTS

The simulations were carried out for a $N = M = 4$ system with QPSK modulation. A rate $1/2$ convolutional code was used, with octal generator polynomials {5,7}; random interleavers were applied. Blocks of 400 bits were used (this implies additional coding delay in case of HE system). Fig.3 and Fig.4 show the coded BER and BLER obtained by means of the studied algorithms using HE and VE respectively; the lower bound (PDC curve) is also given in both cases showing the results attainable if all the interference is perfectly eliminated. We note that VE offers the performance significantly superior to the one attainable with HE. The following observations concerning the studied algorithm were made:

- All the studied algorithm yield similar performance in HE system provided sufficient number of turbo iterations is executed. In T-MRC, the performance deteriorates by 0.5dB in the 6th iteration, compared to the lower bound
- In the first iteration only, T-MMSE-SOFT-SIC with MJ ordering approaches closely the lower-bound limit in the HE system. Note that MW ordering deteriorates the results by more than 1dB. This is because within the current iteration, the reliable LLRs from the decoders of initially detected



a)Bit Error Rate v/s SNR



b) Block Error Rate v/s SNR

Fig. 4. Simulation results for vertically encoded systems

sub-stream $j = 1 : \ldots k-1$ are used to calculate $\mathbf{w}_k$ and b_k. Clearly, for fair comparison with other turbo receivers, this iterative version of V-BLAST should be used.

- For VE, only T-MAP and T-MMSE approach the lower bound. T-MMSE-HARD-SIC looses 2.5dB at BLER=10^{-2} and shows similar performance to the one obtained by means of the T-MMSE-SOFT-SIC algorithm in HE systems.
- The performance attained using the algorithm T-MRC is better for HE than for VE. We explain this phenomena as follows. In the VE system all the sub-streams pass through similar conditions averaged through interleaving, so often none of the blocks will be error free. In the HE system however, some sub-streams have much better conditions (e.g. SINR) than the others and this, after some iterations, benefits the lower-SINR sub-streams due to the PIC procedure included in the algorithm.

5. CONCLUSIONS

This paper, using common notational framework and simulation setup, presents and analyzes various algorithm for the iterative turbo receivers in MIMO systems. From the presented analysis we may conclude that

- The best performance is offered by vertically encoded systems, and in order to attain it with relatively low complexity, the algorithm T-MMSE should be preferred over T-MAP.
- For horizontally encoded systems the performance of all the studied algorithms is similar and close to the lower bound, therefore the lowest complexity solution should be adopted. The algorithm T-MMSE-SOFT-SIC yields satisfactory results already in the first iteration (with small loss of performance with respect to the lower bound) if appropriate detection order is chosen (i.e. MJ - cf. Section 3.4). It should be considered as an alternative to more computationally complex iterative algorithms.

The further study should focus on the impact of channel estimation techniques on the performance of the algorithms and include the convergence analysis techniques such as those based on EXIT [9].

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