

# ADAPTIVE BIT ALLOCATION FOR SPACE-TIME BLOCK CODED OFDM SYSTEM

Mohammad Torabi and M. Reza Soleymani

Electrical and Computer Engineering Department, Concordia University,  
1455 de Maisonneuve Blvd. West, Montreal, Quebec, H3G 1M8, Canada.  
mohammad@ece.concordia.ca

## ABSTRACT

In this paper, a new scheme consisting of a combination of adaptive bit allocation, Space-Time Block Coded-OFDM and antenna selection is presented. The proposed scheme, exploits the benefits of Space-Time Block Codes, OFDM and adaptive bit allocation to provide high quality of transmission for wireless communications over frequency selective multipath channels with enhanced performance in terms of spectral and power efficiency. The system performance of non-adaptive OFDM, adaptive OFDM, non-adaptive STBC-OFDM, and the proposed adaptive STBC-OFDM are evaluated and compared. It is shown that the proposed scheme can greatly improve the performance of non-adaptive STBC-OFDM system.

## 1. INTRODUCTION

There is a growing demand for high-speed, spectrally efficient and reliable communication. Providing high-quality services in wireless environment has several challenges. Recently, the use of multiple transmit and receive antenna systems has been proposed as an efficient solution for future wireless systems. In particular, Space-Time Block Coding (STBC) systems have received much attention since they can greatly improve the system performance over flat fading channels with a reasonable level of complexity [1],[2].

However, in non-flat fading channels such as frequency selective multipath channels, convolution of the channel impulse response with STBC output destroys the orthogonality of the STBC. Using OFDM, the channel impulse response can be considered to be flat within each subcarriers. Hence, STBC with OFDM can be effectively used in non-flat fading channels.

In a hostile wireless environments, the channel dispersion will extensively attenuate some subcarriers of OFDM. Therefore, in the presence of multipath fading some subchannels will have higher attenuation and lower Signal-to-Noise Ratio (SNR) resulting in a higher error probability that decreases the overall system performance. This problem can be mitigated using different signal constellations for different subcarriers, i.e., adaptive modulation / adaptive bit allocation.

Adaptive modulation is a well known and powerful technique for increasing the spectral efficiency and improving the system performance. It has been studied for flat fading channels and multicarrier time-invariant channels. Adaptive modulation scheme for multicarrier systems, the so called Adaptive OFDM (AOFDM), has been studied in [3][4]. The goal of adaptive modulation in an OFDM system is to allocate an appropriate number of bits and to choose the suitable modulation mode for transmission in each

subcarrier, in order to improve the system performance or to keep the overall Bit Error Rate (BER) performance at a desired level. Good performance of these techniques requires accurate channel estimation and a reliable feedback to the transmitter.

In order to exploit the advantages of space-time coding, the combination of STBC and AOFDM has been considered in [5]. In [5] it is shown that the full benefits of AOFDM and STBC can not be exploited at the same time. In this paper, we present a space-time block coded OFDM scheme in conjugation with transmit antenna selection and adaptive modulation that can improve the overall system performance. The performance of the proposed system called A-STBC-OFDM, has been analytically evaluated and has been compared with non-adaptive OFDM, adaptive OFDM and non-adaptive STBC-OFDM. It is shown that using the antenna selection scheme at the transmitter in conjugation with adaptive bit allocation can improve the performance of STBC-OFDM. In addition, the antenna selection scheme with STBC can further improve the efficiency of the AOFDM.

The rest of this paper is organized as follows. The proposed scheme is represented in Section 2. In Section 3, the spectral efficiency of the adaptive and non-adaptive OFDM is evaluated. Then for the STBC-OFDM and the proposed scheme, both BER and spectral efficiency are analytically evaluated. Numerical analysis for the performance evaluation is presented in Section 4. Finally, Section 5 concludes this paper.

## 2. SYSTEM MODEL

Figure 1 depicts a high level block diagram of the proposed system. A block of data is serial-to parallel converted. According to the channel state information (CSI), known at the transmitter, associated subchannels to antenna 1 and 2 are selected and represented as two vectors:

$$A_i = (\alpha_{i,0}, \alpha_{i,1}, \dots, \alpha_{i,N_s})$$

where  $(i = 1, 2)$  and  $\alpha_{i,k} \in [0, 1]$ . Assuming an OFDM with  $N$  subcarriers,  $N_s$  is the number of subbands, that has been chosen to be  $N_s = N/2$ , i.e., each subband includes two adjacent subchannels.

The basic concept of antenna selection is to transmit each subband by the antenna which has the smallest attenuation of the subband. We simply select the antenna with the largest amplitude.

Therefore, the effective subchannel can be considered as follows.

$$H_{k\text{eff}} = \alpha_{1,k} H_{1,k} + \alpha_{2,k} H_{2,k} \quad (1)$$

Then, based on the characteristics of the selected subchannels i.e.  $H_{k\text{eff}}$ , bit allocation will be calculated. This bit allocation can be done by water-filling or other schemes. Here, we consider a simpler method of adaptive bit allocation where, the average BER

This work was supported by Natural Sciences and Engineering Research Council of Canada (NSERC) grant OGPIN011.

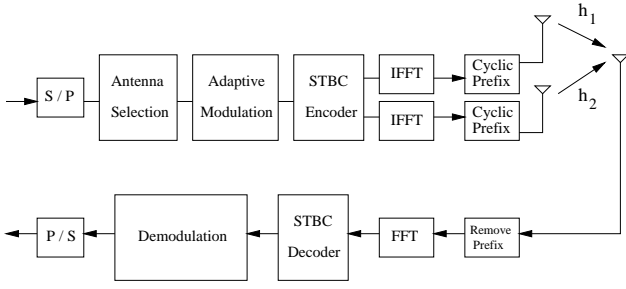


Fig. 1. Block diagram of the proposed system

requirement is set to the desired target. Power adaptation is not considered, for simplicity reasons. Then, all subbands are modulated using M-QAM where M is determined by the number of the allocated bits. Therefore, a signal vector

$$\{s_0, s_1, \dots, s_{N-1}\}$$

is provided as the input for STBC system.

STBC provides two blocks of  $\mathbf{S}_1$  and  $\mathbf{S}_2$  of the length  $N$ , for OFDM systems at transmitters. In order to utilize the space-frequency diversity, the input blocks are encoded as follow:

$$\begin{aligned} \mathbf{S}_1 &= (s_0 \quad -s_1^* \quad s_2 \quad -s_3^* \quad \dots \quad s_{N-2} \quad -s_{N-1}^*)^T \\ \mathbf{S}_2 &= (s_1 \quad +s_0^* \quad s_3 \quad +s_2^* \quad \dots \quad s_{N-1} \quad +s_{N-2}^*)^T \end{aligned} \quad (2)$$

OFDM modulators, generate blocks of  $\mathbf{X}_1$  and  $\mathbf{X}_2$  where they are transmitted by first antenna and second transmit antenna respectively.

Assuming that the guard time interval is longer than the largest delay spread of a multipath channel, to avoid ISI, the received signal will be the convolution of the channel and the transmitted signal. Assuming that the channel is static during an OFDM block, at the receiver side after removing the cyclic prefix, the FFT output as the demodulated received signal can be expressed as:

$$\mathbf{r} = \mathbf{H}_1 \mathbf{S}_1 + \mathbf{H}_2 \mathbf{S}_2 + \mathbf{W} \quad (3)$$

where  $\mathbf{r} = (r_0, \dots, r_{N-1})^T$ ,  $\mathbf{W} = (W_n, \dots, W_{N-1})^T$  denotes AWGN and  $\mathbf{H}_1$  and  $\mathbf{H}_2$  represent diagonal matrices whose elements  $(H_{i,j}, i = 1, 2, j = 0, 1, \dots, N-1)$  are the DFT of the time response of the channels  $h_1$  and  $h_2$ . By knowing the channel information at the receiver, ML detection can be used for decoding of received signal, which can be written as:

$$\begin{aligned} \tilde{s}_{2k} &= \alpha_{1,k} H_{1,2k}^* r_{2k} + \alpha_{2,k} H_{2,2k} r_{2k+1}^* \\ \tilde{s}_{2k+1} &= \alpha_{2,k} H_{2,2k+1}^* r_{2k} - \alpha_{1,k} H_{1,2k+1} r_{2k+1}^* \end{aligned} \quad (4)$$

where  $k = 0, \dots, \frac{N}{2} - 1$ .

Assuming that the channel gains between two adjacent subchannels are approximately equal, i.e.,  $H_{1,2k} = H_{1,2k+1}$  and  $H_{2,2k} = H_{2,2k+1}$ , then by substituting (3) into (4) the decoded signal can be expressed as:

$$\begin{aligned} \tilde{s}_{2k} &= (\alpha_{1,k}^2 |H_{1,2k}|^2 + \alpha_{2,k}^2 |H_{2,2k}|^2) s_{2k} + \\ &\quad \alpha_{1,k} H_{1,2k}^* W_{2k} + \alpha_{2,k} H_{2,2k} W_{2k+1}^* \\ \tilde{s}_{2k+1} &= (\alpha_{1,k}^2 |H_{1,2k}|^2 + \alpha_{2,k}^2 |H_{2,2k}|^2) s_{2k+1} + \\ &\quad \alpha_{2,k} H_{2,2k}^* W_{2k} - \alpha_{1,k} H_{1,2k} W_{2k+1}^* \end{aligned} \quad (5)$$

The above variables provide a diversity gain of order two for every  $s_{2k}$  and  $s_{2k+1}$ , where proper antenna selection, as can be seen, can increase the SNR at the receiver and improve the system performance. It can be concluded that the proposed scheme can provide significant gains in performance over conventional OFDM and conventional STBC-OFDM. At the end, the elements of block  $\{\tilde{s}_j\}_{j=0}^{N-1}$  are adaptively demodulated to extract the information data.

### 3. ADAPTIVE BIT ALLOCATION FOR STBC-OFDM

In this section we determine adaptive bit allocation for maximizing spectral efficiency. Assume MQAM is employed for each subchannel and  $\beta_k$  bits/symbol is assigned for the  $k$ th subchannel, where  $M = 2^{\beta_k}$ . Also the negligible degradation due to the cyclic prefix in OFDM is not considered.

#### 3.1. Adaptive OFDM

The expression for the instantaneous BER of the  $k$ th subchannel in the block of OFDM (square MQAM with Gray bit mapping on each subcarrier) over a frequency selective fading channel can be approximately written as:

$$BER[k] \approx \frac{2}{\beta_k} \left(1 - \frac{1}{\sqrt{2^{\beta_k}}}\right) \text{erfc} \left( \sqrt{\frac{1.5 \gamma_s |H_k|^2}{2^{\beta_k} - 1}} \right) \quad (6)$$

where  $\text{erfc}(x)$  is the complementary error function.

$BER[k]$  can be approximated as [6], [7]:

$$BER[k] \approx 0.2 \exp \left( -\frac{1.6 \gamma_s |H_k|^2}{2^{\beta_k} - 1} \right) \quad (7)$$

where  $\gamma_s = \frac{E_s}{N_0}$ ,  $E_s$  is the symbol energy at the transmitter and  $\frac{N_0}{2}$  is the variance of the real/imaginary part of the AWGN.  $H_k$  is the frequency response of the fading channel.

By inverting (7), the maximum instantaneous data rate  $\beta_k$  that can be transmitted under a target BER ( $BER_t$ ) constraint for a given instantaneous SNR can be represented as:

$$\beta_k \approx \log_2 \left( 1 - \frac{1.6 \gamma_s |H_k|^2}{\ln \left( \frac{BER_t}{0.2} \right)} \right) \quad (8)$$

The average spectral efficiency (number of bits per second per Hz) can be written as:

$$R = E_{H_k} \{\beta_k\} \quad (9)$$

#### 3.2. Non-Adaptive OFDM

In the case of non-adaptive OFDM, the same number of bits is allocated to each subbands, i.e.  $\beta_k = \beta$ . The average BER can be written as:

$$\overline{BER} = E_{H_k} \{BER[k]\} \quad (10)$$

Since  $|H_k|$  is Rayleigh-distributed, we can obtain:

$$\overline{BER} \approx \frac{0.2}{1 + \frac{1.6 \gamma_s}{2^{\beta} - 1}} \quad (11)$$

We can invert (11) to express  $\beta$  as a function of  $\gamma_s$  and the target  $BER_t$  as:

$$\beta \approx \log_2 \left( 1 + \frac{1.6 \gamma_s}{\left( \frac{0.2}{BER_t} - 1 \right)} \right) \quad (12)$$

The average spectral efficiency, in this case is equal to  $\beta$ . Figure 2, shows the spectral efficiencies of non-adaptive OFDM, and adaptive OFDM.

### 3.3. Adaptive STBC-OFDM (A-STBC-OFDM)

According to (5) and (6), the expression for the instantaneous  $BER$  of each subcarrier of MQAM-STBC-OFDM over frequency selective fading channel can be written as:

$$BER[k] \approx \frac{2}{\beta_k} \left( 1 - \frac{1}{\sqrt{2^{\beta_k}}} \right) \times \operatorname{erfc} \left( \sqrt{\frac{1.5 \gamma_s (\alpha_{1,k}^2 |H_{1,k}|^2 + \alpha_{2,k}^2 |H_{2,k}|^2)}{2^{\beta_k} - 1}} \right) \quad (13)$$

This can be approximated as:

$$BER[k] \approx 0.2 \exp \left( - \frac{1.6 \gamma_s (\alpha_{1,k}^2 |H_{1,k}|^2 + \alpha_{2,k}^2 |H_{2,k}|^2)}{2^{\beta_k} - 1} \right) \quad (14)$$

therefore, for the target  $BER_t$  in STBC-OFDM, the maximum instantaneous data rate can be calculated from:

$$\beta_k \approx \log_2 \left( 1 - \frac{1.6 \gamma_s (\alpha_{1,k}^2 |H_{1,k}|^2 + \alpha_{2,k}^2 |H_{2,k}|^2)}{\ln \left( \frac{BER_t}{0.2} \right)} \right) \quad (15)$$

The average spectral efficiency can be written as:

$$R = E_{H_{1,k}, H_{2,k}} \{ \beta_k \} \quad (16)$$

### 3.4. Non-Adaptive STBC-OFDM

In the case of non-adaptive STBC-OFDM, the average  $BER$  can be represented as:

$$\overline{BER} = E_{H_{1,k}, H_{2,k}} \{ BER[k] \} \quad (17)$$

Since  $|H_{1,k}|$  and  $|H_{2,k}|$  are i.i.d Rayleigh-distributed, therefore, by choosing  $\alpha_{1,k}^2 = \alpha_{2,k}^2 = 0.5$ , (without antenna selection) we can obtain:

$$\overline{BER} \approx \frac{0.2}{\left( 1 + \frac{0.8 \gamma_s}{2^{\beta} - 1} \right)^2} \quad (18)$$

Therefore, we can invert (18) to express  $\beta$  as a function of  $\gamma_s$  and the target  $\overline{BER}_t$  as:

$$\beta \approx \log_2 \left( 1 + \frac{0.8 \gamma_s}{\left( \sqrt{\frac{0.2}{\overline{BER}_t}} - 1 \right)} \right) \quad (19)$$

The average spectral efficiency, in this case is equal to  $\beta$ . Figure 3, shows the spectral efficiencies of non-adaptive STBC-OFDM, and A-STBC-OFDM.

According (8) and (15), continuous-rate adaptation can be obtained based on the CSI, where the number of bits/symbol is not restricted to integer values. In practice, numbers of bits/symbol should be integer values regarding the constellation size, the so-called discrete-rate adaptation. Discrete-rate adaptive modulation responds to the instantaneous  $SNR$  by varying the constellation size. We can divide the  $SNR$  range into a number of regions, each representing a constellation size.

The set of switching thresholds/adaptation is then obtained from the requires instantaneous  $SNR$  to achieve the target-BER using M-QAM over an AWGN channel. For example, for a target-BER of  $10^{-4}$ , the thresholds are 8.4, 11.4, 18.2 and 24.3 dB for BPSK (1 bit), 4-QAM (2 bits), 16-QAM (4 bits) and 64-QAM (6 bits) respectively. For  $SNR < 8.4dB$  zero bit will be allocated (no Transmission).

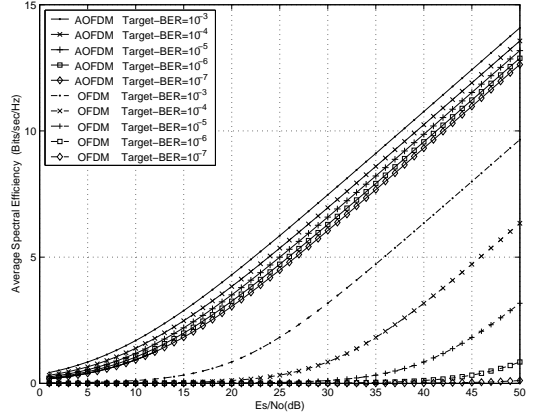


Fig. 2. Spectral efficiency of OFDM and AOFDM

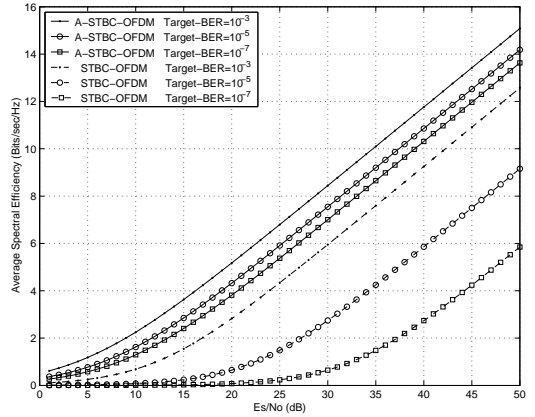


Fig. 3. Spectral efficiency of STBC-OFDM and A-STBC-OFDM

## 4. SIMULATION RESULTS

The performance of the proposed system is evaluated by computer simulation for frequency selective multipath fading channels. The considered non-flat channels are multipath frequency selective channels where at the antenna  $i$ , ( $i = 1, 2$ ) can be expressed as:

$$h_i(t) = \sum_{m=0}^{L-1} \alpha_{m,i}(t) \delta(t - \tau_m(t)) \quad (20)$$

where  $\alpha_{m,i}(t)$  is the tap weight,  $\tau_m(t)$  is the time delay of the  $m$ -th path and  $L$  is the number of total paths. It is assumed that the channel state information (CSI) is available at the receiver and transmitter. Channel taps are complex Gaussian random process with zero mean and variance of  $\frac{1}{L}$  (equal power).

The average spectral efficiencies of non-adaptive OFDM and adaptive OFDM are compared in Figure 2 for different target BERs. It can be seen that adaptive OFDM can greatly improve the performance of OFDM. For example at a target-BER= $10^{-3}$  and spectral efficiency of 6 bits/sec/Hz about 14 dB gain can be obtained, and at a target-BER= $10^{-5}$  and spectral efficiency of 4 bits/sec/Hz about 30 dB gain can be obtained.

Figure 3 compares, the spectral efficiency of the SFBC-OFDM with that of A-SFBC-OFDM. As shown, A-SFBC-OFDM is better than SFBC-OFDM in terms of spectral efficiency for the given target BER. For example, at a spectral efficiency of 6 bits/sec/Hz,

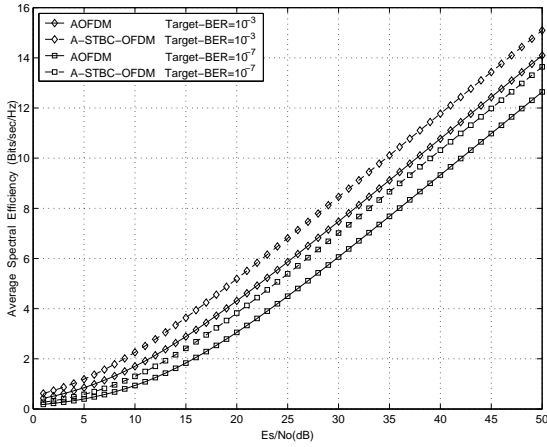


Fig. 4. Spectral efficiency of AOFDM and A-STBC-OFDM

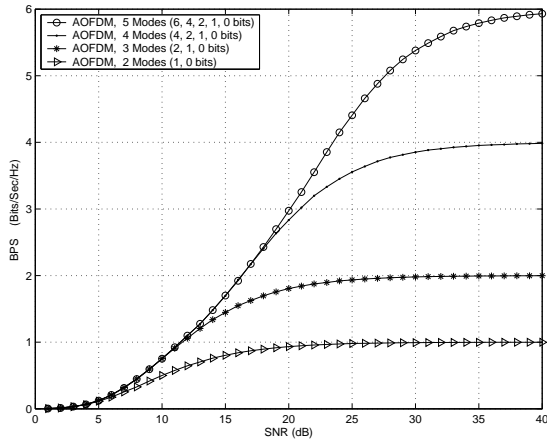


Fig. 5. Spectral efficiency of AOFDM

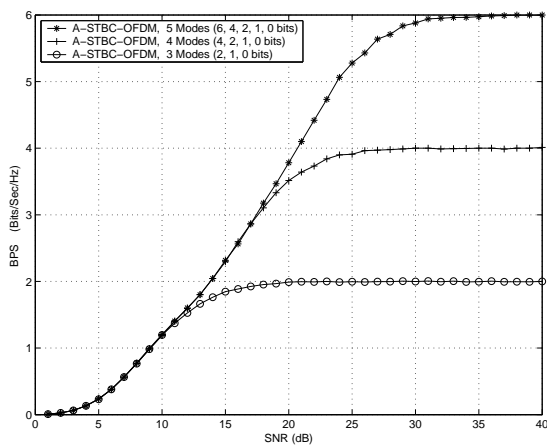


Fig. 6. Spectral efficiency of A-STBC-OFDM

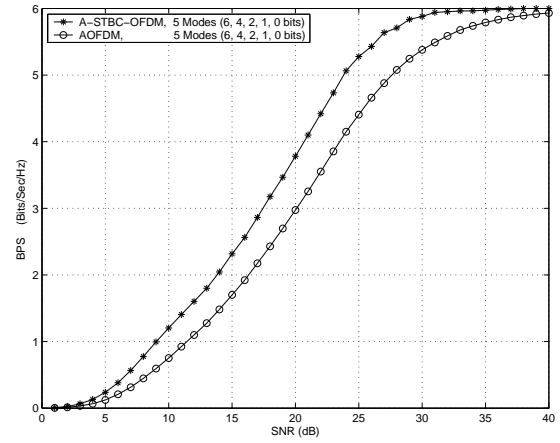


Fig. 7. Spectral efficiency of AOFDM and A-STBC-OFDM

gains of 7.6 dB, 15.16 dB and 23.51 dB can be obtained at a target-BERs of  $10^{-3}$ ,  $10^{-5}$  and  $10^{-7}$ , respectively. Figure 4 compares, the spectral efficiency of the AOFDM with that of A-STBC-OFDM. As shown, A-STBC-OFDM has superior performance than AOFDM. For example, at a spectral efficiency of 6 bits/sec/Hz, gains of 3 dB can be obtained at a target-BERs of  $10^{-3}$  and  $10^{-7}$ .

Figures 5, 6 show the average spectral efficiencies of discrete-rate case of AOFDM, A-STBC-OFDM. And finally, Figure 7 shows that A-STBC-OFDM is better than AOFDM in terms of spectral efficiency.

## 5. CONCLUSION

In order to improve the system performance and spectral efficiency of STBC-OFDM system, we proposed adaptive bit allocation. In the proposed system, according to the CSI available at the transmitter, an appropriate number of bits are allocated to each sub-carrier of STBC-OFDM in order to keep the overall BER performance at a desired level. It is shown that adaptive bit allocation can greatly improve the system performance and efficiency of STBC-OFDM systems. It is also shown that with the proposed scheme, STBC can significantly improve the performance of the adaptive OFDM.

## 6. REFERENCES

- [1] S. M. Alamouti, "A Simple Transmit Diversity Technique for Wireless Communications", *IEEE Journal on Selected areas in Comm.* Vol.16, No.8, Oct. 1998.
- [2] V. Tarokh, H. Jafarkhani and A.R. Calderbank, "Space Time Block Codes from orthogonal designs", *IEEE Trans. Information Theory*, vol. 45, no.5, pp. 1456-1467, July 1999.
- [3] T. Keller and L. Hanzo, "Adaptive multicarrier modulation: a convenient frame work for time-frequency processing in wireless communications," *Proc. of the IEEE*, vol. 88, pp. 611-640, May 2000.
- [4] A. Czylik, "Adaptive OFDM for wideband radio channels", in *Proc. Globecom 1996*.
- [5] T. H. Liew and L. Hanzo, "Space-Time Block coded Adaptive Modulation Aided OFDM," in *Proc. Globecom 2001*.
- [6] S. T. Chung and A.J. Goldsmith, "Degrees of freedom in adaptive modulation: a unified view," *IEEE Trans. on Comm.*, vol. 49, pp. 1561-1571, Sept. 2001.
- [7] S. Ye, R. S. Blum and L. J. Cimini, "Adaptive modulation for variable-rate OFDM systems," in *proc. VTC spring 2001*.