

A GENERALIZED EXPONENTIAL BER BOUND FOR POWER ALLOCATION WITH IMPERFECT CHANNEL ESTIMATES

Francesc Rey, Meritxell Lamarca, Gregori Vázquez

Department of Signal Theory and Communications, Polytechnic University of Catalonia
UPC Campus Nord - Mòdul D5, c/Jordi Girona 1-3, 08034 Barcelona (Spain)
e-mail:{frey,xell,gregori}@gps.tsc.upc.es

ABSTRACT

A beamforming for OFDM modulation is proposed that allocates power over subchannels and subcarriers. The design follows a minimum BER criterion taking into account the different reliability of CSI estimates at both ends of the transmitter link. The transmitter design is based on a generalized exponential BER bound that is tighter than the Chernoff bound, whereas the receiver that minimizes the BER is implemented by the MAP detector. The resulting design reconfigures itself, from the known solution when perfect CSI is available to the open-loop solution depending on the channel uncertainty.

1. INTRODUCTION

The use of closed-loop schemes with MIMO systems, increases the link performance either in terms of capacity or in terms of reliability. However, the potential of these schemes can only be fully accomplished when perfect Channel State Information (CSI) is available. In real scenarios, CSI is always imperfect, and its quality is unbalanced between transmitter and receiver. The transmitter has only an *a priori* channel state knowledge, based on a quantized feedback channel and the prediction of the future channel state from previous CSI [1]. On the contrary, the receiver is able to take advantage of the observation at the channel output to measure the channel state *a posteriori*. Accordingly, since neither prediction, nor quantized error degrades CSI at the receiver, the channel knowledge is always superior at this side.

This paper aims to design robust transmitter and receiver algorithms when the quality of CSI is unequal, following the same Bayesian approach proposed in [2]. The transmitter, employing the predicted CSI, allocates the available power among all subcarriers and antennas in a MIMO-OFDM channel, minimizing the Bit Error Rate (BER) when channel estimates are noisy. Unlike other proposed algorithms that minimize the Chernoff bound as alternative to the $Q(\cdot)$ function [3], this paper introduces a tighter generalized exponential BER bound adjusted, for each subcarrier, to the nominal SNR. At the receiver a robust solution is also implemented based on a maximum a posteriori (MAP) detector, making use of all the available information. The

resulting design reconfigures itself as a function of the channel uncertainty degree: when perfect CSI is available robust and non robust solutions converge, whereas when the quality in CSI diminishes robust algorithm tends to the open-loop solution outperforming non-robust one. This design is similar to that one presented in [4], where partial CSI is used at the transmitter to combine the benefits of beamforming with predetermined space-time codes. Although the robust algorithm presented in this paper is also focused on beamforming (i.e. only the most dominant channel mode is used to transmit symbols), it can be generalized to transmit distinct symbols in the larger channel modes increasing the data throughput [5].

2. PROBLEM STATEMENT

This section describes the signal model for an OFDM MIMO communications system and summarizes the Bayesian approach to design power allocation strategies robust to CSI errors in [2]. The MIMO configuration consists of M_T transmitter and M_R receiver antennas. The application of OFDM modulation with Q subcarriers allows to decouple the frequency-selective MIMO channel into Q MIMO frequency-flat channels if a certain structure is imposed in the transmitter and receiver.

Let $\mathbf{x} = [x(1) \dots x(Q)]^T$ be the $Q \times 1$ vector that contains Q information symbols to be transmitted in one OFDM symbol, assumed to be i.i.d, with zero mean and variance $E\{\mathbf{x}\mathbf{x}^H\} = \sigma_x^2 \mathbf{I}$. The input-output relation, once the cyclic prefix has been removed, can be written in terms of matrices that involve only one subcarrier each as:

$$\mathbf{r}_k = \mathbf{H}_k \mathbf{f}_k x_k + \mathbf{n}_k \quad k = 1 \dots Q \quad (1)$$

where \mathbf{r}_k is the $M_R \times 1$ vector that contains the symbols received through the different antennas for the k th subcarrier; \mathbf{H}_k is a $M_R \times M_T$ matrix containing the frequency responses of the MIMO channels; \mathbf{f}_k is a $M_T \times 1$ vector that allocates the power over the M_T antennas; and \mathbf{n}_k is the noise vector after the FFT, which has the same Gaussian statistic as its time-domain counterpart, zero mean and variance $E\{\mathbf{n}_k \mathbf{n}_k^H\} = \sigma_n^2 \mathbf{I}$.

Making use of the identity $\text{vec}(\mathbf{ABC}) = (\mathbf{C}^T \otimes \mathbf{A}) \text{vec}(\mathbf{B})$ [6], the received vector in (1) can also be written as:

$$\mathbf{r}_k = \mathbf{F}_k \mathbf{h}_k x_k + \mathbf{n}_k \quad k = 1 \dots Q \quad (2)$$

where \mathbf{h}_k vector, which reflects the MIMO channel responses at the k th subcarrier, and \mathbf{F}_k are defined as:

$$\mathbf{h}_k = \text{vec}\{\mathbf{H}_k\} \quad ; \quad \mathbf{F}_k = \mathbf{f}_k^T \otimes \mathbf{I} \quad (3)$$

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Similarly, all subcarrier channel vectors are grouped in a single vector \mathbf{h} that will denote the whole MIMO channel response:

$$\mathbf{h} = [\mathbf{h}_1^T \quad \dots \quad \mathbf{h}_Q^T]^T \quad (4)$$

The performance of any power allocation strategy is very sensitive to CSI errors. Accordingly, as channel response is never completely known, an optimal design of \mathbf{f}_k vector can never be fulfilled assuming perfect CSI at the transmitter. Instead, the design of robust solutions that introduce the statistics of the channel uncertainty in the cost function exhibit lower sensitivity and provide better performances.

The robust design of \mathbf{f}_k will follow a Bayesian approach based on the estimate channel model $\hat{\mathbf{h}}$:

$$\hat{\mathbf{h}} = \mathbf{h} + \boldsymbol{\varepsilon} \quad (5)$$

where the estimation errors $\boldsymbol{\varepsilon}$ are modelled as a zero mean Gaussian random variable, independent of the true Rayleigh channel \mathbf{h} .

Assuming uncorrelation between antennas and between channel taps in the channel impulse response, and assuming that the power delay profile of the channel impulse response is identical for all subchannels, the covariance matrix of the channel \mathbf{h} is:

$$E\{\mathbf{h}\mathbf{h}^H\} = \mathbf{P} \otimes \mathbf{I} \quad (6)$$

where \mathbf{P} is the circulant matrix built as a Hermitian Toeplitz matrix whose first row is $[P(1) \quad \dots \quad P(Q)]$, and $P(k) \quad k = 1 \dots Q$ is the DFT of the power delay profile. Similarly, the covariance of the channel error $\boldsymbol{\varepsilon}$ extended to all subcarriers and antennas becomes:

$$E\{\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^H\} = \mathbf{E} \otimes \mathbf{I} \quad (7)$$

where matrix \mathbf{E} , with the same structure that \mathbf{P} , contains the DFT of the variance in the channel estimation error for each tap.

As the channel estimate $\hat{\mathbf{h}}$ only provides partial information on the true channel \mathbf{h} , the impact of the channel uncertainty is mitigated by averaging the function to minimize over the real channel, given the channel estimate. Assuming that $\hat{\mathbf{h}}$ and \mathbf{h} vectors are jointly Gaussian, the conditional p.d.f. $f_{\mathbf{h}|\hat{\mathbf{h}}}(\mathbf{h})$ required to compute previous expectation, is also a Gaussian random variable whose mean and covariance are given by [2]:

$$\begin{aligned} \mathbf{m}_{\mathbf{h}|\hat{\mathbf{h}}} &= (\mathbf{P}(\mathbf{P} + \mathbf{E})^{-1} \otimes \mathbf{I}) \hat{\mathbf{h}} \\ \mathbf{C}_{\mathbf{h}|\hat{\mathbf{h}}} &= (\mathbf{P}(\mathbf{P} + \mathbf{E})^{-1} \mathbf{E}) \otimes \mathbf{I} \end{aligned} \quad (8)$$

2.1. The equivalent channel

The conditional mean (8) can be rewritten as a linear combination of the estimated channels for all subcarriers:

$$\mathbf{h}_k^{eq} = \mathbf{m}_{\mathbf{h}_k|\hat{\mathbf{h}}} = \sum_{j=1}^Q \beta_k(j) \hat{\mathbf{h}}_j \quad (9)$$

where $\beta_k(j)$ is the j -th element of vector $\boldsymbol{\beta}_k$ defined as:

$$\boldsymbol{\beta}_k = (\mathbf{P}^T + \mathbf{E}^T)^{-1} \mathbf{p}_k \quad (10)$$

and \mathbf{p}_k is the k th column of \mathbf{P}^T .

According to (9), \mathbf{h}_k^{eq} can be defined as an equivalent channel that exploits the correlation between subcarriers, and the channel uncertainty structure, to mitigate the mismatch between the real and the estimated channel (for an extended interpretation on this equivalent channel see [2]).

3. TRANSMITTER DESIGN

This section designs the linear transformation \mathbf{f}_k to optimally allocate the power over all antennas and subcarriers, minimizing the uncoded BER for each subcarrier. Assuming QPSK modulation, the exact BER for the k th subcarrier, and the averaged BER over all subcarriers become:

$$BER_k = Q(\sqrt{SNR_k}) ; \quad \overline{BER} = \frac{1}{Q} \sum_{k=1}^Q BER_k \quad (11)$$

As the use of the exact $Q(\cdot)$ function derives in complex solutions, this paper proposes an expansion of the $Q(\cdot)$ function in the neighborhood of the nominal SNR_k for each particular subcarrier. By a generalized exponential expansion of the BER, the $Q(\cdot)$ function can be expressed as:

$$Q(\sqrt{x}) \simeq D e^{-Ax} \quad (12)$$

where constants A and D can be chosen to set certain constraints. The Chernoff bound can be regarded as a particular case of (12) for $A = 1/2$ and $D = 1/2$. However, a more tight bound can be obtained following a Taylor expansion of $\ln(Q(\sqrt{x}))$ in the neighborhood of the point $x = a$. In this case A and D constants are given by:

$$A = \frac{e^{-|a|/2}}{2\sqrt{2\pi a} Q(\sqrt{a})} ; \quad D = Q(\sqrt{a}) e^{Aa} \quad (13)$$

It can be shown that (13) provides a lower bound for $Q(\cdot)$, that is very tight in a wide range of values of x around a .

According to (12), the exact BER for the k th subcarrier in the neighborhood of $a = SNR_k$ can be approximated by:

$$BER_k = Q(\sqrt{SNR_k}) \simeq \delta_k e^{-\alpha_k SNR_k} \quad (14)$$

where α_k and δ_k constants, which depend on the k th subcarrier, refers to A and D in (13).

The SNR_k for the k th subcarrier is given by:

$$SNR_k = \frac{\sigma_x^2}{\sigma_n^2} \mathbf{h}_k^H \mathbf{F}_k^H \mathbf{F}_k \mathbf{h}_k \quad (15)$$

and the exponent in (14) can be expressed as a function of vector \mathbf{h} defined in (4) as:

$$\alpha_k SNR_k = \mathbf{h}^H \mathbf{M}_k \mathbf{h} \quad (16)$$

where \mathbf{M}_k is the $QM_T M_R \times QM_T M_R$ matrix:

$$\mathbf{M}_k = \gamma_k \begin{bmatrix} \mathbf{0}_1 \\ \vdots \\ \mathbf{I}_k \\ \vdots \\ \mathbf{0}_Q \end{bmatrix} \mathbf{F}_k^H \mathbf{F}_k \begin{bmatrix} \mathbf{0}_1 & \dots & \mathbf{I}_k & \dots & \mathbf{0}_Q \end{bmatrix} \quad (17)$$

and \mathbf{I}_i denotes a $M_T M_R$ identity matrix allocated in the i th position, $\mathbf{0}_i$ is an all zero matrix, and $\gamma_k = \alpha_k \frac{\sigma_x^2}{\sigma_n^2}$.

Substituting (16) and (14) in the averaged uncoded BER (11), the robust function to minimize, subject to a power constraint, is obtained computing the expectation of (11) over the real channel given the channel estimate:

$$\xi = E_{\mathbf{h}/\hat{\mathbf{h}}} \{ \overline{BER} \} = \frac{1}{Q} E_{\mathbf{h}/\hat{\mathbf{h}}} \left\{ \sum_{k=1}^Q \delta_k e^{-\mathbf{h}^H \mathbf{M}_k \mathbf{h}} \right\} \quad (18)$$

subject to $\sum_{k=1}^Q \mathbf{f}_k^H \mathbf{f}_k = P_0$

3.1. Cost function

Using the results in (8) to expand the Gaussian p.d.f. $f_{\mathbf{h}/\hat{\mathbf{h}}}$, the averaged cost function in (18) becomes:

$$\xi = \frac{1}{Q} \sum_{k=1}^Q \frac{\delta_k}{\pi^Q |\mathbf{C}_{\mathbf{h}/\hat{\mathbf{h}}}|} \int_{\mathbf{h} \in \mathbb{C}} e^{-\mathbf{h}^H \mathbf{M}_k \mathbf{h}} e^{-(\mathbf{h} - \mathbf{h}^{eq})^H \mathbf{C}_{\mathbf{h}/\hat{\mathbf{h}}}^{-1} (\mathbf{h} - \mathbf{h}^{eq})} d\mathbf{h} \quad (19)$$

where \mathbf{h}^{eq} , as defined in (9), denotes the equivalent channel over all subcarriers. Previous integral can be easily solved rewriting its integrand as:

$$\int_{\mathbf{h} \in \mathbb{C}} e^{-(\mathbf{h} - \mu)^H \beta (\mathbf{h} - \mu) - \eta} d\mathbf{h} \quad (20)$$

where:

$$\begin{aligned} \beta &= \mathbf{M}_k + \mathbf{C}_{\mathbf{h}/\hat{\mathbf{h}}}^{-1} \\ \mu &= \beta^{-1} \mathbf{C}_{\mathbf{h}/\hat{\mathbf{h}}}^{-1} \mathbf{h}^{eq} \\ \eta &= \mathbf{h}^{eqH} (\mathbf{M}_k \mathbf{C}_{\mathbf{h}/\hat{\mathbf{h}}} + \mathbf{I})^{-1} \mathbf{M}_k \mathbf{h}^{eq} \end{aligned} \quad (21)$$

The solution to the integral in (20) can be found by comparing its integrand with a complex Gaussian p.d.f., whose integral equals to one. Accordingly, (19) becomes:

$$\xi = \frac{1}{Q} \sum_{k=1}^Q \frac{\delta_k}{|\mathbf{C}_{\mathbf{h}/\hat{\mathbf{h}}} \mathbf{M}_k + \mathbf{I}|} e^{-\mathbf{h}^{eqH} (\mathbf{M}_k \mathbf{C}_{\mathbf{h}/\hat{\mathbf{h}}} + \mathbf{I})^{-1} \mathbf{M}_k \mathbf{h}^{eq}} \quad (22)$$

As the matrix \mathbf{M}_k only contains non-zero values in a block of elements as defined in (17), previous expression can be simplified following:

$$\begin{aligned} |\mathbf{C}_{\mathbf{h}/\hat{\mathbf{h}}} \mathbf{M}_k + \mathbf{I}| &= 1 + \gamma_k \omega |\mathbf{f}_k|^2 \\ (\mathbf{M}_k \mathbf{C}_{\mathbf{h}/\hat{\mathbf{h}}} + \mathbf{I})^{-1} \mathbf{M}_k &= \frac{1}{1 + \gamma_k \omega |\mathbf{f}_k|^2} \mathbf{M}_k \end{aligned} \quad (23)$$

where ω , obtained from the conditioned covariance matrix $\mathbf{C}_{\mathbf{h}/\hat{\mathbf{h}}}$ (8), denotes the (k, k) th element of $(\mathbf{P}(\mathbf{P} + \mathbf{E})^{-1} \mathbf{E})$, and is independent of the k th subcarrier since \mathbf{E} and \mathbf{P} are circulant matrices.

Finally substituting (23) and (3) in (22), and applying $vec(\mathbf{ABC}) = (\mathbf{C}^T \otimes \mathbf{A}) vec(\mathbf{B})$ [6], the function to minimize, subject to a power constraint, becomes:

$$\xi = \frac{1}{Q} \sum_{k=1}^Q \frac{\delta_k}{1 + \gamma_k \omega |\mathbf{f}_k|^2} e^{-\gamma_k \mathbf{f}_k^H \mathbf{H}_k^{eqH} \mathbf{H}_k^{eq} \mathbf{f}_k \left(\frac{1}{1 + \gamma_k \omega |\mathbf{f}_k|^2} \right)} \quad (24)$$

3.2. Robust power allocation

This subsection presents a solution for the optimization of (24). The minimization is obtained selecting \mathbf{f}_k to be in the direction of the right singular vector \mathbf{v}_k associated to the largest singular value λ_k^{eq} of the equivalent channel \mathbf{H}_k^{eq} :

$$\mathbf{f}_k = \mathbf{v}_k \phi_k \quad (25)$$

where the scalar ϕ_k is the new parameter to design, and denotes the power allocated to the k th subcarrier. In order to proof (25) it is sufficient to see that all the terms in the summation of the cost function in (24) are positive. Then, the way to minimize the function is to minimize each term independently. According to the SVD of the equivalent channel, previous minimization is achieved maximizing the argument of the exponential term, focusing the transmitted symbols in the direction of the singular vector associated to the maximum singular value of the channel.

Substituting (25) in (24) the new cost function, including the power constraint (18), becomes:

$$\frac{1}{Q} \sum_{k=1}^Q \frac{\delta_k}{1 + \gamma_k \omega |\phi_k|^2} e^{-\frac{\gamma_k |\phi_k|^2 |\lambda_k^{eq}|^2}{1 + \gamma_k \omega |\phi_k|^2}} - \mu \left[\sum_{k=1}^Q |\phi_k|^2 - P_0 \right] \quad (26)$$

where μ is the Lagrange multiplier. The minimization problem follows deriving the gradient $\nabla_{\phi_k^*}$ and equaling it to zero. After some manipulations on the gradient, the optimal power allocation can be obtained by solving for each subcarrier:

$$\begin{aligned} &-\frac{\gamma_k |\phi_k|^2 |\lambda_k^{eq}|^2}{1 + \gamma_k \omega |\phi_k|^2} - 2 \ln(1 + \gamma_k \omega |\phi_k|^2) + \\ &\ln \left(\omega + \frac{|\lambda_k^{eq}|^2}{1 + \gamma_k \omega |\phi_k|^2} \right) = \mu - \ln(\delta_k \gamma_k) \end{aligned} \quad (27)$$

A closed form solution for this identity can not be derived. However under the assumption that the channel uncertainty is low¹, we approximate $\ln(1 + x) \simeq x$ and $1/(1 + x) \simeq 1$, and (27) is simplified:

$$-\gamma_k |\lambda_k^{eq}|^2 |\phi_k|^2 - 2\gamma_k \omega |\phi_k|^2 + \ln(\omega + |\lambda_k^{eq}|^2) = \mu - \ln(\delta_k \gamma_k) \quad (28)$$

obtaining a closed form solution for $|\phi_k|^2$:

$$|\phi_k|^2 = \left[-\frac{\mu - \ln(\delta_k \gamma_k) - \ln(\omega + |\lambda_k^{eq}|^2)}{\gamma_k (2\omega + |\lambda_k^{eq}|^2)} \right]^+ \quad (29)$$

and μ is determined forcing the power constraint (18).

4. THE OPTIMUM RECEIVER

Making use of all the available information at the receiver side (i.e the designed \mathbf{f}_k for all subcarriers, and CSI at the receiver) the optimum receiver becomes the MAP detector. Under the assumption that all transmitted symbols are equally likely, the MAP criterion and ML make the same decisions. Accordingly, in the case of imperfect channel estimates, the decision rule is based on finding the transmitted symbols that maximize the conditional likelihood function $f_{\mathbf{r}_k/\hat{\mathbf{h}}}(\mathbf{r}_k/x_k)$. As the conditional ML function is Gaussian, obtaining the maximum of the likelihood function over x_k is equivalent to finding the symbols x_k that maximize:

$$\mathbf{D}(\mathbf{r}_k, x_k) = \frac{2}{\sigma_x^2 |\phi_k|^2 \omega + \sigma_n^2} Re \left\{ \mathbf{r}_k^H \mathbf{H}_k^{eq} \mathbf{f}_k x_k \right\} \quad (30)$$

where constants and irrelevant terms have been omitted, and conditional mean and covariance have been obtained following (8).

¹If this assumption does not hold, an iterative solution can be derived using the gradient $\nabla_{\phi_k^*}$.

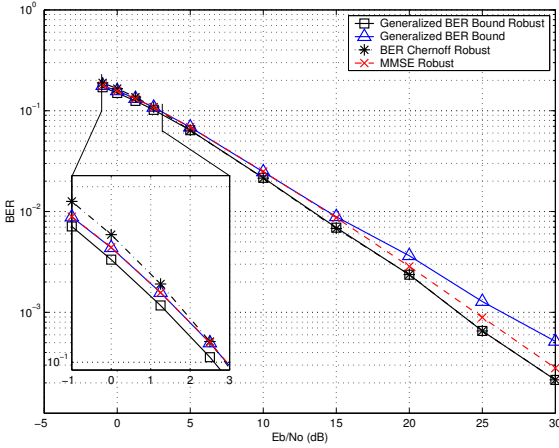


Figure 1: BER comparison between different power allocation strategies. $M_T=1$, $M_R=1$. Tx uncertainty: $\sigma_\epsilon^2 = 0.02$.

5. SIMULATION RESULTS

Computer simulation were carried out to illustrate the robustness of the proposed beamformer and MAP receiver as a function of the MIMO configuration. The bit stream to be transmit was mapped into a QPSK constellation, and one symbol per subcarrier was transmitted over the M_T antennas. The channel obeyed an exponential power delay profile with 50ns of delay spread. It was assumed that the channel estimation error had the same variance σ_ϵ^2 for all taps of the impulse channel response. Channel uncertainty at the receiver was assumed to be proportional to the noise variance σ_n^2 , as would be the case in a linear channel estimator. On the contrary, channel uncertainty at the transmitter included a proportional to σ_n^2 term modelling channel prediction error in time-varying channels.

Figure 1 evaluates the BER (11) for different power allocation criteria: the generalized BER bound solution (α_k and δ_k according to (13)), the Chernoff bound solution ($\alpha_k = \delta_k = 1/2$) and the MMSE solution [2]. As it was expected, robust solutions have best performance than non-robust ones, and the relative difference grows when SNR increases. Moreover, the proposed exponential BER bound always outperforms others, even at low SNR's, where Chernoff bound exhibits a lower performance. Accordingly, the proposed bound (12) becomes an appropriate alternative to the $Q(\cdot)$ function extensive to any SNR ratio.

Figure 2 shows the minimum EbNo to achieve a $\overline{BER} \leq 10^{-3}$ for 1x1 and 2x1 MIMO configurations. The target EbNo is plotted as a function of the channel uncertainty degree ρ defined as:

$$\rho = E \left\{ \epsilon^H \epsilon \right\} / E \left\{ \hat{\mathbf{h}}^H \hat{\mathbf{h}} \right\} \quad (31)$$

Accordingly, $\rho = 0$ denotes perfect CSI, whereas $\rho = 1$ means no channel knowledge. Two simulations are plotted for different channel uncertainty degrees at the receiver: perfect CSI and $\sigma_\epsilon^2 = \sigma_n^2/4$. The point $\rho = 0$, plotted as a reference, was always simulated with perfect CSI at transmitter and receiver. The robustness of the proposed solution is evidenced comparing robust and non-robust algorithms for different channel uncertainties. When perfect CSI is available, both algorithms have the same performance. Nevertheless, when the CSI quality degrades the losses of the non-robust solution are significant. It is also

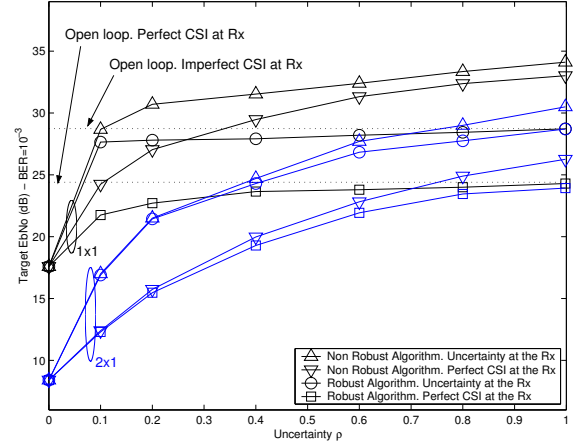


Figure 2: Minimum EbNo that achieves $\overline{BER} \leq 10^{-3}$. Tx. uncertainty defined by ρ . Rx. uncertainty $\sigma_\epsilon^2 = \sigma_n^2/4$.

worth to remark that the robust algorithm tends to the open-loop solution when CSI degrades.

6. CONCLUSIONS

This paper presented the design of a robust power allocation strategy focused on the minimization of the uncoded BER, and an optimum MAP detector, in the presence of channel uncertainties. Unlike other solutions that consider the Chernoff bound to evaluate the BER, this paper introduced a more tight generalized exponential BER bound. Numerical results demonstrate that the robust solution, according to a Bayesian formulation, outperforms algorithms that assume perfect channel knowledge, with a complexity similar to that one of existing techniques. Moreover, the proposed solution reconfigures itself as a function of the channel uncertainty degree, achieving the open-loop solution when no channel information is available.

7. REFERENCES

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