TIME-DOMAIN METHOD FOR TRACKING DISPERSIVE CHANNELS IN MIMO OFDM SYSTEMS

Timo Roman, Mihai Enescu and Visa Koivunen

Signal Processing Laboratory
Helsinki University of Technology
P.O. Box 3000
FIN-02015 HUT, Finland
email: {troman,mihai,visa}@wooster.hut.fi

ABSTRACT

In this paper we address the problem of channel estimation for Multiple-Input Multiple-Output OFDM systems for mobile users. Channel tracking and equalization method stemming from Kalman filtering is proposed for time-frequency selective channels. Tracking of MIMO channel matrix is performed in time-domain and equalization in frequency domain. Computational complexity is significantly reduced by applying the matrix inversion lemma. Simulation results are presented using realistic channel model in typical urban scenarios.

1. INTRODUCTION

Radio spectrum is a scarce resource in wireless communication. A very high spectral efficiency in terms of $bits/s/Hz$ may be achieved by employing multiple transmitters and receivers in both ends of the link. Such systems can be modeled using Multiple-Input Multiple-Output (MIMO) system model. High capacity is obtained by exploiting different sources of diversity such as rich scattering environment.

Multicarrier systems, including orthogonal frequency division multiplexing (OFDM) systems play an important role in future beyond 3G wireless communication services. A key benefit of OFDM is its ability to turn a frequency selective channel into a set of parallel narrowband channels, which leads to very simple equalization since the transmission becomes free of Intersymbol Interference (ISI). However, the theoretical benefits of MIMO and OFDM systems may not be fully achieved in broadband mobile applications because the channels are both time and frequency selective.

In this paper we address the problem of tracking time-varying channels in MIMO-OFDM systems. Both frequency and time selectivity of the channel are taken into account. A time-domain MIMO channel matrix tracking algorithm stemming from Kalman filter is derived. Time-domain channel tracking has recently been employed in a Single-Input Single-Output (SISO) OFDM system using the H_∞ approach [2]. The high computational complexity in Kalman gain calculation is significantly reduced by applying the matrix inversion lemma. Complexity is reduced to be proportional to channel length, not only to the number of subcarriers. The channel estimation is followed by equalization in frequency domain. Both zero-forcing and minimum mean square error (MMSE) equalizers are presented.

The time-domain approach introduced in this paper shows highly reliable tracking performance and robustness. It tracks accurately both the amplitudes and phases of MIMO channels. Time-domain method exploits the inherent frequency correlation among the taps. Hence, it is robust to estimation errors that are spread over the complete frequency band.

The rest of the paper is organized as follows. In the next section, we briefly present the MIMO-OFDM system model. In Section 3 we introduce the channel tracking and equalization scheme. In Section 4 we present simulation results using mobile 2-transmitter 2-receiver MIMO-OFDM system in a typical urban scenario. Finally, Section 5 concludes the paper.

This work has been supported by the Academy of Finland and Nokia.

2. SYSTEM MODEL

The MIMO-OFDM transmission model used in this paper is presented in Figure 1. In the following, the transmission process is described in a formal way, using the notation introduced in [6]. A 2-transmit / 2-receive antenna configuration is considered. The extension to the M-transmit / M-receive antenna case is a straightforward generalization. The k^{th} modulated OFDM block at transmit antenna i is written as $\tilde{\boldsymbol{x}}_i(k) = \boldsymbol{F}_N\, \boldsymbol{a}_i(k)$, where $\boldsymbol{F}_N$ is the $N \times N$ inverse discrete Fourier transform (IDFT) matrix, N being the total number of subcarriers and $\boldsymbol{a}_i(k)$ is the $N \times 1$ complex symbol vector sent from antenna i.

The received $2N \times 1$ signal block after cyclic prefix insertion, followed by transmission on the wireless channel and cyclic prefix removal is expressed as:

$$\begin{bmatrix} \boldsymbol{r}_1(k) \\ \boldsymbol{r}_2(k) \end{bmatrix} = \begin{bmatrix} \widetilde{\boldsymbol{H}}_{11}(k) & \widetilde{\boldsymbol{H}}_{21}(k) \\ \widetilde{\boldsymbol{H}}_{12}(k) & \widetilde{\boldsymbol{H}}_{22}(k) \end{bmatrix} \cdot \begin{bmatrix} \tilde{\boldsymbol{x}}_1(k) \\ \tilde{\boldsymbol{x}}_2(k) \end{bmatrix} + \begin{bmatrix} \boldsymbol{w}_1(k) \\ \boldsymbol{w}_2(k) \end{bmatrix}, \quad (1)$$

or, in a more compact matrix form:

$$\boldsymbol{r}(k) = \widetilde{\boldsymbol{H}}(k)\, \tilde{\boldsymbol{x}}(k) + \boldsymbol{w}(k), \quad (2)$$

where $\boldsymbol{r}_j(k)$ is the k^{th} received block of size $N \times 1$ at antenna j and $\widetilde{\boldsymbol{H}}_{ij}$ is the $N \times N$ channel convolution matrix, modeling the wireless environment between the i^{th} transmit and j^{th} receive antenna, $i, j = 1, 2$.

Due to cyclic prefix insertion and removal operations, $\widetilde{\boldsymbol{H}}_{ij}$ matrices are circulant, with the (r, l)th entry given by $h_{i,j,(r-l) \mod N}$. The channel taps $\{h_{i,j,l}\}_{l=0,\ldots,L-1}$ are assumed to be invariant over the duration of one ODFM block, and are also supposed to vary independently in time. Furthermore, channel impulse responses are considered to be uncorrelated one from each other. This means that the scattering environment is rich and the antennas are placed further than the coherence distance apart.

The maximum channel length is assumed to be L, the length of the cyclic prefix, in order to avoid inter-block interference. The

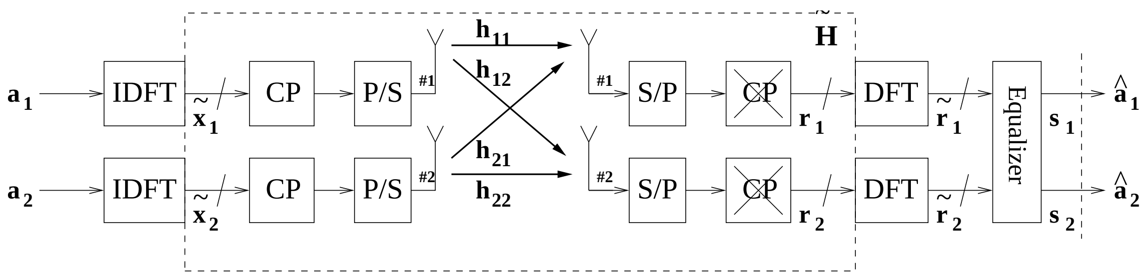

Fig. 1. MIMO OFDM transmission.

noise term $\boldsymbol{w}$ is assumed to be circular white Gaussian with variance σ^2.

Since circulant matrices implement circular convolutions, they are diagonalized by DFT and IDFT operations and thus after having performed the discrete Fourier transform of $\boldsymbol{r}_j(k)$ we obtain:

$$\begin{aligned}\tilde{\boldsymbol{r}}_j(k) &= \boldsymbol{F}_N^H \widetilde{\boldsymbol{H}}_{1j}(k)\boldsymbol{F}_N\,\boldsymbol{a}_1(k) + \boldsymbol{F}_N^H \widetilde{\boldsymbol{H}}_{2j}(k)\boldsymbol{F}_N\,\boldsymbol{a}_2(k) + \\ &\quad \boldsymbol{F}_N^H\,\boldsymbol{w}_j(k) \\ &= \boldsymbol{D}_{1j}(k)\,\boldsymbol{a}_1(k) + \boldsymbol{D}_{2j}(k)\,\boldsymbol{a}_2(k) + \tilde{\boldsymbol{n}}_j(k), \end{aligned} \tag{3}$$

for $j = 1, 2$, and where $\tilde{\boldsymbol{n}}_j(k) = \boldsymbol{F}_N^H\,\boldsymbol{w}_j(k)$, $\boldsymbol{F}_N^H$ is unitary DFT matrix, and the diagonal matrices

$$\begin{aligned}\boldsymbol{D}_{ij}(k) &= \boldsymbol{F}_N^H \widetilde{\boldsymbol{H}}_{ij}(k)\boldsymbol{F}_N \\ &= diag\left\{\sum_{r=0}^{N-1} h_{i,j,r}(k)\exp\left(-j\frac{2\pi nr}{N}\right)\right\}_{n=0,\ldots,N-1}\end{aligned} \tag{4}$$

contain the frequency response of the channel $\boldsymbol{h}_{ij}(k)$, evaluated at the subcarrier frequencies. Equations in (3) for $j = 1, 2$ can be merged into a single one:

$$\begin{bmatrix}\tilde{\boldsymbol{r}}_1(k)\\ \tilde{\boldsymbol{r}}_2(k)\end{bmatrix} = \begin{bmatrix}\boldsymbol{D}_{11}(k) & \boldsymbol{D}_{21}(k)\\ \boldsymbol{D}_{12}(k) & \boldsymbol{D}_{22}(k)\end{bmatrix}\begin{bmatrix}\boldsymbol{a}_1(k)\\ \boldsymbol{a}_2(k)\end{bmatrix} + \begin{bmatrix}\tilde{\boldsymbol{n}}_1(k)\\ \tilde{\boldsymbol{n}}_2(k)\end{bmatrix}, \tag{5}$$

which can be written in a more compact matrix form as:

$$\tilde{\boldsymbol{r}}(k) = \boldsymbol{\mathcal{D}}(k)\,\boldsymbol{a}(k) + \tilde{\boldsymbol{n}}(k). \tag{6}$$

Finally comes the equalization stage, operating in the frequency plane as follows:

$$\begin{aligned}\boldsymbol{s}(k) &= \boldsymbol{G}(k)\,\tilde{\boldsymbol{r}}(k) &(7)\\ &= \boldsymbol{G}(k)\,\left(\boldsymbol{\mathcal{D}}(k)\,\boldsymbol{a}(k) + \tilde{\boldsymbol{n}}(k)\right), &(8)\end{aligned}$$

where $\sigma_{\tilde{n}}^2$ is the variance of the noise and $\boldsymbol{G}(k)$ is the $2N \times 2N$ equalizer matrix, at OFDM block time k. Then, decisions are carried out on $\boldsymbol{s}(k)$ in order to obtain the symbol estimate $\hat{\boldsymbol{a}}(k)$. With the linear model given in (6), the Zero Forcing (ZF) equalizer is easily derived as:

$$\boldsymbol{G}_{ZF}(k) = \boldsymbol{\mathcal{D}}^{-1}(k), \tag{9}$$

whereas MMSE equalization is performed using:

$$\boldsymbol{G}_{MMSE}(k) = \boldsymbol{R}_{\boldsymbol{a},\tilde{\boldsymbol{r}}}\,\boldsymbol{R}_{\tilde{\boldsymbol{r}},\tilde{\boldsymbol{r}}}^{-1}, \tag{10}$$

where

$$\boldsymbol{R}_{\tilde{\boldsymbol{r}},\tilde{\boldsymbol{r}}} = E\left[\tilde{\boldsymbol{r}}\tilde{\boldsymbol{r}}^H\right] = \sigma_a^2\boldsymbol{\mathcal{D}}(k)\boldsymbol{\mathcal{D}}^H(k) + \sigma^2\boldsymbol{I}, \tag{11}$$

and

$$\boldsymbol{R}_{\boldsymbol{a},\tilde{\boldsymbol{r}}} = E\left[\boldsymbol{a}\tilde{\boldsymbol{r}}^H\right] = \sigma_a^2\boldsymbol{\mathcal{D}}^H, \tag{12}$$

σ_a^2 being the average symbol energy. A closer look to $\boldsymbol{G}_{ZF}$ and $\boldsymbol{G}_{MMSE}$ matrices reveals a structure similar to

$$\boldsymbol{\mathcal{D}} = \begin{bmatrix}\boldsymbol{D}_{11} & \boldsymbol{D}_{21}\\ \boldsymbol{D}_{12} & \boldsymbol{D}_{22}\end{bmatrix},$$

defined in (6), each of the blocks $\boldsymbol{D}_{11}, \boldsymbol{D}_{12}, \boldsymbol{D}_{21}$ and $\boldsymbol{D}_{22}$ being diagonal. Exploiting this special matrix structure, significantly lower complexity can be achieved in the equalization stage since direct matrix inversion in (10) is avoided. Furthermore, this highlights an important property of OFDM types of transmission: the initial frequency selective channel has been turned into a set of N frequency flat channels. The equalizer is needed here to compensate the flat fading experienced on each subcarrier and also to de-multiplex the two transmitted streams $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$.

3. TIME-DOMAIN CHANNEL TRACKING

In mobile communications, the channels are time-varying. Hence, the channel needs to be tracked and equalizer coefficients updated periodically. In this section, the proposed MIMO-OFDM channel tracking algorithm is described in detail.

Equation (2) can be rewritten as:

$$\begin{aligned}\boldsymbol{r}(k) &= \begin{bmatrix}\widetilde{\boldsymbol{X}}_1(k) & \widetilde{\boldsymbol{X}}_2(k) & \boldsymbol{0}_{N\times L} & \boldsymbol{0}_{N\times L}\\ \boldsymbol{0}_{N\times L} & \boldsymbol{0}_{N\times L} & \widetilde{\boldsymbol{X}}_1(k) & \widetilde{\boldsymbol{X}}_2(k)\end{bmatrix}\begin{bmatrix}\boldsymbol{h}_{11}(k)\\ \boldsymbol{h}_{21}(k)\\ \boldsymbol{h}_{12}(k)\\ \boldsymbol{h}_{22}(k)\end{bmatrix}\\ &+ \boldsymbol{w}(k)\\ &= \widetilde{\boldsymbol{X}}(k)\,\boldsymbol{h}(k) + \boldsymbol{w}(k),\end{aligned} \tag{13}$$

where $\widetilde{\boldsymbol{X}}_i(k)$ is a $N \times L$ circulant matrix formed with the modulated block $\tilde{\boldsymbol{x}}_i(k)$ and $\boldsymbol{h} = [\boldsymbol{h}_{11}, \boldsymbol{h}_{21}, \boldsymbol{h}_{12}, \boldsymbol{h}_{22}]^T$ is the $4L \times 1$ state vector obtained by stacking the individual channel vectors $\boldsymbol{h}_{ij}(k) = [h_{i,j,0}(k), h_{i,j,1}(k), \ldots, h_{i,j,L-1}(k)]^T$, $i, j = 1, 2$. The time evolution of the channel taps can be described with:

$$\boldsymbol{h}(k) = \boldsymbol{A}\,\boldsymbol{h}(k-1) + \boldsymbol{v}(k), \tag{14}$$

where $\boldsymbol{A}$ is the state transition matrix and $\boldsymbol{v}$ is the state noise. In this paper the state transition matrix is considered to be close to the identity matrix. This matrix can be also estimated from the received data, see e.g., [3]. Equations (13) and (14) form the state-space model describing our transmission system. Considering that the model is linear and the noise is Gaussian, Kalman filtering can be applied to estimate the state vector $\boldsymbol{h}$. The expression for the Kalman gain is:

$$\boldsymbol{K}(k) = \boldsymbol{P}(k|k-1)\tilde{\boldsymbol{X}}^H(k)\left[\tilde{\boldsymbol{X}}(k)\boldsymbol{P}(k|k-1)\tilde{\boldsymbol{X}}^H(k) + \boldsymbol{R}\right]^{-1}, \tag{15}$$

where $\boldsymbol{R} = \sigma^2 \boldsymbol{I}$ is the covariance matrix of the measurement noise $\boldsymbol{w}$, which was assumed to be uncorrelated. $\boldsymbol{P}(k|k-1)$, the covariance matrix of the prediction error, is updated as follows:

$$\boldsymbol{P}(k|k-1) = \boldsymbol{A}\boldsymbol{P}(k-1|k-1)\boldsymbol{A}^H + \boldsymbol{Q}, \tag{16}$$

$\boldsymbol{Q}$ being the covariance matrix of the state noise. In our simulation studies in Section 4, $\boldsymbol{Q}$ is chosen manually. However, it can be reliably estimated, for example by using [3, 4]. The update equation for $\boldsymbol{P}(k|k)$, the covariance matrix of the estimated error, is:

$$\boldsymbol{P}(k|k) = \boldsymbol{P}(k|k-1) - \boldsymbol{K}(k)\widetilde{\boldsymbol{X}}(k)\boldsymbol{P}(k|k-1). \tag{17}$$

To finish with, the estimated channel at time k is expressed as:

$$\hat{\boldsymbol{h}}(k|k) = \hat{\boldsymbol{h}}(k|k-1) + \boldsymbol{K}(k)\left[\boldsymbol{r}(k) - \tilde{\boldsymbol{X}}^H(k)\,\hat{\boldsymbol{h}}(k|k-1)\right], \tag{18}$$

where $\hat{\boldsymbol{h}}(k|k-1) = \boldsymbol{A}\,\hat{\boldsymbol{h}}(k-1|k-1)$.

The overall structure of channel tracking algorithm is illustrated in Figure 2. The algorithm works as follows. For OFDM symbol at time k:

1. Decode the received vector $\boldsymbol{r}(k)$ and obtain the symbol estimate $\hat{\boldsymbol{a}}(k)$, using $\hat{\boldsymbol{h}}(k-1|k-1)$, i.e. the filtered estimate of the channel at symbol time $k-1$.
2. Re-modulate $\hat{\boldsymbol{a}}(k)$: $\hat{\tilde{\boldsymbol{x}}}(k) = \boldsymbol{F}_N\,\hat{\boldsymbol{a}}(k)$.
3. Build the estimate $\hat{\widetilde{\boldsymbol{X}}}(k)$ using $\hat{\tilde{\boldsymbol{x}}}(k)$.
4. Run the Kalman filter to finally obtain $\hat{\boldsymbol{h}}(k|k)$.

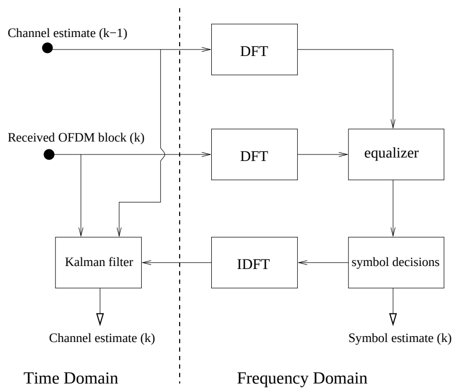

Fig. 2. Time-domain channel estimation and tracking, frequency domain equalization.

Except for the first OFDM symbol, the algorithm works in a decision directed mode. The major computational cost lies in the calculation of the matrix inversion in the Kalman gain expression (15). By applying the matrix inversion lemma, the number of operations can be reduced to $O(LN^2)$ when tracking is done in time-domain. The complexity is of order $O(N^3)$ when the processing takes place in the frequency domain. In practice $L \ll N$, hence significantly lower complexity is achieved.

4. SIMULATIONS

This section presents the simulation results, using the time-domain channel estimation and tracking algorithm in a MIMO-OFDM system. A 2-transmit / 2-receive antenna setup is considered. The carrier frequency is $f_0 = 2.4$ GHz and the number of subcarriers is set to $N = 128$. The available bandwidth is chosen equal to $B = 1$ MHz. The subcarrier symbol rate is of 7.8 KHz. BPSK symbol modulation is employed. Wireless channels from each transmit to each receive antenna experience Rayleigh fading with independent propagation paths. The receiver speed is of $v = 30$ km/h. The Doppler spectrum is Jake's and the power loss and delay profiles are: $[0, -1, -3, -9]$ [dB] and $[0, 1, 2, 3]$ $[\mu s]$ which correspond to a Typical Urban type of scenario. No correlation is considered neither at the transmitter nor at the receiver side, which means the four channel impulse responses $\boldsymbol{h}_{11}$, $\boldsymbol{h}_{21}$, $\boldsymbol{h}_{12}$ and $\boldsymbol{h}_{22}$ fade independently. The MMSE equalizer (10) was chosen, as it does not suffer from noise enhancement as the zero forcing approach (9) does.

Frequency offsets between transmitters and receivers are assumed to be compensated for. Channels are considered to remain stationary during the OFDM block time.

The performance of the proposed method is first shown in time, where tracking is performed. Real and imaginary parts of $\boldsymbol{h}_{11}$ are plot in Figure 3, in the case of 30 km/h velocity of the mobile terminal and at 15 dB SNR. Time variations of the four channel taps are accurately tracked, even for those with low average power.

Since the equalization stage operates in the frequency domain, accuracy in estimating frequency responses of the channels at the subcarrier frequencies needs to be investigated. Figure 5 and 6 respectively show amplitude and phase responses, for real and estimated channel, at a given OFDM block time. Since time-domain estimation performs well, channel transfer functions are consequently also modeled accurately, in both amplitude and phase. Re-training symbols are sent periodically (every 100 blocks, in our simulations), in order to avoid loosing the track, which could occur if all the channels in MIMO system happen to fall in a deep fade simultaneously. This would be a problem if the MIMO channels are highly correlated and rank of the channel matrix is low.

Tracking in time turns out to be more robust to estimation errors because the frequency correlation of the taps can be efficiently exploited. Furthermore, estimation errors are spread over the whole transmission spectrum, and not concentrated on a given set of subcarriers. Frequency domain tracking [1], without any AR modeling, suffers from decision errors: each subcarrier being tracked separately, a single decision error induces an erroneous feedback, provoking a permanent loss of both the channel track and the data stream on the corresponding subcarrier. The final performance criterion is the bit error rate as a function of noise variance, presented

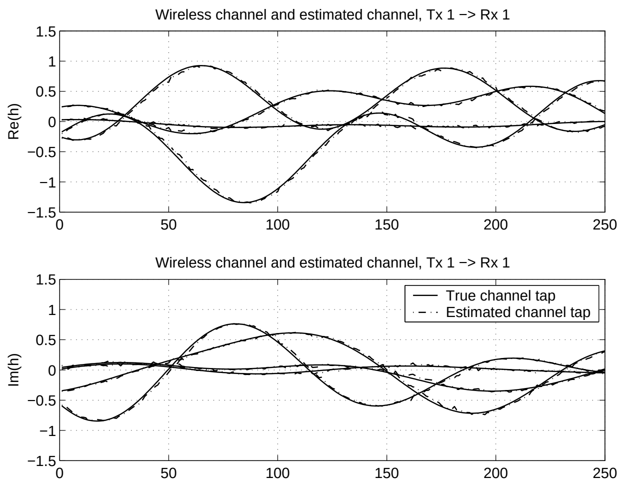

Fig. 3. Time-domain tracking (SNR = 15 dB, v = 30 km/h).

in Figure 4 for a terminal velocity equal to 30 km/h. A lower bound for the performance of the tracking algorithm is given by using the ideal channel state information (CSI), i.e. perfectly known channel at the receiver side. As shown by simulation curves, tracking in time-domain provides us with results close to the ones obtained with known channel. Lower terminal velocities lead to even better performance, closer to the CSI case.

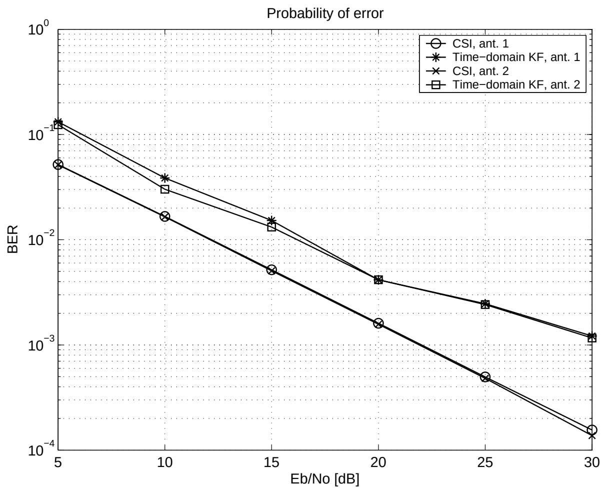

Fig. 4. Bit error rate performance (over 10000 blocks).

5. CONCLUSIONS

In this paper, a channel estimation and tracking algorithm was derived for mobile MIMO-OFDM systems. A Kalman filter running in time-domain is at the core of the proposed method. Superior tracking performance and robustness are achieved, in comparison to existing frequency domain tracking algorithms, mainly due to the ability to exploit the frequency correlation between channel taps. Even though additional DFT's are needed, complexity requirements are also lowered. The reliable performance of the method was demonstrated for mobile user in typical urban scenario. Future work includes studying the performance of the method in the face of correlated channels in MIMO systems.

6. REFERENCES

[1] Bulumulla, S.B., Kassam, S.A., Venkatesh, S.S., "An adaptive diversity receiver for OFDM in fading channels", International Conference on Communications (ICC 1998), Vol. 3, Page(s): 1325 -1329.

[2] Cai J., Shen X., Mark J.W., "Robust channel estimation for OFDM wireless communication systems - An H_∞ approach", Vehicular Technology Conference, 2002. Proceedings. VTC 2002-Fall. 2002 IEEE 56th, Volume: 4, 2002 Page(s): 2106 -2110.

[3] M. Enescu, V. Koivunen, "On the estimation of state transition matrix and noise statistics in state-space models", Vehicular Technology Conference, 2002. Proceedings. VTC 2002-Fall. 2002 IEEE 56th , Volume: 4 , 2002 Page(s): 2192 -2196.

[4] M. Enescu, M. Sirbu, V. Koivunen "Resursive Estimation of Noise Statistics in Kalman Filter Based MIMO Equalization." In the proceedings of XXVIIth General Assembly of the International Union of Radio Science (URSI), Maastricht the Netherlands, 17-24 August 2002.

[5] Molisch, A.F., "Wideband wireless digital communications", Prentice Hall, 2000.

[6] Z. Wang, G.B. Giannakis, "Wireless multicarrier communications", IEEE Signal Processing Magazine, Vol. 17 Issue: 3 , May 2000, Page(s): 29-48.

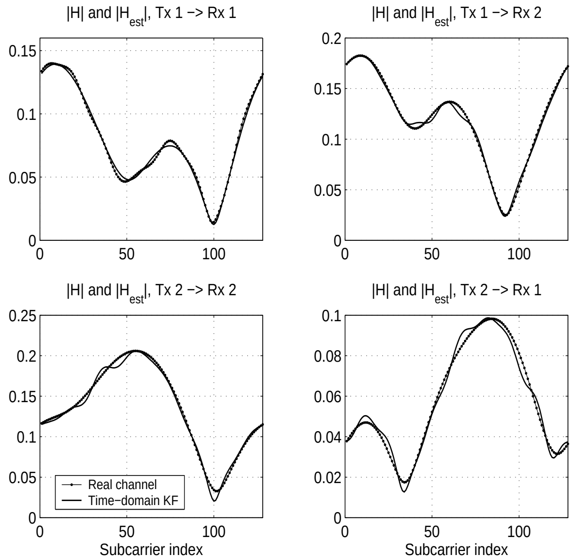

Fig. 5. Amplitude responses (SNR = 15 dB, v = 30 km/h).

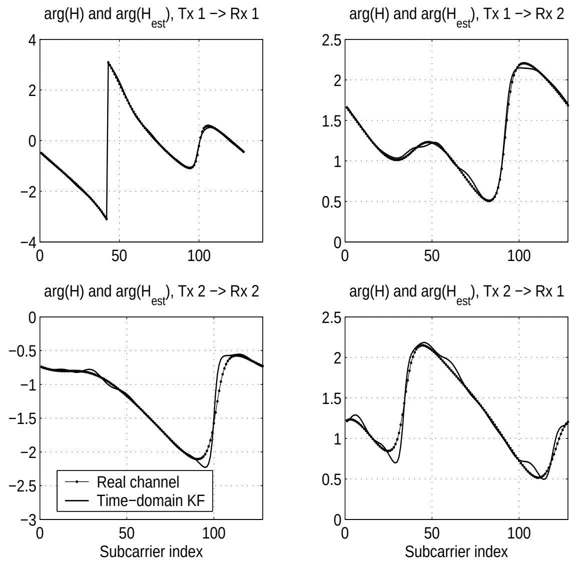

Fig. 6. Phase responses (SNR = 15 dB, v = 30 km/h).