AN EM-BASED CHANNEL ESTIMATION ALGORITHM FOR SPACE-TIME AND SPACE-FREQUENCY BLOCK CODED OFDM

Xiaoqiang Ma, Hisashi Kobayashi, and Stuart C. Schwartz

Electrical Engineering Department, Princeton University
Princeton, New Jersey 08544-5263
Email: xma, hisashi, stuart@ee.princeton.edu

ABSTRACT

The combination of multiple-antenna and orthogonal frequency division multiplexing (OFDM) provides reliable communications over frequency selective fading channels. We investigate this approach and focus on the application of space-time block codes (STBC) and space-frequency block codes (SFBC) in OFDM systems. We compare the performance of maximum likelihood (ML), zero forcing (ZF) and conventional detection algorithms. We show that ZF provides a good trade-off between computational complexity and performance. The problem of channel estimation in STBC-OFDM and SFBC-OFDM system is also studied, including the derivation of the Cramer-Rao lower bound (CRLB). Since knowledge of the channel is required to coherently decode STBC-OFDM and SFBC-OFDM, we propose an iterative channel estimation algorithm based on the EM algorithm that requires very few pilot symbols. The CRLB can be achieved by the channel estimation algorithm.

1. INTRODUCTION

The combination of MIMO and OFDM is a strong candidate for the fourth-generation wireless communications [8]. It can provide very high spectrum efficiency and high data rate with reasonable complexity. In this paper we will investigate the combination of STBC and SFBC with OFDM. In particular, we will focus on the Alamouti STBC [1] with one receive antenna as an example. The same idea can be extended to STBC-OFDM or SFBC-OFDM with more transmit or receive antennas. Lee and Williams [6][7], studied STBC-OFDM and SFBC-OFDM. But their study was based on the the conventional decoding scheme, which assumed the channel response during one codeword is constant. We will investigate the effect of time or frequency variation on the performance using different decoding schemes. The channel estimation problem for MIMO-OFDM was first stuided by Li [3]. A corresponding simplified algorithm was proposed in [4]. The main drawback of the above algorithm is that it is not suitable in a system with channel variation from frame to frame. Lee and Williams [5] proposed a multirate pilot-symbol-assisted channel estimator for OFDM with multiple transmit antennas. However, the percentage of pilot symbols is quite high, which decreases the spectrum efficiency. Another shortcoming is the delay caused by the time domain filtering. We propose an EM-based channel estimation algorithm for STBC-OFDM and SFBC-OFDM, which is a modified version of [2]. It is a frame based algorithm, which means that time domain filtering is not necessary. Therefore, the detection delay is minimized.

The rest of the paper is organized as follows. In Section 2 we will describe the baseband OFDM system model with transmit antenna diversity. In Section 3 we will discus several STBC decoding algorithms. The CRLB and modified CRLB (MCRB) for channel estimation in MIMO-OFDM will be derived in Section 4. An EM-based channel estimation algorithm is discussed in Section 5. Finally, we draw some conclusions in Section 6.

2. SYSTEM MODEL

The schematic diagram of Fig. 1 is a baseband equivalent representation of an OFDM system with two transmit antennas and one receive antenna.

2.1. STBC-OFDM

STBC-OFDM encoding scheme involves two OFDM frames. At the m^{th} subcarrier, symbol $X_1(m)$ is transmitted from antenna 1 and symbol $X_2(m)$ is transmitted from antenna 2 at time instant 1. At time instant 2, $-X_2^*(m)$ and $X_1^*(m)$ are transmitted from antenna 1 and antenna 2, respectively. Denote the frequency response from transmit antenna 1 to the receive antenna as $H_{11}(m)$ and $H_{12}(m)$ at time instant 1 and 2, respectively, and the frequency response from transmit antenna 2 to the receive antenna as $H_{21}(m)$ and $H_{22}(m)$ at time instant 1 and 2, respectively. The received signals are given as

$$Y_1(m) = H_{11}(m)X_1(m) + H_{21}(m)X_2(m) + N_1(m),$$

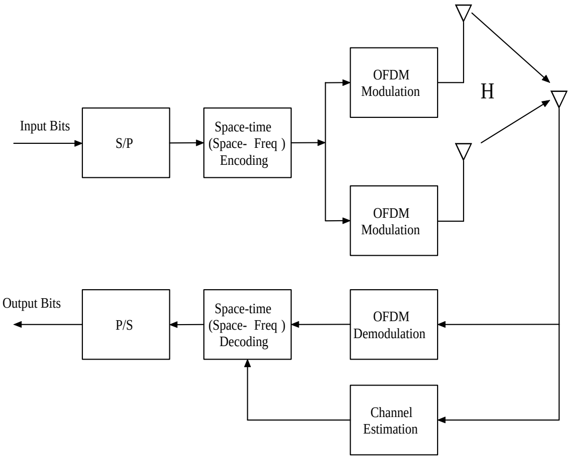


Fig. 1. Baseband OFDM system model with transmit diversity

$$Y_2(m) \quad = \quad -H_{12}(m)X_2^*(m) + H_{22}(m)X_1^*(m) + N_2(m).$$

2.2. SFBC-OFDM

The rationale for the use of SFBC in OFDM is that the channel frequency response of adjacent subcarriers remains almost constant in such a system. It only involves one OFDM frame. In the $2m^{th}$ subcarrier, symbol $X_1(2m)$ is transmitted from antenna 1 and symbol $X_2(2m)$ is transmitted from antenna 2. At the same time, $-X_2^*(2m)$ and $X_1^*(2m)$ are transmitted from antenna 1 and antenna 2 respectively in the $(2m+1)^{th}$ subcarrier. Denote the frequency response from transmit antenna 1 to the receive antenna as $H_1(2m)$ and $H_1(2m+1)$ of the $2m^{th}$ and the $(2m+1)^{th}$ subcarriers, respectively, and the frequency response from transmit antenna 2 to the receive antenna as $H_2(2m)$ and $H_2(2m+1)$ of the $2m^{th}$ and the $(2m+1)^{th}$ subcarriers, respectively. The received signals can be expressed in a similar way.

3. STBC DECODING SCHEMES

STBC decoding is discussed in [1]. It assumes the channel is constant during one codeword. We call it the conventional decoding method in this paper. For the case of STBC with changing channels and SFBC in OFDM, the assumption no longer holds. The channel coefficient matrix is not a scaled unitary matrix in general. Therefore, the conventional decoding method is not optimal. The optimal decoding method in this case is the maximum likelihood (ML) algorithm. The main shortcoming of the optimal decoding algorithm is the high computation burden that grows exponentially as the number of constellation points increases.

A suboptimal decoding approach is the so called zero-forcing (ZF) approach. It is less computationally complex and in the case of two transmit antennas, the matrix inversion is quite simple.

Simulation results (omitted due to limited space) show that the performance of the ZF approach is only slightly worse than the performance of the optimal ML approach. Therefore, the ZF approach is a good tradeoff between performance and complexity.

4. CRAMER-RAO LOWER BOUND

In this section we will derive the CRLB and MCRB for the channel estimate in general MIMO-OFMDM systems. In particular, we will focus on MIMO-OFDM systems with two transmit antennas and one receive antenna.

The MIMO-OFDM system model with two transmit antennas and one receive antenna can be written in vector form

$$\underline{Y} = \mathbf{X}_1\mathbf{W}_L\underline{h}_1 + \mathbf{X}_2\mathbf{W}_L\underline{h}_2 + \underline{N}. \tag{1}$$

where $\underline{Y}$ is the received signal, $\mathbf{X}_1$ and $\mathbf{X}_2$ are transmitted signals from antennas 1 and 2 in diagonal matrix form, $\underline{h}_1$ and $\underline{h}_2$ are CIRs from transmit antenna 1 and 2 to the receive antenna, respectively. $\underline{N}$ is the Gaussian noise vector and each element is zero mean and variance $\sigma^2/2$ for each dimension. $\mathbf{W}_L$ is the submatrix of the FFT matrix with first L columns.

The parameter vector here is obviously $\underline{\theta} = [\underline{h}_1^T \quad \underline{h}_2^T]^T$. The CRLB gives a lower bound for the variance of an unbiased estimate

$$CRLB(\underline{\theta}_i) = I^{-1}(\underline{\theta})_{ii}, \quad 1 \le i \le 2L, \tag{2}$$

where L is the channel delay spread and $I(\theta)$ is Fisher information matrix

$$I(\underline{\theta}) = \mathsf{E}\left\{ \frac{\partial}{\partial\underline{\theta}} \log f(\underline{Y}|\underline{\theta}) \left(\frac{\partial}{\partial\underline{\theta}} \log f(\underline{Y}|\underline{\theta}) \right)^H \right\} \tag{3}$$

Following (1) we have the pdf of $\underline{Y}$ given $\underline{\theta}$

$$f(\underline{Y}|\underline{\theta}) = \frac{1}{(\pi\sigma^2)^M} \exp\left\{ -\frac{1}{\sigma^2} ||\underline{Y} - \sum_{k=1}^{2} \mathbf{X}_k\mathbf{W}_L\underline{h}_k||^2 \right\}, \tag{4}$$

where we assume the data matrices $\mathbf{X}_1$ and $\mathbf{X}_2$ are known so that they do not appear as conditioned variables.

The Fisher information matrix can be partitioned into four small blocks $I(\underline{\theta}) = \begin{bmatrix} I_1 & I_2 \\ I_3 & I_4 \end{bmatrix}$, where

$$\begin{aligned} I_1 &= \frac{1}{\sigma^2}\mathbf{W}_L^H\mathbf{X}_1^H\mathbf{X}_1\mathbf{W}_L, I_2 = \frac{1}{\sigma^2}\mathbf{W}_L^H\mathbf{X}_2^H\mathbf{X}_1\mathbf{W}_L \\ I_3 &= \frac{1}{\sigma^2}\mathbf{W}_L^H\mathbf{X}_1^H\mathbf{X}_2\mathbf{W}_L, I_4 = \frac{1}{\sigma^2}\mathbf{W}_L^H\mathbf{X}_2^H\mathbf{X}_2\mathbf{W}_L \end{aligned}$$

We define the CRLB of $\underline{h}_1$ and $\underline{h}_2$ as

$$CRLB(\underline{h}_1) = \sum_{i=1}^{L} I^{-1}(\underline{\theta})_{ii}, \quad (5)$$

$$CRLB(\underline{h}_2) = \sum_{i=L+1}^{2L} I^{-1}(\underline{\theta})_{ii}. \quad (6)$$

These CRLBs are obviously signal dependent since the transmitted signals $\mathbf{X}_1$ and $\mathbf{X}_2$ appear in the Fisher information matrix. In order to eliminate the dependency of the transmitted signals, we take expectations of I_i with respect to $\mathbf{X}_1$ and $\mathbf{X}_2$. We also make an additional assumption that transmitted signals from different antennas are independent, i.e., $\mathsf{E}\{X_1(m)X_2^*(m)\} = 0$. And, we also assume $\mathsf{E}\{|X(m)|^2\} = A$. This assumption is valid for general MIMO-OFDM systems with independent transmitted signals from different transmit antennas as well as STBC-OFDM and SFBC-OFDM. Therefore, we have the following expectations $\mathsf{E}\{I_1\} = \mathsf{E}\{I_4\} = \frac{MA}{\sigma^2} I_L$ and $\mathsf{E}\{I_2\}\mathsf{E}\{I_3\} = 0$. The CRLBs becomes modified CRLB (MCRB) after taking the above expectations. The MCRBs can easily be calculated as

$$MCRB(\underline{h}_1) = MCRB(\underline{h}_2) = \frac{L\sigma^2}{MA}, \quad (7)$$

where M is the number of subcarriers. The above derivation can easily be extended to the case of using D frames of data, assuming the channel is constant during these D frames. The result becomes

$$MCRB(\underline{h}_1) = MCRB(\underline{h}_2) = \frac{L\sigma^2}{MDA}, \quad (8)$$

which is D times smaller than the case that only one frame of data are observed. Further extensions to arbitrary number of transmit or receive antenna is straightforward.

5. AN EM-BASED CHANNEL ESTIMATION ALGORITHM FOR STBC-OFDM AND SFBC-OFDM

If the system is perfectly time and frequency synchronized for STBC-OFDM and SFBC-OFDM, channel estimation error is the main source of performance degradation. In this section, we propose an EM-based channel estimation algorithm for STBC-OFDM and SFBC-OFDM, which is a modified version of [2].

First we give the EM-based channel estimation algorithm for SFBC-OFDM which only involves one OFDM frame. The SFBC-OFDM system model is given by 1 with $X_1(2m+1) = -X_2^*(2m)$ and $X_2(2m+1) = X_1^*(2m)$.

Following [9], a natural choice for "complete" data $\underline{Z}_1$ and $\underline{Z}_2$ is obtained by decomposing the observed data $\underline{Y}$ into 2 components, i.e.,

$$\underline{Z}_i = \mathbf{X}_i \mathbf{W}_L \underline{h}_i + \underline{N}_i, \quad i = 1, 2, \quad (9)$$

where $\underline{N}_i, i = 1, 2$ are obtained by arbitrarily decomposing the total noise $\underline{N}$ into 2 components such that $\underline{N}_1 + \underline{N}_2 = \underline{N}$. Thus, the relation between the "complete" data $(\underline{Z}_1, \underline{Z}_2)$ and "incomplete" data $\underline{Y}$ is given by $\underline{Y} = \underline{Z}_1 + \underline{Z}_2$.

It is easy to show the above described EM-base channel estimation algorithm takes the following form:

$$\underline{h}_i^{(p+1)} = \mathbf{W}_L^H \mathbf{X}_i^{-1} \hat{\underline{Z}}_i^{(p)}, \quad i = 1, 2, \quad (10)$$

where

$$\hat{\underline{Z}}_i^{(p)} = \underline{Z}_i^{(p)} + \beta_i \left(\underline{Y} - \sum_{j=1}^{2} \underline{Z}_j^{(p)} \right), \quad (11)$$

$$\underline{Z}_i^{(p)} = \mathbf{X}_i \mathbf{W}_L \underline{h}_i^{(p)}. \quad (12)$$

Observe that β_i can be arbitrarily selected due to the arbitrarily decomposition of the independent noise components $\underline{N}_i$. The only constraint is $\beta_1 + \beta_2 = 1$. A typical value is $\beta_1 = \beta_2 = 0.5$ for the case of two transmit antennas.

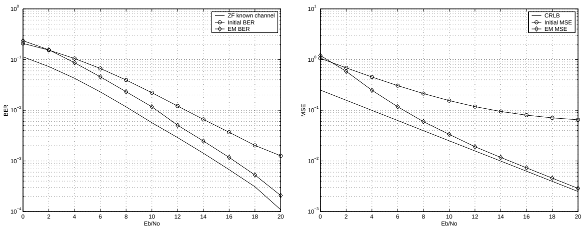
Note that the above result can only be applied for pilot frames, i.e, the transmitted signals $\underline{X}_1$ and $\underline{X}_2$ are known at the receiver, which is exactly the case in [2]. However, in the case of signal transmission, we don't know all the transmitted signals in these OFDM frames except for some pilot symbols. Thus, to make it feasible we adopt the p^{th} estimates $\underline{X}_1^{(p)}$ and $\underline{X}_2^{(p)}$ instead of the actual values in the algorithm. In SFBC-OFDM systems, we propose to use the ZF detection approach to obtain the estimates of transmit signals. Consequently, the algorithm is actually decision-directed and (10) and (12) become

$$\underline{h}_i^{(p+1)} = \mathbf{W}_L^H (\mathbf{X}_i^{(p)})^{-1} \hat{\underline{Z}}_i^{(p)}, \quad i = 1, 2, \quad (13)$$

$$\underline{Z}_i^{(p)} = \mathbf{X}_i^{(p)} \mathbf{W}_L \underline{h}_i^{(p)}. \quad (14)$$

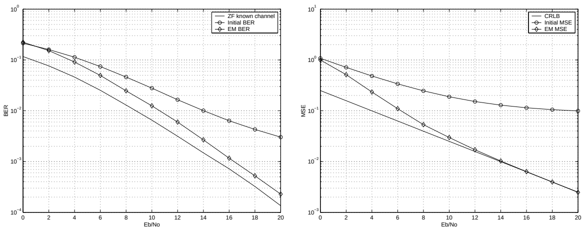
The above algorithm can easily be applied to STBC-OFDM systems. The only difference is that in STBC-OFDM systems the algorithm involves two OFDM frames and only one OFDM frame is involved in SFBC-OFDM systems. Thus, the algorithm should be carried out twice in each iteration for those two OFDM frames. The ZF detection approach is also used to detect the transmitted signals in these two frames.

From Figs. 2 and 3 we can observe that the EM-based channel estimation algorithm can reduce the BER and MSE. Furthermore, it can achieve a BER performance close to the case where the channel characteristic is completely known at the receiver in the high SNR region. However, there is still a BER gap between the lower bound and the BER of the EM-based algorithm. The MSE also is very close to the MCRB when the SNR increases. Comparing Figure 2(b) with 3(b) we find that the MCRB can be achieved by increasing the SNR in a SFBC-OFDM system, while this is not the case in a STBC-OFDM system. It turns out that the



(a) BER (b) MSE

Fig. 2. BER and MSE of the EM-based channel estimation algorithm for STBC-OFDM systems.



(a) BER (b) MSE

Fig. 3. BER and MSE of the EM-based channel estimation algorithm for SFBC-OFDM systems.

number of iterations per frame of STBC-OFDM systems is slightly larger than that of SFBC-OFDM systems. This is observed from Fig. 4. In general, the proposed EM-based channel estimation algorithm works both for STBC-OFDM and SFBC-OFDM systems. Similar performance, including BER and MSE, can be achieved by the algorithm. However, it is more suitable for SFBC-OFDM systems due to the lower MSE and smaller iterations per frame.

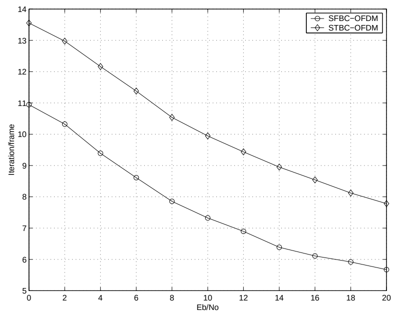


Fig. 4. Iterations per OFDM frame of the EM-based channel estimation algorithm for STBC-OFDM and SFBC-OFDM systems.

6. CONCLUSION

In this paper we investigated applications of STBC and SFBC in OFDM systems. Channel estimation in STBC-OFDM and SFBC-OFDM is studied, including the derivation of the CRLB and MCRB. We proposed an iterative channel estimation algorithm based on the EM algorithm with only a few pilot symbols required. Simulation results show that it is a very promising channel estimation technique, which can achieve near optimum BER and the CRLB when the SNR becomes large. Furthermore, it appears to be more suitable for SFBC-OFDM in terms of lower MSE and smaller required iterations.

Note: Additional details in a longer paper and larger figures can be found at http://www.ee.princeton.edu/~xma/mimo.pdf.

7. REFERENCES

[1] S. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE JSACs*, vol. 16, no. 8, pp. 1451-1458, Oct. 1998.

[2] Y. Xie and C. N. Georghiades, "An EM-based channel estimation algorithm for OFDM with transmitter diversity," in *Proc. of Globecom 2001*, pp. 871-875.

[3] Ye(Geoffrey) Li, N. Seshadri, and S. Ariyavisitakul, "Channel estimation for OFDM systems with transmitter diversity in mobile wireless channels," *IEEE JSAC*, vol. 17, no. 3, pp. 451-460, Mar. 1999.

[4] Ye(Geoffrey) Li, "Simplified channel estimation for OFDM systems with multiple transmit antennas," *IEEE Trans. on Wireless Comm.*, vol. 1, no. 1, pp. 67-75, Jan. 2002.

[5] K. F. Lee and D. B. Williams, "A multirate pilot-symbol-assisted channel estimator for OFDM transmitter diversity systems," in *Proc. of ICASSP 2001*, vol. 4, pp. 2409 -2412, 2001.

[6] K. F. Lee and D. B. Williams, "A space-time coded transmitter diversity technique for frequency selective fading channels," in *Proc. of 2000 IEEE Sensor Array and Multichannel Signal Processing Workshop*, pp. 149 -152, 2000.

[7] K. F. Lee and D. B. Williams, "A space-frequency transmitter diversity technique for OFDM systems," in *Proc. of Globlecom 2000*, vol. 3, pp. 1473 -1477, 2000.

[8] H. Sampath, S. Talwar, J. Tellado, V. Erceg and A. Paulraj, "A fourth-generation MIMO-OFDM broadband wireless systems: design, performance, and field trial results," *IEEE Comm. Mag.*, vol. 40, no. 9, pp. 143-149, Sep. 2002.

[9] Meir Feder and Ehud Weinstein, "Parameter estimation of superimposed signals using the EM algorithm," *IEEE Trans. on Acoustic, Speech, and Signal Processing*, Vol. 36, No. 4, pp. 477-489, Apr. 1989.