THE EFFECTS OF CHANNEL CORRELATION ON MAXIMAL RATIO COMBINING PERFORMANCE IN THE PRESENCE OF COCHANNEL INTERFERERS

Yeliz Tokgoz and Bhaskar D. Rao

University of California, San Diego
Center for Wireless Communications
La Jolla, CA 92093-0407

ABSTRACT

In this paper, we analyze the effects of channel correlation on the performance of Maximal Ratio Combining (MRC) in receive antenna diversity systems. In the analysis, the channel is modelled as flat Rayleigh fading, slowly varying and an arbitrary number of cochannel interferers are assumed to be present. In an interference limited scenario, an exact closed form expression for signal to interference ratio (SIR) distribution is obtained for a dual antenna system. The degradation in the outage probability as a function of the correlation severity is investigated.

1. INTRODUCTION

In receive antenna diversity systems, the signals at the multiple antenna elements are combined to improve system performance by combatting cochannel interference and channel impairments such as multipath fading. Maximal Ratio Combining (MRC) is one of the well known linear combining techniques in which the spatial combiner weights are chosen so that the output signal to (thermal) noise ratio (SNR) is maximized. There is a reduction in the diversity gain of such a system when there is correlation among the antenna elements possibly due to insufficient antenna spacing. It is of interest to determine the performance degradation caused by this channel correlation.

The performance of an MRC system with independent flat Rayleigh fading model is analyzed in [1] and [2], where closed form SINR distribution and outage probability expressions are provided. The effect of fading correlation among antenna elements have also been investigated. However, most of the work have been limited to single user systems [3, 4, 5]. In [6], systems with multiple interferers are considered and simulation results are presented. An analytical approach to performance analysis is given in [7], but it is assumed that either only the desired user or only the interferers experience correlated fading. Even then, the error probability expressions provided are not in closed from, but in the form of statistical expectations. In this paper, we consider an MRC system with arbitrary number of co-channel interferers and correlated flat Rayleigh fading across the antenna elements. For a dual diversity system, we provide an exact expression for the signal to interference ratio (SIR). We also investigate the performance degradation as a function of the correlation severity.

The paper is organized as follows: The system model is introduced in Section 2. In Section 3, the SIR distribution of MRC is obtained. Simulation results are presented in Section 4.

This research was supported by CoRe research grant Core00-10074.

2. SYSTEM MODEL

Consider the uplink of a wireless communication system with M antennas at the base station. The received signal consists of components from the desired user, the N interfering users and thermal noise.

$$\mathbf{r(t)} = \sqrt{P_0}\cdot\mathbf{c_0}\cdot d_o(t) + \sum_{k=1}^{N}\sqrt{P_k}\cdot\mathbf{c_k}\cdot d_k(t) + \mathbf{n(t)} \quad (1)$$

The desired and interfering user powers are denoted by P_k, where k is the user index. d_k corresponds to the information bits of the k^{th} user and it is assumed to be zero-mean and unit variance. The $M \times 1$ noise vector $\mathbf{n}$ is complex white (both temporally and spatially) Gaussian with zero-mean and variance σ_n^2. Without loss of generality, σ_n^2 is set to one. Non-unity variances can be readily accommodated by scaling the user powers by $1/\sigma_n^2$. The channel is modelled as flat Rayleigh fading, and the circularly Gaussian vector $\mathbf{c_k}$ corresponds to the fading coefficients of the k^{th} user. The fading coefficients at each antenna are assumed to be correlated, *i.e.* $E[\mathbf{c_k}\mathbf{c_k}^H] = \Sigma$, which in practice could be due to insufficient antenna spacing. The channel is also assumed to be slowly-varying, so that the fading coefficients remain unchanged over the frame. Hence, a quasi-static analysis is applicable.

The signal to interference plus noise ratio (SINR) for this system model is expressed as [7]

$$SINR = \frac{P_0 \cdot |\mathbf{w}^H \mathbf{c_0}|^2}{\mathbf{w}^H \left(\sum_{k=1}^{N} P_k \cdot \mathbf{c_k} {\mathbf{c_k}}^H + I \right) \mathbf{w}} \tag{2}$$

where **w** is the spatially combining weight vector.

3. MAXIMAL RATIO COMBINING

Maximum Ratio Combining (MRC) is one of the most common linear combining techniques in receive diversity systems. In MRC, the spatial combiner weights are chosen to maximize the output signal to (thermal) noise ratio (SNR). The corresponding spatial combiner weights, **w**, are the fading coefficients of the desired user [8]. In other words, MRC projects the received M dimensional signal onto the direction of the desired user. It is the optimum linear combining technique in the presence of thermal noise.

The output SINR for an MRC system can be written as

$$SINR = \frac{P_0 \cdot ({\mathbf{c_0}}^H \mathbf{c_0})^2}{{\mathbf{c_0}}^H \left(\sum_{k=1}^{N} P_k \cdot \mathbf{c_k} {\mathbf{c_k}}^H + I \right) \mathbf{c_0}} \tag{3}$$

The distribution of the output SINR have been investigated in [1] for an uncorrelated fading system and a closed form SINR distribution expression have been derived. However, with the introduction of channel correlation, the numerator and the denominator of the SINR expression can no longer be statistically independent. This fact makes the analysis of the correlated fading system a much more complicated problem. In order to continue with the analysis, a different approach has to be taken along with some simplifying assumptions.

The first simplifying assumption we make in this paper is to ignore the effect of thermal noise. Consequently, the results are valid in an interference limited environment. We further assume that all the interfering users have equal power levels, *i.e.* $P_k = P_I \;\; \forall \; k = 1, \ldots, N$. With these assumptions, the signal to interference ratio (SIR) can now be expressed as

$$SIR = \frac{P_0 \cdot ({\mathbf{c_0}}^H \mathbf{c_0})^2}{P_I \sum_{k=1}^{N} ({\mathbf{c_0}}^H \mathbf{c_k})({\mathbf{c_0}}^H \mathbf{c_k})^H} \tag{4}$$

We define two new random variables, w_k and K.

$$K = P_I \cdot \sum_{k=1}^{N} |w_k|^2, \quad \text{where} \quad w_k = \frac{{\mathbf{c_0}}^H \mathbf{c_k}}{\sqrt{\mathbf{c_0} \Sigma \mathbf{c_0}}} \tag{5}$$

where K is weighted chi-square distributed with $2N$ degrees of freedom and has the following distribution.

$$f_K(k) = \begin{cases} \dfrac{1}{\Gamma(N) P_I^N} \cdot k^{N-1} \cdot e^{-k/P_I} & for\ k \geq 0, \\ 0 & for\ k < 0 \end{cases} \tag{6}$$

Substituting K into the SIR expression, we have

$$SIR = \frac{P_0}{K} \cdot \frac{({\mathbf{c_0}}^H \mathbf{c_0})^2}{{\mathbf{c_0}}^H \Sigma \mathbf{c_0}} \tag{7}$$

It can be shown that the w_k's conditioned on the value of $\mathbf{c_0}$ are complex Gaussian distributed with zero mean and unit variance ($w_k | \mathbf{c_0} \sim \mathcal{N}(0,1)$). Since the conditional distribution of w_k is not a function of the conditioning variable $\mathbf{c_0}$, we can conclude that the w_k's (and therefore K) are independent from $\mathbf{c_0}$. We have now expressed SIR as the product of two independent terms. In order to find the SIR distribution, we first condition it on K and obtain the conditional distribution in closed form. We then derive the unconditional distribution by averaging over the conditioning variable.

For the sake of analytical tractability, we only consider dual antenna diversity systems, where $\mathbf{c_k} = [x_{k,1} + jy_{k,1} \; x_{k,2} + jy_{k,2}]^T$. In accordance with the Rayleigh fading assumption, the $x_{k,i}$'s and $y_{k,i}$'s are Gaussian random variables ($\sim \mathcal{N}(0, 1/2)$) and $E[x_{k,i}\, y_{k,i}] = 0$. As shown in [6], if we assume that the mobile's signal reaches all the antenna array elements at the same time, the real and imaginary parts of the fading coefficients at different antenna elements are also independent from one another ($E[x_{k,1}\, y_{k,2}] = E[x_{k,2}\, y_{k,1}] = 0$). Hence, the correlation among the fading coefficients can be modelled by a single real valued parameter, r.

$$E[\mathbf{c_k} {\mathbf{c_k}}^H] = \Sigma = \begin{bmatrix} 1 & r \\ r & 1 \end{bmatrix} \tag{8}$$

When $r = 0$, we have the uncorrelated channel case for which the exact SINR distribution is already computed in [1] for an arbitrary number of antennas, M.

$$\begin{aligned} f_{SINR}(s) &= \frac{e^{-s/P_0}}{\Gamma(N)} \cdot \frac{P_0^{-M} s^{M-1}}{(M-1)!} \\ &\quad \times \sum_{k=0}^{M} \binom{M}{k} P_I^k \cdot \frac{\Gamma(k+N)}{\left(s \dfrac{P_I}{P_0} + 1 \right)^{k+N}} \end{aligned} \tag{9}$$

The closed from expression for the outage probability of the uncorrelated channel have also been derived in [2]. When $r = 1$, it means that both antenna elements observe the exact same fading coefficient and the system performance is the same as a Rayleigh fading channel with no diversity. The SINR distribution and the outage probability can be calculated using the uncorrelated channel expressions for a single antenna system, $M = 1$.

For $0 < r < 1$, we need to transform the SIR expression (7) to a form which is analytically more tractable. The eigenvalue decomposition of Σ can be written as

$$\Sigma = U\Lambda U^H = U \begin{bmatrix} 1+r & 0 \\ 0 & 1-r \end{bmatrix} U^H$$

Using the eigenvalue decomposition of Σ, we rewrite the SIR in terms of a new spatially white vector ν such that $\nu = \Lambda^{-1/2} U^H \mathbf{c_0}$.

$$SIR = \frac{P_0}{K} \cdot \frac{(\nu^H \Lambda \nu)^2}{\nu^H \Lambda^2 \nu}$$

where $\nu = [\nu_1 \, \nu_2]^T$ and $\nu \sim \mathcal{N}(0, I)$. We further define two new random variables u and w such that $u = (\nu^H \Lambda \nu)^2$ and $w = \nu^H \Lambda^2 \nu$. Now, the SIR is expressed as

$$SIR = \frac{P_0}{K} \cdot \frac{u^2}{w} \tag{10}$$

where

$$\begin{aligned} u &= (1+r) \cdot |\nu_1|^2 + (1-r) \cdot |\nu_2|^2 \\ w &= (1+r)^2 \cdot |\nu_1|^2 + (1-r)^2 \cdot |\nu_2|^2 \end{aligned} \tag{11}$$

for $0 < r < 1$.

Since $2 \cdot |\nu_1|^2$ and $2 \cdot |\nu_2|^2$ are both chi-square distributed with two degrees of freedom, the joint distribution of u and w can be computed using functions of random variables.

$$f_{U,W}(u, w) = \frac{1}{2r(1-r^2)} \cdot e^{-\frac{2}{1-r^2} u} \cdot e^{\frac{1}{1-r^2} w} \tag{12}$$

where

$$0 \le (1-r)u \le w \le (1+r)u$$

We initially use the above joint distribution to calculate the conditional cumulative distribution function (cdf) of the SIR.

$$\begin{aligned} F_{Z|K}(z|k) &= Pr\left[\frac{P_0}{k} \cdot \frac{u^2}{w} \le z\right] = Pr\left[w \ge \frac{P_0}{k} \cdot \frac{u^2}{z}\right] \\ &= \int_0^{\frac{kz(1+r)}{P_0}} \frac{1}{2r} \cdot e^{-\frac{u}{1+r}} du - \int_0^{\frac{kz(1-r)}{P_0}} \frac{1}{2r} \cdot e^{-\frac{u}{1-r}} du \\ &- \int_{\frac{kz(1-r)}{P_0}}^{\frac{kz(1+r)}{P_0}} \frac{1}{2r} \cdot \exp\left(\frac{P_0}{kz(1-r^2)} u^2 - \frac{z}{1-r^2} u\right) du \end{aligned} \tag{13}$$

Denoting the third integral on the right hand side of the above expression as I, we write the conditional probability distribution function (pdf) as the derivative of the conditional cdf with respect to z.

$$\begin{aligned} f_{Z|K}(z|k) &= \frac{k(1+r)}{P_0} \cdot \frac{1}{2r} \cdot e^{-kz/P_0} \\ &- \frac{k(1-r)}{P_0} \cdot \frac{1}{2r} \cdot e^{-kz/P_0} - \frac{\partial I}{\partial z} \\ &= \frac{k}{P_0} \cdot e^{-kz/P_0} - \frac{\partial I}{\partial z} \end{aligned}$$

where

$$\begin{aligned} \frac{\partial I}{\partial z} &= \left[\frac{\sqrt{k\pi(-1+r^2)}}{4r\sqrt{P_0 z}} + \frac{k^{3/2}\sqrt{\pi z}}{2rP_0^{3/2}\sqrt{-1+r^2}}\right] \\ &\times e^{-\frac{kz}{P_0(1-r^2)}} \cdot erf\left(\sqrt{\frac{kz}{P_0(-1+r^2)}} \cdot r\right) \\ &+ \frac{k}{2P_0} \cdot e^{-\frac{zk}{P_0}} \end{aligned}$$

The unconditional SIR distribution is then computed by averaging the conditional distribution over the conditioning variable, K.

$$\begin{aligned} f_Z(z) &= \int_0^\infty f_{Z|K}(z|k) \cdot f_K(k) \cdot dk \\ &= \frac{NP_I P_0^N}{(zP_I + P_0)^{N+1}} - \frac{1}{\Gamma(N) P_I^N} \\ &\times \int_0^\infty \frac{\partial I}{\partial z} \cdot k^{N-1} \cdot e^{-k/P_I} \cdot dk \end{aligned}$$

After numerous steps of algebra [9], the above distribution can be written in closed form in terms of hypergeometric functions.

$$\begin{aligned} f_Z(z) &= \frac{NP_I P_0^N}{(zP_I + P_0)^{N+1}} - \frac{N \cdot P_0^N \cdot (1-r^2)^{N+1}}{P_I^N \cdot r^{2N+2} \cdot z^{N+1} \cdot (-1)^N} \\ &\times \mathcal{R}e\left\{H(P_0, P_I, N, r)\right\} + \frac{NP_I P_0^N}{2(zP_I + P_0)^{N+1}} \end{aligned} \tag{14}$$

where

$$\begin{aligned} H(P_0, P_I, N, r) &= \frac{1}{2(2N+1)} \cdot {}_2F_1\Bigg(N + \frac{1}{2}, N+1; \\ & N + \frac{3}{2}, \frac{zP_I + P_0(1-r^2)}{P_I r^2 z}\Bigg) + \frac{N+1}{r^2(2N+3)} \\ &\times {}_2F_1\left(N + \frac{3}{2}, N+2; N + \frac{5}{2}, \frac{zP_I + P_0(1-r^2)}{P_I r^2 z}\right) \end{aligned} \tag{15}$$

The hypergeometric function ${}_2F_1(a, b; c, d)$ is defined as follows [9]:

$$\begin{aligned} & {}_2F_1\left(\frac{\nu}{2}, \frac{\nu+1}{2}; \frac{\nu}{2} + 1, \frac{\mu^2}{\beta^2}\right) = \\ & \frac{\sqrt{\pi} \cdot \nu \cdot \beta^\nu}{\Gamma\left(\frac{\nu+1}{2}\right)} \cdot \int_0^\infty x^{\nu-1} \cdot e^{\mu^2 x^2} \cdot [1 - erf(\beta x)] \, dx \end{aligned} \tag{16}$$

when $\mathcal{R}e\{\beta^2\} > \mathcal{R}e\{\mu^2\}$ and $\mathcal{R}e\{\nu^2\} > 0$. The values of ${}_2F_1(a, b; c, d)$ are readily available in software packages such as `Matlab`.

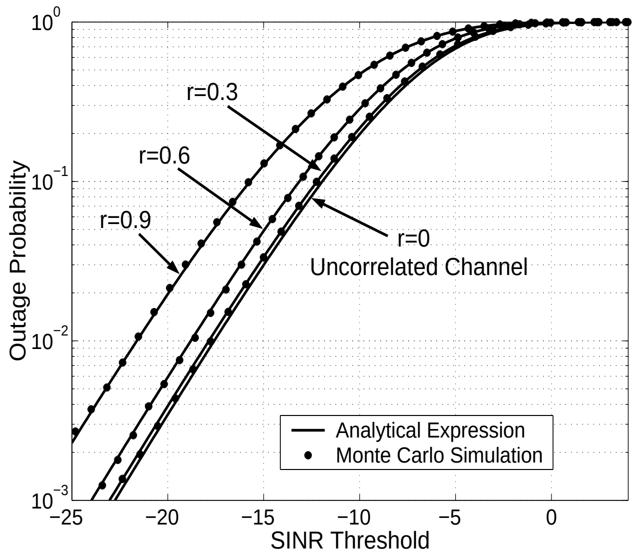

Fig. 1. Outage probability of a dual antenna MRC system with $N = 8$ interferers for $P_0 = P_I = 10$ and $r = \{0, 0.3, 0.6, 0.9\}$

4. NUMERICAL RESULTS

In this section, we present numerical results that validate the closed form expression derived for the SIR distribution. The results also demonstrate the effect of channel correlation on system performance. We consider a dual antenna system with 8 interfering users. The desired user and the interferers are of equal power, *i.e*, $P_0 = P_I = 10$. The SIR distribution is computed both by using the analytical expression and through Monte Carlo simulations. Then, these distributions are numerically integrated from zero to a specified SIR threshold to obtain the outage probability.

In Figure 1, the outage probabilities calculated using simulation results and the analytical SIR distribution expression are presented for various values of the correlation coefficient, $r = \{0.3, 0.6, 0.9\}$. The outage probability of the uncorrelated channel ($r = 0$) is also plotted for comparison purposes [2]. The results indicate that for small values of r, the performance degradation is not significant. It is only when r exceeds the values of 0.4-0.5 that the system suffers from the correlation among the antenna elements.

5. CONCLUSION

In this paper, we considered a receive antenna diversity system that employs maximal ratio combining. The channel was assumed to be correlated Rayleigh fading. For an interference limited scenario, we obtained a closed form expression for the SIR distribution of a dual antenna system (with arbitrary number of cochannel interferers) as a function of the correlation coefficient. We also investigated the effects of correlation on the system performance and concluded that the system can withstand correlation coefficients of up to 0.4-0.5.

6. REFERENCES

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