Analysis of Diversity, Beamforming and Hybrid Diversity-Beamforming Systems

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Abstract— ***In this paper we compare Shannon capacities of diversity, beamforming, and hybrid diversity-beamforming systems for reverse and forward links. Our results show that the reverse link capacity of a diversity system is higher than that of a beamforming system, while the beamforming system can provide higher capacity than the diversity system in the forward link. A hybrid diversity-beamforming system can provide comparable capacity to the diversity system in the reverse link and to the beamforming system in the forward link. We also compare outage capacities of these three systems and our results show that the diversity system provides higher outage capacity both in reverse and forward links. Estimation of forward link channel covariance matrix is of vital importance for sub-optimum design of forward link beamformer and is addressed in the last section of the paper. We propose two methods for estimating the forward link channel covariance matrix based on reverse link measurements for a frequency-division-duplex (FDD) system.***

I. Introduction

An antenna array can be used at a base station to enhance the capacity of wireless communication systems either by exploiting diversity or beamforming [1]- [5]. Diversity techniques can be employed to overcome the multipath fading problem by receiving/transmitting uncorrelated signals; however, correlation among received/transmitted signals degrades the performance of the diversity systems. On the other hand, beamforming can be employed to provide gain through directional transmission, directional reception and interference cancellation; however, optimum design of a beamformer needs the channel information at the transmitter and/or receiver for suitable directional transmission/reception and interference suppression. For reverse link, reverse channel information can be measured at the base station and an optimum beamformer be designed to gain from directional reception and interference cancellation. For forward link directional transmission and null beamforming, forward link channel information should be provided at the base station. Recently [7] [8] proposed the hybrid diversity-beamforming system, which provides desirable features of both diversity and beamforming systems.

In capacity analysis part of this paper as in [9], we assume that channel information is available at the transmitter for optimum design of a beamformer in single user situations, resulting in comparisons of Shannon and outage capacities of diversity, beamforming, and hybrid diversity-beamforming systems. For Time-Division-Duplex (TDD) systems, in which reverse and forward links use the same carrier frequencies, the forward link beamformer can be designed based on reverse link channel information; however, for Frequency-Division-Duplex (FDD) systems in which reverse and forward links use different carrier frequencies, reverse and forward link channels are uncorrelated and therefore, design of forward link beamformer based on reverse link channel information degrades the performance of the system [6]. In last section of the paper we propose two methods for estimating the forward link channel covariance matrix based on reverse link measurements. The channel covariance matrix can be used to design a sub-optimum forward link beamformer [6] [11]. The first method needs a preliminary estimate of Direction-of-Arrival (DOA) of the desired signal and the second method is based on the idea that if two different arrays are employed in close proximity and they are scaled versions of each other with the scaling factor proportional to the ratio of two wavelengths, the reverse and forward link channels are the same [10]. This method spatially resamples the reverse link channel and the formulation shows that the covariance matrix of the forward link channel can be estimated. This method eliminates the need of having two antenna arrays for reverse and forward links and also avoids the DOA estimation of the desired signal.

This paper is organized as follows. In the second section, channel model and antenna architecture for diversity, beamforming, and hybrid diversity-beamforming systems are given. In the third section, Shannon and outage capacities of these systems are derived and computer simulation results are provided.

In the fourth section, we discuss the implementation issues for the CDMA2000 and propose two methods for estimating the forward link channel covariance matrix based on reverse link measurements.

Notation & Definition: $E\{.\}$ denotes the expectation operator, $(.)^\dagger$ denotes the transpose, $(.)^H$ denotes the Hermitian transpose, a random variable is directly denoted by its probability distribution function, χ_j^2 denotes a chi-square random variable with j degrees of freedom. The $x\%$ outage capacity is defined as the capacity offered by the system with probability $(1-x)$.

II. Systems Model and Antenna Architectures

Assume that the base station is equipped with an M-element ($M = kn$) antenna array and the mobile station is equipped with a single antenna. The diversity system can be realized by spacing the base station antenna elements far apart (approximately 10λ where λ is the carrier wavelength) so that each antenna element receives or transmits signals with uncorrelated fading. We assume that the channel is Rayleigh and therefore the channel response vector can be written as

$$h_d = [\alpha_1, \cdots, \alpha_M]$$

where h_d entries are i.i.d., complex, zero mean, unit variance Gaussian random variables.

The beamforming system can be realized by spacing antenna ele-

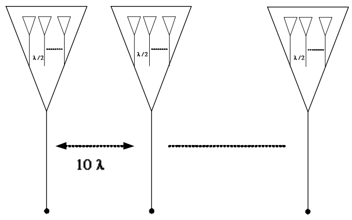

Fig. 1. Antenna architecture for hybrid diversity-beamforming system

ments closely together (approximately $\frac{\lambda}{2}$) so that each antenna receives or transmits the same signal except for the phase shift which depends on antenna array configuration and direction of arrival of the desired signal. For a uniform linear array the channel vector can be written as

$$h_b = \alpha[1, e^{\frac{-j2\pi d \sin(\theta)}{\lambda}}, \cdots, e^{\frac{-j2\pi (M-1) d \sin(\theta)}{\lambda}}]$$

where α is a complex, zero mean, unit variance Gaussian random variable, d is the interelement spacing, θ is the direction of arrival, and λ is the carrier wavelength.

The hybrid diversity-beamforming system can be realized by combining the diversity and the beamforming systems. Antenna elements are spaced in k far apart groups and in each group, n antenna elements are spaced closely together [7]. Antenna architecture for diversity-beamforming system is depicted in Figure 1. Assuming that antenna elements are spaced uniformly in each group the channel vector can be written as

$$h_{d-b} = [h_1, \cdots, h_k]$$

where

$$h_j = \alpha_j[1, e^{\frac{-j2\pi d \sin(\theta)}{\lambda}}, \cdots, e^{\frac{-j2\pi (n-1) d \sin(\theta)}{\lambda}}]$$

for $j = 1, \cdots, k$

III. Reverse Link Capacity Performance

In this section we consider the reverse link capacity performance. For the diversity system the random channel capacity is given by [13]

$$c = \log_2(1 + \rho.\chi^2_{2M})$$

where c is the random channel capacity measured in bps/Hz and ρ is the average SNR at each receiving antenna element. The Shannon capacity is the expected value of the random capacity and can be written as

$$E\{c\} = E\{\log_2(1 + \rho.\chi^2_{2M})\}$$

It can be easily shown through the transformation of random variable [12] that c is distributed as

$$f_C(c) = \frac{\ln(2)}{\rho} 2^c f_{\chi^2_{2M}}(\frac{2^c - 1}{\rho}) \quad (1)$$

where $f_C(c)$ is the probability density function (PDF) of the channel capacity, and $f_{\chi^2_{2M}}(x)$ is the PDF of a χ^2 random variable with $2M$ degrees of freedom.

For the beamforming system with optimum beamformer weights, the random capacity can be written as

$$c = \log_2(1 + M.\rho.\chi^2_2) \quad (2)$$

and Shannon capacity is the expected value of (2). The PDF of channel capacity can be written as

$$f_C(c) = \frac{\ln(2)}{M.\rho} 2^c f_{\chi^2_2}(\frac{2^c - 1}{M.\rho}) \quad (3)$$

For the hybrid diversity-beamforming system with optimum beamformer weights in each group, the random capacity can be written as

$$c = \log_2(1 + n.\rho.\chi^2_{2k}) \quad (4)$$

and Shannon capacity is the expected value of (4). The PDF of channel capacity can be written as

$$f_C(c) = \frac{\ln(2)}{n.\rho} 2^c f_{\chi^2_{2k}}(\frac{2^c - 1}{n.\rho}) \quad (5)$$

Figure 2 depicts Shannon capacity for different number of antennas. For the hybrid diversity-beamforming system antenna elements are spaced into two equal number groups. The diversity system provides the highest capacity and the beamforming system provides the lowest capacity. Figure 3 depicts 1% outage capacity using equations (1), (3) and (5) for different number of antennas. The diversity system provides the highest outage capacity and the beamforming system provides the lowest one.

IV. Forward Link Capacity Performance

In this section we consider the forward link capacity performance. For the diversity system the random channel capacity is given by [13]

$$c = \log_2(1 + \frac{\rho}{M}.\chi^2_{2M}) \quad (6)$$

and Shannon capacity is the expected value of (6). The PDF of channel capacity can be written as

$$f_C(c) = \frac{M.\ln(2)}{\rho} 2^c f_{\chi^2_{2M}}(M.\frac{2^c - 1}{\rho}) \quad (7)$$

For the beamforming system with optimum beamformer weights, the random capacity is given by [9]

$$c = \log_2(1 + M.\rho.\chi^2_2) \quad (8)$$

and Shannon capacity is the expected value of (8). The PDF of channel capacity can be written as

$$f_C(c) = \frac{\ln(2)}{M.\rho} 2^c f_{\chi^2_2}(\frac{2^c - 1}{M.\rho}) \quad (9)$$

For the hybrid diversity-beamforming system with optimum beamformer weights in each group, the random capacity can be written as

$$c = \log_2(1 + \frac{n\rho}{k}\chi^2_{2k}) \quad (10)$$

and Shannon capacity is the expected value of (10). The PDF of channel capacity can be written as

$$f_C(c) = \frac{k.\ln(2)}{n.\rho} 2^c f_{\chi^2_{2k}}(k.\frac{2^c - 1}{n\rho}) \quad (11)$$

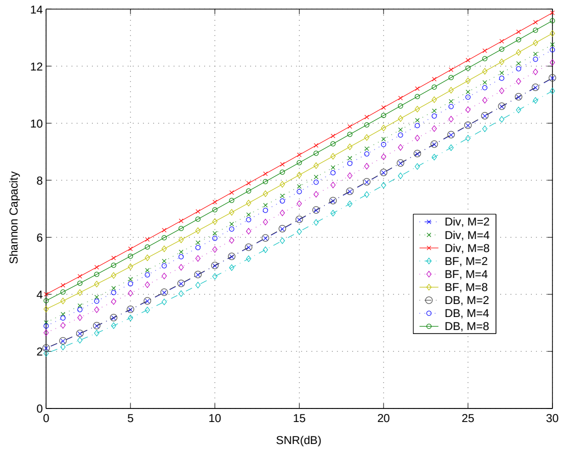

Fig. 2. Shannon capacity for reverse link of diversity, beamforming and hybrid beamforming-diversity systems

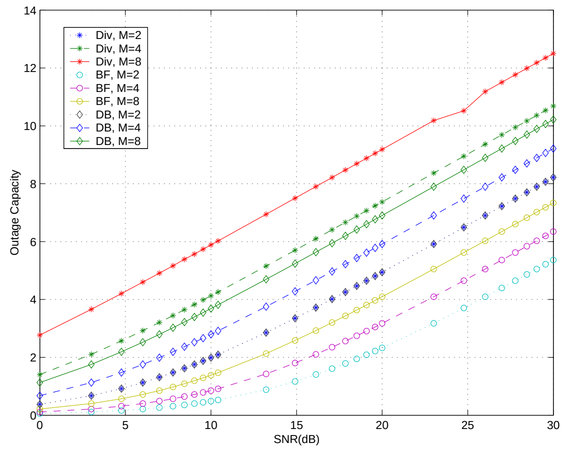

Fig. 3. Outage capacity for reverse link of diversity, beamforming and hybrid beamforming-diversity systems

Figure 4 depicts Shannon capacity for different number of antennas. For the hybrid diversity-beamforming system antenna elements are spaced into two equal number groups. The beamforming system provides the highest capacity and the diversity system provides the lowest one. Figure 5 depicts 1% outage capacity using equations (7), (9) and (11) for different number of antennas. The diversity system provides the highest outage capacity and the beamforming system provides the lowest one.
Figure 6 depicts the reverse and forward links Shannon capacities for an 8-elements antenna array for different number of groups. The four group architecture provides a good capacity in both reverse and forward links.

V. Application and Discussion

The CDMA2000 standard supports a transmit diversity system known as space-time coding [16] where symbols are transmitted via two far apart antennas [17]. This diversity system does not need the forward link channel information available at the transmitter. This standard also supports beamforming; however, because the reverse and forward links use different carrier frequencies, the forward link channel should be provided at the base station for optimum design of a beamformer. Therefore, the use of feedback from the mobile station which makes the knowledge of the forward link channel available at the base station is proposed in [18]; however, this method wastes bandwidth and also the standard does not support it. In the following we propose two methods for providing the forward link channel covariance matrix at the base station for sub-optimum design of a forward link beamformer. The first method needs a preliminary estimate of DOA of the desired signal, while the second method takes advantage of the geometry of uniform linear array (ULA) to adjust the spatial sampling frequency as a function of the carriers wavelength. The reverse link channel response can be written as

$$h_r = \alpha[1, e^{\frac{-j2\pi d_r \sin(\theta)}{\lambda_r}}, \cdots, e^{\frac{-j2\pi(M-1)d_r \sin(\theta)}{\lambda_r}}]^{\dagger} \qquad (12)$$

where d_r is the interelement spacing, θ is the direction of arrival, λ_r is the reverse link carrier wavelength. For a FDD system only the DOAs remain unchanged from reverse to forward link transmissions [18] [14]. Thus, the forward link channel response can be written as

$$h_f = \beta[1, e^{\frac{-j2\pi d_r \sin(\theta)}{\lambda_f}}, \cdots, e^{\frac{-j2\pi(M-1)d_r \sin(\theta)}{\lambda_f}}]^{\dagger} \qquad (13)$$

where λ_f is the forward link carrier wavelength. The reverse link and forward link channel covariance matrices can be written as

$$R_r = E\{h_r {h_r}^H\} \qquad R_f = E\{h_f {h_f}^H\} \qquad (14)$$

For most mobile radio channel models $E\{\alpha^2\}$ is equal to $E\{\beta^2\}$ [15] . Wang and Kaveh [19] proved the existence of a coherent signal-subspace transformation matrix which satisfies

$$T(\theta)v_r = v_f \qquad (15)$$

where

$$v_r = [1, e^{\frac{-j2\pi d_r \sin(\theta)}{\lambda_r}}, \cdots, e^{\frac{-j2\pi(M-1)d_r \sin(\theta)}{\lambda_r}}]^{\dagger} \qquad (16)$$

and

$$v_f = [1, e^{\frac{-j2\pi d_r \sin(\theta)}{\lambda_f}}, \cdots, e^{\frac{-j2\pi(M-1)d_r \sin(\theta)}{\lambda_f}}]^{\dagger} \qquad (17)$$

and T is a $M \times M$ matrix defined as follow.

$$\begin{bmatrix} t_{0,0} & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & t_{M-1,M-1} \end{bmatrix} \qquad (18)$$

where $t_{i,i} = \frac{e^{\frac{-j2\pi i d_r sin(\theta)}{\lambda_f}}}{e^{\frac{-j2\pi i d_r sin(\theta)}{\lambda_r}}}$ for $i = 0, \cdots, M-1$, which is the ith element of v_f divided by the ith element of v_r. It can be easily shown that R_r and R_f are related as

$$R_f = TR_rT^H \qquad (19)$$

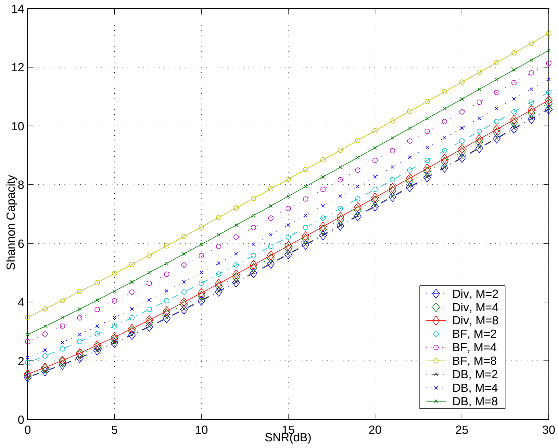

Fig. 4. Shannon capacity for forward link of diversity, beamforming and hybrid beamforming-diversity systems

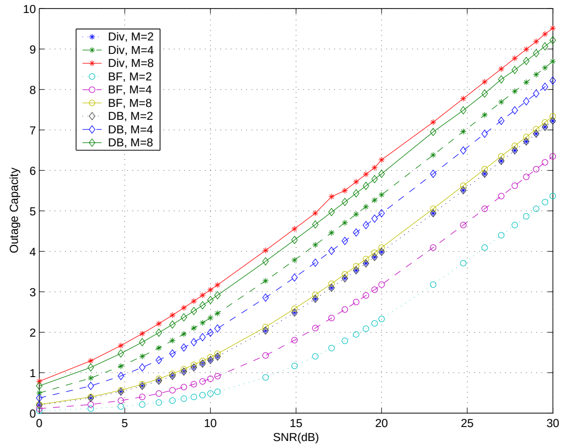

Fig. 5. Outage capacity for forward link of diversity, beamforming and hybrid beamforming-diversity systems

This transformation is robust to small DOA estimation errors.

The second method is simply to adjust the spatial frequency interval, d_r, as a function of the carriers wavelength. The mth element of the reverse link channel, h_r^m, can be written as

$$h_r^m = \alpha e^{\frac{-j2\pi m d_r \sin(\theta)}{\lambda_r}} \tag{20}$$

for $m = 0, \cdots, M-1$ and the nth element of the forward link channel, h_f^n, can be written as

$$h_f^n = \beta e^{\frac{-j2\pi n d_r \sin(\theta)}{\lambda_f}} \tag{21}$$

for $n = 0, \cdots, M-1$. Let h^l denotes the channel response upon a continuous array positioned along l-axis operating at wavelength λ_r and be written as

$$h^l = \alpha e^{\frac{-j2\pi l sin(\theta)}{\lambda_r}} \tag{22}$$

Then h_r^m can be viewed as the result of spatially sampling h^l at positions $l = md_r$ and $m = 0, \cdots, M-1$. It can be seen that if the reverse link channel response is resampled at positions $l = nd_r \frac{\lambda_r}{\lambda_f}$ and $n = 0, \cdots, M-1$, the forward link channel covariance matrix can be estimated.

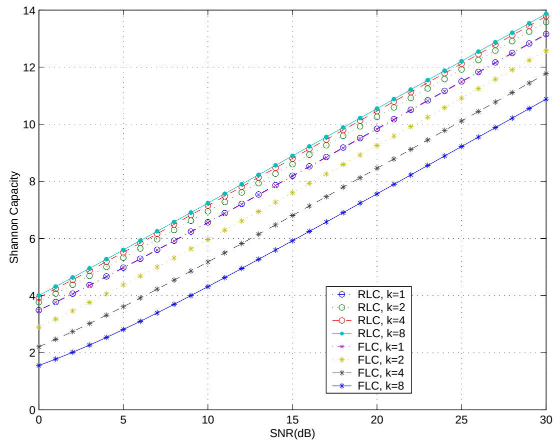

Fig. 6. Reverse and forward link Shannon capacity for different grouping

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