OPTIMALITY RANGE OF TRANSMISSION OVER DIFFERENT TERMINALS IN COOPERATIVE MULTIANTENNA SYSTEMS

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ABSTRACT

In our work we study a cooperative system using multiplexing. The key idea of a *cooperative system* is, that spatially separated stations assist each other to build up a *distributed smart antenna* and thereby to improve the efficiency of their transmissions. Here, the multiple antenna link will be exploited by *multiplexing*.

In order to evaluate the multiple antenna link, we introduce a system model considering one cooperative station. From this, we present the capacity and some properties of system, which differ form ordinary MIMO systems. Furthermore we derive the optimal amplification factors, which specify the optimal power allocation. Finally we compare the ergodic capacity between simple and cooperative transmission.

1. INTRODUCTION

The increasing demand for wireless services makes new and sophisticated concepts in wireless communication necessary. Recent work showed that the throughput of ad-hoc and multihop systems decreases with the number of nodes [1][2]. However, improved performance can be expected form cooperative diversity concepts [3][4].

But systems with multiple transmit and receive antennas can be also exploited by the multiple input/multiple output (MIMO) concept *multiplexing*, i.e. the data is split into several data streams and transmitted over distinct channels. At the latest since Telatar [5] it is known that such systems also increase the capacity.

In this work, we consider the system depicted in figure 1. We show that cooperative systems using multiplexing behave different than ordinary MIMO systems. It turns out that under certain conditions, it is optimal to abstain from the cooperative station which was used at less power. However, the optimal number of used channels of an instantaneous channel realization depend on the total transmit power correspondingly to the water-filling principle in ordinary MIMO systems [6][7]. It is important to notice that already the inspection of a system with only one cooperating station provides knowledge about system properties even with more cooperating stations. In a system with more than one cooperating station, our results about the optimality range when only one station transmits are adaptable to the station with the best channel to the receiver. The continuing characteristic depend on the correlation and need therefore additional analysis.

Since the transmit stations are spatially separated, the cooperating station has to be provided with data. Here, we will not further elaborate this problem and modulate this process by an additive noise term at the cooperating station.

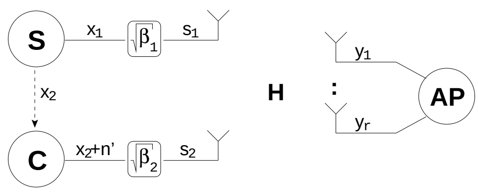

Fig. 1. System model with one cooperating transmit station

In our work, we derive the optimal amplification factors in respect to the total transmission power of the MIMO link, where optimal refers to the system capacity. Furthermore, we determine the conditions when its optimal, that only one station transmits the whole information. Referring to this, we consider the cases when only the transmission willing station is transmitting or the whole information will be forwarded by the cooperating station. The first case corresponds to the simple transmit situation without cooperation, the second corresponds to the relay case.

1.1. System Model

We consider the system model illustrated in figure 1. One station, depicted by the information source S, wants to transmit information to an access point AP. Therefore, the transmission desiring station cooperates with a separated but reachable station C in its vicinity, both are equipped with one antenna and build up together a distributed smart antenna. The access point is equipped with r receive antennas, so that the resulting communication scenario corresponds to the common *multiple access channel*, which exploits the $(2 \times r)$-MIMO channel.

Using our communication model we examine the simple cooperative concept of *amplify and forward*, where the cooperating station just retransmits the amplified received signal. To fulfill this, the information source divides its information into two independent signals x_1 and x_2 of power P. Since we want to examine the properties of the MIMO link between the distributed smart antenna and the receiver, we will not further specify the link between information source and cooperating station, we only assume that the signal x_2 gets disturbed by an additive white Gaussian noise $n^{'}$.

The information source and cooperative station amplify their signals x_1 and $x_2 + n^{'}$ by the factors $\sqrt{\beta_1}$ and $\sqrt{\beta_2}$ and transmit the amplified signals s_1 and s_2 respectively to the access point. Further we assume a flat-fading MIMO channel $\boldsymbol{H} = (\boldsymbol{h}_1, \boldsymbol{h}_2)$ between transmitters and receiver, where $\boldsymbol{h}_1$ and $\boldsymbol{h}_2$ denote the channel between information source and cooperative station to the access point respectively. At each receive antenna element, the receive signals $y_i\ i = 1, .., r$ get disturbed by additive white Gaussian noise $n_i^{''}$, both stacked in vectors $\boldsymbol{y}$ and $\boldsymbol{n}^{''}$ respectively. So we can post the system equation in vector notation as follows:

$$\begin{aligned} \boldsymbol{y} &= \boldsymbol{H}\boldsymbol{s} + \boldsymbol{n}^{''} = \boldsymbol{h}_1\, s_1 + \boldsymbol{h}_2\, s_2 + \boldsymbol{n}^{''} \\ &= \sqrt{\beta_1} x_1\, \boldsymbol{h}_1 + \sqrt{\beta_2}(x_2 + n^{'})\, \boldsymbol{h}_2 + \boldsymbol{n}^{''}. \end{aligned} \tag{1}$$

In eq.(1) and the following, we use boldface capitals and small letters to signify matrices and vectors respectively. $|\boldsymbol{A}|$ and $\boldsymbol{A}^T$ indicate the determinant and transpose of a matrix $\boldsymbol{A}$ and $\boldsymbol{a}^H$ the Hermitian of a vector $\boldsymbol{a}$. $\boldsymbol{I}_n$ denotes the identity matrix of size $n \times n$.

We define the total transmission power P_t used for the MIMO link:

$$P_t = \beta_1\, P + \beta_2(P + \sigma^2_{n^{'}}). \tag{2}$$

Note that we do not take into account the necessary power for distributing the signal x_2 form the information source to the cooperating station. This may be object to realization considerations, but we are intended to investigate the properties of the MIMO link established by the distributed smart antenna.

For the following consideration, we also define the instantaneous correlation ρ between channel $\boldsymbol{h}_1$ and $\boldsymbol{h}_2$:

$$\rho = \frac{|\boldsymbol{h}_1^H \boldsymbol{h}_2|}{||\boldsymbol{h}_1||\, ||\boldsymbol{h}_2||}, \tag{3}$$

where $||\cdot||$ denotes the Euclidean norm.

2. SYSTEM CAPACITY

We assume $n^{'} \in \mathbb{C}$ to be zero mean independent identically distributed (iid) Gaussian noise with power $\sigma^2_{n^{'}}$ and likewise $\boldsymbol{n}^{''} \in \mathbb{C}^r$ to be a zero mean iid noise vector with each entry of power $\sigma^2_{n^{''}}$ and covariance matrix $\boldsymbol{N}^{''} = \sigma^2_{n^{''}} \boldsymbol{I}_r$. For the following, we will define a noise vector $\boldsymbol{n}^{'} = (0, n^{'})^T$ with covariance matrix $\boldsymbol{N}^{'} = \mathcal{E}[\boldsymbol{n}^{'} \boldsymbol{n}^{'H}]$, where $\mathcal{E}[\cdot]$ denotes the expectation value. Also, let $\boldsymbol{B}$ denote the amplification matrix, which only contains the amplification factors on its diagonal.

The capacity of the system can be achieved by transmitting a circularly symmetric complex Gaussian distributed vector $\boldsymbol{x} \in \mathbb{C}^2$ with zero mean and covariance matrix $\boldsymbol{Q} = \mathcal{E}[\boldsymbol{x}\boldsymbol{x}^H]$. Accordingly, the system capacity depending on the amplification factors arises as follows:

$$\begin{aligned} C(\beta_1, \beta_2) &= \log_2 \frac{|\boldsymbol{H}\boldsymbol{B}(\boldsymbol{N}^{'} + \boldsymbol{Q})\boldsymbol{B}^H\boldsymbol{H}^H + \boldsymbol{N}^{''}|}{|\boldsymbol{H}\boldsymbol{B}\boldsymbol{N}^{'}\boldsymbol{B}^H\boldsymbol{H}^H + \boldsymbol{N}^{''}|} = \\ &\log_2 \frac{|(\boldsymbol{N}^{'} + \boldsymbol{Q})^{\frac{1}{2}} \boldsymbol{B}^H \boldsymbol{H}^H \boldsymbol{H}\boldsymbol{B}(\boldsymbol{N}^{'} + \boldsymbol{Q})^{\frac{1}{2}} + \sigma^2_{n^{''}} \boldsymbol{I}_2|}{|\boldsymbol{N}^{'\frac{1}{2}} \boldsymbol{B}^H \boldsymbol{H}^H \boldsymbol{H}\boldsymbol{B}\boldsymbol{N}^{'\frac{1}{2}} + \sigma^2_{n^{''}} \boldsymbol{I}_2|}, \end{aligned} \tag{4}$$

where we used the determinant property $|\boldsymbol{I} + \boldsymbol{A}\boldsymbol{B}| = |\boldsymbol{I} + \boldsymbol{B}\boldsymbol{A}|$. The determinants in last term of eq.(4) can be easily calculated, so that some interesting properties of the capacity can be derived. In particular, the limit analysis shows that for an arbitrary, but fixed β_1, the capacity converges to a constant depending on β_1 with β_2 increasing. Further, the discussion of the derivative reveals more important properties, which are listed in the following table:

C increases logarithmically with increasing β_1	
C increases with increasing β_2	$(\rho^2 < \frac{P}{P+\sigma^2_{n^{'}}}) \vee$ $\big((\rho^2 > \frac{P}{P+\sigma^2_{n^{'}}}) \wedge (\beta_1 < \hat{\beta}_1)\big)$
C decreases with increasing β_2	$(\rho^2 > \frac{P}{P+\sigma^2_{n^{'}}}) \wedge (\beta_1 > \hat{\beta}_1)$

Table 1. Properties of system capacity

In table 1 $\vee$ and $\wedge$ denote the logically *or* and *and* respectively and

$$\hat{\beta}_1 = \frac{||\boldsymbol{h}_2||^2\, \sigma^2_{n^{''}}}{\sigma^2_{n^{'}}\, |\boldsymbol{h}_1^H\, \boldsymbol{h}_2\,|^2 - P(||\boldsymbol{h}_1||^2\, ||\boldsymbol{h}_2||^2 - |\boldsymbol{h}_1^H\, \boldsymbol{h}_2\,|^2)}. \tag{5}$$

It is interesting to observe that in a certain case the capacity decreases with increasing β_2. This is caused by the noise $n^{'}$ and high correlation between the channels.

In figure 2 the capacity depending on β_1 and β_2 for a Rayleigh channel realization where C increases with β_1 and β_2 is plotted.

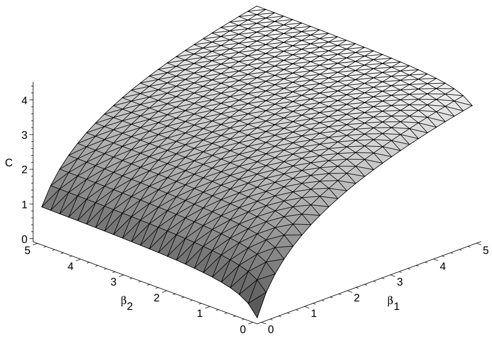

Fig. 2. Capacity depending on β_1, β_2 for $P = \sigma^2_{n'} = \sigma^2_{n''} = 1$ and a Rayleigh channel realization with $\rho^2 < 0.5$

2.1. Optimal Amplification

The system capacity and total transmission power depend on the amplification factors β_1 and β_2. Therefore there exist amplification factors which maximize the capacity with respect to a maximum total transmission power P_t^{max}. According to this we can state the optimization problem as follows:

$$C^{max}(P_t^{max}) = \max_{\boldsymbol{b}=(\beta_1,\beta_2)^T \in \mathbb{R}^2_+} C(\beta_1, \beta_2) \quad \text{subject to } \beta_1 P + \beta_2(P + \sigma^2_{n'}) \leq P_t^{max} \tag{6}$$

This is an optimization problem with affine inequality constraints and a feasible set $\mathscr{F} = \{\boldsymbol{b} \in \mathbb{R}^2 : \boldsymbol{G}\boldsymbol{b} \leq \boldsymbol{r}\}$ with $\boldsymbol{G} = (-\boldsymbol{I}_2, (P, P + \sigma^2_{n'})^T)^T$ and $\boldsymbol{r} = (0, 0, P_t^{max})^T$. Notice that even though eq.(4) can be written as a difference of two convex functions, it needs not to be convex itself, just consider the properties of the capacity listed in table 1. Therefore we have to spend some thoughts to derive the optimum.

It is always optimal to transmit with maximum power, e.g. $P_t = P_t^{max}$. This is easily seen considering the following contradiction: Assuming $\tilde{\boldsymbol{b}} \in \mathscr{F}$ is optimal with $P_t < P_t^{max}$, than using the rest power to increase β_1 will increase the capacity, thus $\tilde{\boldsymbol{b}}$ is not optimal.

Since we know, that in the optimum the system transmits maximum power P_t^{max}, we can restate the optimization problem in a much easier fashion:

$$C^{max}(P_t^{max}) = \max_{t \in [0,1]} C\left((1-t)\frac{P_t^{max}}{P}, t\frac{P_t^{max}}{P + \sigma^2_{n'}}\right). \tag{7}$$

Solving eq.(7) shows, that there exists only one $t^{max} \in [0, 1]$ which solves $\frac{dC(t)}{dt} = 0$, if and only if

$$-\mathcal{A}\left(\frac{P_t^{max}}{P + \sigma^2_{n'}}\right)^2 - \mathcal{B}\frac{P_t^{max}}{P + \sigma^2_{n'}} \leq \mathcal{C} \leq 0, \tag{8}$$

where $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ are defined as follows:

$$\begin{aligned}
\mathcal{A} &= \sigma^2_{n'}(P + \sigma^2_{n'})^2\, ||\boldsymbol{h}_2||^2 (||\boldsymbol{h}_1||^2\, ||\boldsymbol{h}_2||^2 - |\boldsymbol{h}_1^H\, \boldsymbol{h}_2\,|^2) \\
\mathcal{B} &= 2\,\sigma^2_{n''}(P + \sigma^2_{n'})^2 (||\boldsymbol{h}_1||^2\, ||\boldsymbol{h}_2||^2 - |\boldsymbol{h}_1^H\, \boldsymbol{h}_2\,|^2) \\
\mathcal{C} &= \sigma^2_{n''}\, P_t^{max}((P + \sigma^2_{n'})\,|\boldsymbol{h}_1^H\, \boldsymbol{h}_2\,|^2 - P\,||\boldsymbol{h}_1||^2\, ||\boldsymbol{h}_2||^2) \\
&\quad + \sigma^4_{n''}(P(||\boldsymbol{h}_1||^2 - ||\boldsymbol{h}_2||^2) + \sigma^2_{n'}\, ||\boldsymbol{h}_1||^2),
\end{aligned}$$

otherwise the maximum has to be a boundary maximum. Using t^{max} we can post the optimal amplification factors $\boldsymbol{b}^{max} = ({\beta_1}^{max}, {\beta_2}^{max})^T \in \mathscr{F}$:

$$\begin{aligned}
{\beta_2}^{max} &= -\frac{\mathcal{B}}{2\mathcal{A}} + \sqrt{\left(\frac{\mathcal{B}}{2\mathcal{A}}\right)^2 - \frac{\mathcal{C}}{\mathcal{A}}} \\
{\beta_1}^{max} &= \frac{P_t^{max} - {\beta_2}^{max}(P + \sigma^2_{n'})}{P}
\end{aligned} \tag{9}$$

In the next, we will specify the boundary maximums $\boldsymbol{b}^{max} = \left(\frac{P_t^{max}}{P}, 0\right)^T$ and $\boldsymbol{b}^{max} = \left(0, \frac{P_t^{max}}{P+\sigma^2_{n'}}\right)^T$, which correspond to the cases when its optimal to transmit the whole information over channel $\boldsymbol{h}_1$ or $\boldsymbol{h}_2$ respectively. In order to derive conditions to identify the boundary maximum we solved the problems $\frac{dC(t)}{dt}\big|_{t=0} \leq 0$ and $\frac{dC(t)}{dt}\big|_{t=1} \geq 0$. The results of the calculation are presented in table 2.

It is optimal to transmit only over channel $\boldsymbol{h}_1$.	$\left(\frac{\lVert\boldsymbol{h}_1\rVert^2}{\lVert\boldsymbol{h}_2\rVert^2} \geq \frac{P}{P+\sigma^2_{n'}}\right) \wedge \left(\rho^2 > \frac{P}{P+\sigma^2_{n'}}\right)$ (illustrated in fig. 3 upper right)
	$\left(\frac{\lVert\boldsymbol{h}_1\rVert^2}{\lVert\boldsymbol{h}_2\rVert^2} \geq \frac{P}{P+\sigma^2_{n'}}\right) \wedge \left(\rho^2 < \frac{P}{P+\sigma^2_{n'}}\right) \wedge \left(P_t^{max} \leq \hat{P}_t^{max}\right)$ (illustrated in fig. 3 upper left)
	$\left(\frac{\lVert\boldsymbol{h}_1\rVert^2}{\lVert\boldsymbol{h}_2\rVert^2} \leq \frac{P}{P+\sigma^2_{n'}}\right) \wedge \left(\rho^2 > \frac{P}{P+\sigma^2_{n'}}\right) \wedge \left(P_t^{max} \geq \hat{P}_t^{max}\right)$ (illustrated in fig. 3 lower left)
It is optimal to transmit only over channel $\boldsymbol{h}_2$.	$\left(\frac{\lVert\boldsymbol{h}_1\rVert^2}{\lVert\boldsymbol{h}_2\rVert^2} \leq \frac{P}{P+\sigma^2_{n'}}\right) \wedge \left(P_t^{max} \leq \sigma^2_{n''} \cdot \left(-\frac{\mathcal{E}}{2\mathcal{D}} + \sqrt{\left(\frac{\mathcal{E}}{2\mathcal{D}}\right)^2 + \frac{\mathcal{F}}{\mathcal{D}}}\right)\right)$ (illust. in fig. 3 lower left and right)

Table 2. Conditions for boundary maximums

In table 2 we used following variables:

$$\begin{aligned}
\hat{P}_t^{max} &= \sigma^2_{n''}\, \frac{||\boldsymbol{h}_1||^2(P + \sigma^2_{n'}) - ||\boldsymbol{h}_2||^2\, P}{||\boldsymbol{h}_1||^2\, ||\boldsymbol{h}_2||^2\, P - |\boldsymbol{h}_1^H\, \boldsymbol{h}_2\,|^2(P + \sigma^2_{n'})} \\
\mathcal{D} &= \sigma^2_{n'}\, ||\boldsymbol{h}_2||^2(||\boldsymbol{h}_1||^2\, ||\boldsymbol{h}_2||^2 - |\boldsymbol{h}_1^H\, \boldsymbol{h}_2\,|^2) \\
\mathcal{E} &= ||\boldsymbol{h}_1||^2\, ||\boldsymbol{h}_2||^2(P + 2\,\sigma^2_{n'}) - |\boldsymbol{h}_1^H\, \boldsymbol{h}_2\,|^2(P + \sigma^2_{n'}) \\
\mathcal{F} &= ||\boldsymbol{h}_2||^2\, P - ||\boldsymbol{h}_1||^2(P + \sigma^2_{n'})
\end{aligned}$$

In lower left of figure 3, it is interesting to observe that with increasing the total transmission power from a certain point, the amplification factor β_2 decreases unless it is zero.

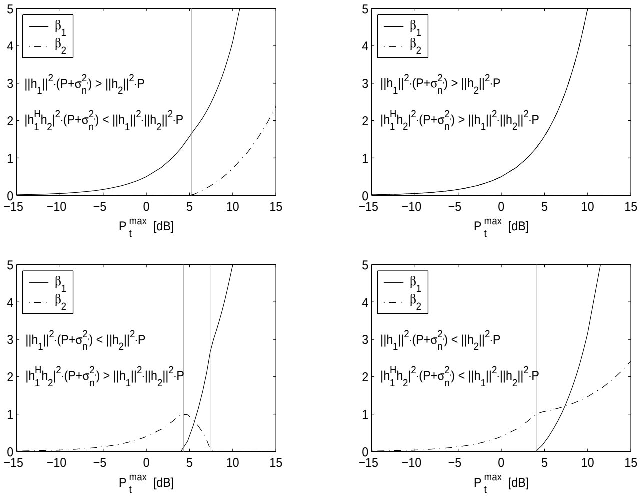

Fig. 3. Exemplary regime of optimal amplification factors

2.2. Ergodic capacity

The maximum ergodic capacity is defined as the expectation value of the maximum capacity C^{max} over channel realizations.

The results presented come form Monte Carlo Simulations with 100000 runs. Thereby, we assumed a system with $P = 1$ and $\sigma^2_{n'} = 10^{-1}$, which results in a good SNR of 10dB at cooperative station and $\sigma^2_{n''} = 10^{-3}$ at the receiver. We modeled the small scale fading on both channels by iid Rayleigh and the large scale fading on channel $\boldsymbol{h}_1$ by a path loss of -40dB.

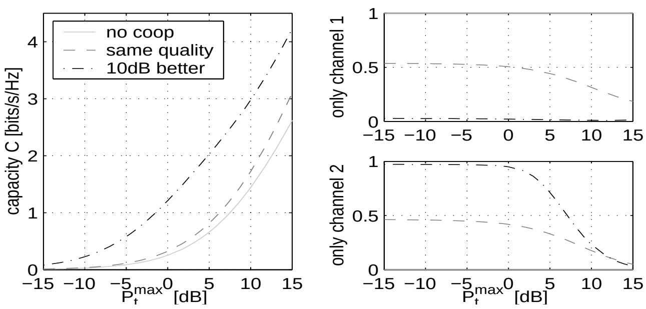

Fig. 4. Comparison between simple and cooperative transmission varying the pass loss $\boldsymbol{h}_2$ in relation to $\boldsymbol{h}_1$

In figure 4 we compare the simple transmission with the cooperative transmission when the path loss of channel $\boldsymbol{h}_2$ is equal -40dB and -30dB. The comparison of the capacity (left figure) is misleading since we did not consider the resource requirement for the link to the cooperative station. But even when we assume time devision multiplexing, thereby we have to multiply the capacity by one half, benefits can be expected when the cooperative station has a better channel.

On the right of figure 4, the probability that only channel one or channel two is used is depicted. It follows that in the low P_t^{max} range either channel one or channel two is used. Further, that with increasing power it is more probable that both channels are allocated, this behavior corresponds to the water-filling principle. In the case of the better channel $\boldsymbol{h}_2$, it also shows that in the low P_t^{max} range almost always channel two is used. Then the cooperative station works as a relay.

3. CONCLUSION

We showed that the MIMO link in cooperative systems using multiplenxing differ from ordinary an MIMO link. When the cooperative station has a better, but highly correlated channel, from a certain power it optimal to abstain from the channel, which was optimal to use at less power.

Nevertheless, the ergodic capacity results showed that benefits can be expected when the coopertive station has a better channel in principle.

4. REFERENCES

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