MULTIPLE TRELLIS CODED DIFFERENTIAL UNITARY SPACE-TIME MODULATION IN RAYLEIGH FLAT FADING

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ABSTRACT

In this paper, we propose multiple trellis-coded differential unitary space-time modulation (MTC-DUSTM), by assigning $k \geq 2$ singal blocks to a single trellis branch. We present the derivation of the formulae for the pairwise error-event probability (PEP) and the bit error probability (BEP) for MTC-DUSTM. Design criteria and a systematic code design have also been proposed. When compared with the trellis coded differential unitary space-time modulation where $k = 1$ signal block is assigned to a branch, we demonstrate that MTC-DUSTM has a better BEP performance when the two schemes have the same number of states.

1. INTRODUCTION

Unitary space-time modulation (USTM) [1], has received much attention for its potential in realizing high data rate transmission in a multiple-input multiple-output (MIMO) wireless communication system, where the channel state information (CSI) is unknown both at the transmitter and the receiver. Later, a differential USTM (DUSTM) scheme was proposed in [2]. By making use of Ungerboeck's idea of "mapping by set partitioning" [3], a power and bandwidth efficient trellis-coded USTM (TC-USTM) was proposed in [7, 8, 9]. In [10], a trellis-coded DUSTM (TC-DUSTM) was proposed. In all these trellis coded schemes, a *single* unitary space-time signal block is assigned to a trellis branch.

Assigning *multiple* ($k \geq 2$) signals to a branch was first proposed for single-input single-output (SISO) systems [4, 5]. In this paper, we extend it to MIMO systems and propose multiple trellis-coded DUSTM (MTC-DUSTM) by assigning multiple signal blocks to a branch. The system model is shown in Fig. 1. We first present the derivation of the decision metric for the coded sequence detection in an ideally interleaved Rayleigh flat fading channel. The formulae for the pairwise error-event probability (PEP) and the bit error probability (BEP) are also derived, which suggest the design criteria for the MTC-DUSTM encoder. Accordingly, a systematic code design method has been proposed. Through computer simulations, we demonstrate that MTC-DUSTM outperforms the TC-DUSTM when the two schemes have the same number of states.

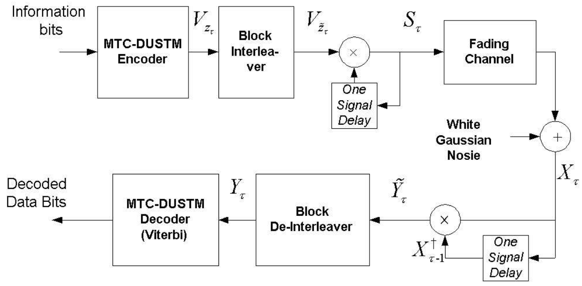

Fig. 1. Block diagram for MTC-DUSTM.

The rest of this paper is organized as follows. In Section 2, DUSTM is briefly reviewed. Performance analyses are given in Section 3, with design criteria proposed accordingly. Systematic code design and examples are presented in Section 4 and 5, respectively. Conclusion is made in Section 6.

2. DIFFERENTIAL UNITARY SPACE-TIME MODULATION

Consider a MIMO system with M transmitter and N receiver antennas. In [2], the unitary space-time signal constellation $\mathbf{V}_L = \{V_l, l = 0, \cdots, L-1\}$ is designed to be an Abelian group under matrix multiplication. Each signal of $\mathbf{V}_L$ is constructed cyclically according to $V_l = V_0^l, l = 0, \cdots, L-1$. $L = 2^{RM}$ denotes the signal size, where R is the information rate in bits per channel use. V_0 is a $M \times M$ diagonal matrix with elements $exp[i(2\pi/L)u_m]$, where $u_m \in \{0, \cdots, L-1\}, 1 \leq m \leq M$ is a set of integers found by making the *dissimilarity* $\zeta_{l,l'} = \frac{1}{2}\prod_{m=1}^{M}\sigma_m(V_{l'} - V_l)^{\frac{1}{M}}$ as large as possible for all $l \neq l'$, where $\sigma_m(V_{l'} - V_l)$ denotes the mth singular value of the matrix $V_{l'} - V_l$. As $\zeta_{l,l'} = |\prod_{m=1}^{M} sin(\pi u_m(l-l')/L)|^{1/M}$ [2], $\zeta_{l,l'}$ is a function of the *signal index interval* $\Delta_{l,l'} = (l'-l)modL$ and can be denoted as $\zeta_{l,l'} = \zeta(\Delta_{l,l'})$. $\zeta = min_{l,l' \in \{0,\cdots,L-1\}}\zeta_{l,l'}$ determines the average block error probability and is referred to as the *diversity product* in [2].

The general idea of TC-DUSTM [10] is to partition $\mathbf{V}_L$ into a series of subsets such that the ζ increases after successive set partitioning. Therefore a "mapping by set partitioning" rule analogous to that in [3] can be applied. After combining with a convolutional encoder with memory ν, a rate $\frac{b}{b+1}$ TC-DUSTM encoder with 2^ν states is formed, where $b = RM$ denotes the information bits per signal block.

3. PERFORMANCE ANALYSIS AND DESIGN CRITERIA FOR MTC-DUSTM

3.1. ML Sequence Detection

As shown in Fig.1, suppose a multiple trellis-coded signal sequence $\mathbf{V}^K = \{V_{z_\tau}, \tau = 1, \cdots, K\}$ is fed into the interleaver, with the output sequence $\tilde{\mathbf{V}}^K = \{V_{\tilde{z}_\tau}, \tau = 1, \cdots, K\}$. τ is the time index for each signal block and $z_\tau \in \{0, \cdots, L-1\}$ is the data represented by each coded singal. $K = q \cdot k$, where q is an integer. The differentially transmitted signal S_τ and the received signal X_τ are [2]

$$S_\tau = V_{z_\tau} S_{\tau-1}, \tag{1}$$

$$X_\tau = \sqrt{\rho} S_\tau H + W_\tau. \tag{2}$$

The initially transmitted signal $S_0 = I_M$ is a $M \times M$ identity matrix. $h_{i,j}, 1 \le i \le M, 1 \le j \le N$ in the channel matrix H are assumed to change continuously according to Jake's model [6]. For different i or j, $h_{i,j}$ are assumed to be independent. For a slow Rayleigh flat-fading channel, H is assumed to be constant during two consecutive signal block intervals. In this period, $h_{i,j}$ are independent and identically distributed (i.i.d) complex Gaussian random variables with zero mean and 0.5 variance in each dimension, denoted as $CN(0,1)$. The additive white Gaussian noise at each receiver antenna in W_τ is also $CN(0,1)$ distributed. ρ is the SNR at each receive antenna. The differentially detected signal sequence is

$$\tilde{Y}_\tau = X_\tau X_{\tau-1}^\dagger, \tau = 1, \cdots, K \tag{3}$$

where $\dagger$ denotes conjugate transpose. Substituting (1) and (2) into (3), we have

$$\tilde{Y}_\tau \approx \rho V_{\tilde{z}_\tau} S_{\tau-1} H H^\dagger S_{\tau-1}^\dagger + W'_\tau, \tag{4}$$

where

$$W'_\tau = \sqrt{\rho} V_{\tilde{z}_\tau} S_{\tau-1} H W_{\tau-1}^\dagger + \sqrt{\rho} W_\tau H^\dagger S_{\tau-1}^\dagger. \tag{5}$$

As $S_{\tau-1}$ is a deterministic unitary matrix, $S_{\tau-1}H$ and H have the same distribution. Let $\mathbf{H} = S_{\tau-1}H$. Entries $\mathbf{h}_{i,j}, 1 \le i \le M, 1 \le j \le N$ in $\mathbf{H}$ are also i.i.d as $CN(0,1)$. As $E\{\mathbf{h}_{m,n}\mathbf{h}_{p,q}\} = \delta_{m,p}\delta_{n,q}$, where δ denotes the Dirac product function, we have $E\{\mathbf{H}\mathbf{H}^\dagger\} = NI_M$ and $E\{\tilde{Y}_\tau | V_{\tilde{z}_\tau}\} = \rho N V_{\tilde{z}_\tau}$. The variance of $\tilde{Y}_\tau$ given $V_{\tilde{z}_\tau}$ can be derived as $\Lambda = \rho MN(\rho+2)I_M$. Therefore the pdf of $\tilde{Y}_\tau$ given $V_{\tilde{z}_\tau}$ is

$$p(\tilde{Y}_\tau | V_{\tilde{z}_\tau}) = \frac{exp\{-tr(\Lambda^{-1}(\tilde{Y}_\tau - \rho N V_{\tilde{z}_\tau})(\tilde{Y}_\tau - \rho N V_{\tilde{z}_\tau})^\dagger)\}}{\pi^{M\times N} det^N(\Lambda)}. \tag{6}$$

Assuming an ideal interleaving, elements in $\mathbf{Y}^K = \{Y_\tau, \tau = 1, \cdots, K\}$ are independent. The joint pdf of $\mathbf{Y}^K$ given $\mathbf{V}^K$ is $p(\mathbf{Y}^K | \mathbf{V}^K) = \prod_{\tau=1}^K p(Y_\tau | V_{z_\tau})$. Note $p(Y_\tau | V_{z_\tau}) = p(\tilde{Y}_\tau | V_{\tilde{z}_\tau})$. The ML sequence decoder can be developed as

$$\mathbf{V}_{ml}^K = \arg \max_{V_{z_\tau} \in \{V_0, \cdots, V_{L-1}\}} \sum_{\tau=1}^K tr\{Y_\tau V_{z_\tau}^\dagger + V_{z_\tau} Y_\tau^\dagger\}. \tag{7}$$

3.2. Performance Analysis and Design Criteria

At the receiver end, suppose the decoder selects a different sequence $\hat{\mathbf{V}}^K = \{V_{\hat{z}_\tau}, \tau = 1, \cdots, K\}$. The length of an error event is defined as the number of places for which the two coded sequences differ, i.e., the Hamming distance between $\mathbf{V}^K$ and $\hat{\mathbf{V}}^K$. From (7), the PEP $p(\mathbf{V}^K \to \hat{\mathbf{V}}^K)$ of mistaking $\mathbf{V}^K$ for $\hat{\mathbf{V}}^K$ is

$$p(\mathbf{V}^K \to \hat{\mathbf{V}}^K) = p(\sum_{\tau=1}^K D_\tau > 0) \tag{8}$$

where $D_\tau = tr\{Y_\tau (V_{\hat{z}_\tau} - V_{z_\tau})^\dagger + (V_{\hat{z}_\tau} - V_{z_\tau}) Y_\tau^\dagger\}$. In [2], D_τ is the quadratic form used to derive the PBEP between V_{z_τ} and $V_{\hat{z}_\tau}$. When $V_{z_\tau} \ne V_{\hat{z}_\tau}$, the characteristic function of D_τ is [1, 2]

$$V_{D_\tau}(i\omega) = \prod_{m=1}^M \left[\frac{1+2\rho}{\rho^2\sigma_{m,\tau}^2[(\omega - i/2)^2 + a_{m,\tau}^2]}\right]^N \tag{9}$$

where

$$a_{m,\tau} = \sqrt{\frac{1}{4} + \frac{1+2\rho}{\rho^2\sigma_{m,\tau}^2}} \tag{10}$$

and $\sigma_{m,\tau}$ is the mth singular value of the matrix $V_{z_\tau} - V_{\hat{z}_\tau}$. When $V_{z_\tau} = V_{\hat{z}_\tau}$, $V_{D_\tau}(i\omega) = 1$. Assuming an ideal interleaving, the quadratic forms D_τ's are independent for different τ's. Therefore the characteristic function of $D = \sum_{\tau=1}^K D_\tau$ is $V_D(i\omega) = \prod_{\tau\in\eta} V_{D_\tau}(i\omega)$, where η is the set of τ for which $V_{z_\tau} \ne V_{\hat{z}_\tau}$. We can invert the characteristic function to find the pdf of D and the PEP can be derived as

$$p(\mathbf{V}^K \to \hat{\mathbf{V}}^K) = \frac{1}{4\pi}\int_{-\infty}^{\infty} d\omega \frac{1}{\omega^2 + 1/4} \prod_{\tau\in\eta}\prod_{m=1}^M \left[\frac{1+2\rho}{\rho^2\sigma_{m,\tau}^2(\omega^2 + a_{m,\tau}^2)}\right]^N \tag{11}$$

At sufficiently high SNR, the PEP is upper bounded by

$$p(\mathbf{V}^K \to \hat{\mathbf{V}}^K) \le \frac{1}{2^{\ell_{min}}}\left(\frac{8}{\rho}\right)^{MN\ell_{min}} \prod_{\tau\in\eta_{min}}\prod_{m=1}^M \sigma_{m,\tau}^{-2N} \tag{12}$$

where η_{min} denotes η along the shortest error event. (12) indicates that the PEP error curve varies as $\rho^{-MN\ell_{min}}$. To minimize the PEP, we have the following design criterion:

Criterion 1. ℓ_{min} should be maximized.

As there are bk information bits per k signal blocks in a trellis branch, there are 2^{bk} paths emanating from each state. If $bk > \nu$, there are parallel paths between consecutive states and $\ell_{min} \leq k$. In this paper, we focus on this case. Therefore it is desirable that $\ell_{min} = k$.

We define $\xi = \prod_{\tau\in\eta_{min}} \zeta_{z_\tau,\hat{z}_\tau}$ as the *effective diversity product*. Thus $\prod_{\tau\in\eta_{min}} \prod_{m=1}^{M} \sigma_{m,\tau}^{-2N} = 2\xi^{-2MN}$. To minimize (12), the minimum of ξ of all the parallel branches, denoted as ξ_{min}, should be maximized. Therefore we have:

Criterion 2. ξ_{min} should be maximized in the set formed by the parallel k-tuples.

It is also desirable that the PEP upper bound in (12) is the same when the parallel paths emanate from or end at different states. Due to the fact that ξ_{min} determines the PEP at high SNR, the following criterion should be satisfied:

Criterion 3. ξ_{min} should be the same for the different sets formed by the parallel k-tuples emanating from or ending at different states.

An asymptotic BEP formula can be expressed as

$$P_b \approx \frac{1}{bk} \sum_{\ell=\ell_{min}}^{\ell'} \sum_{j=1}^{J(\ell)} m_{\ell,j}\, p(\mathbf{V}_{\ell,j}^K \rightarrow \hat{\mathbf{V}}_{\ell,j}^K) \qquad (13)$$

where $J(\ell)$ is the number of all the possible error events having the same length ℓ. $m_{\ell,j}$ is the bit-errors associated with the jth error event with length ℓ and $p(\mathbf{V}_{\ell,j}^K \rightarrow \hat{\mathbf{V}}_{\ell,j}^K)$ is the PEP between these two coded sequences along that path. ℓ' is chosen so that most of the dominant error events are included.

4. SYSTEMATIC CODE DESIGN

We make use of the k-fold *Cartesian products* for the set partitioning for MTC-DUSTM.

First consider the case of $k = 2$. The Cartesian product of $\mathbf{V}_L$ with itself $\mathbf{V}_L \times \mathbf{V}_L$ forms the set $\{(V_0, V_0)$, (V_0, V_1), $\cdots$, (V_{L-1}, V_{L-2}), $(V_{L-1}, V_{L-1})\}$, with dimension L^2. Let $\mathbf{\Psi}_L^0$ denote the set $\{V_{m_0}, \cdots, V_{m_{L-1}}\}$, where $(m_0, \cdots, m_{L-1})$ is a permutation of the sequence $(0, \cdots, L-1)$. Then $\mathbf{\Psi}_L^i = \{V_{m_0\oplus i}, \cdots, V_{m_{L-1}\oplus i}\}$, $i = 0, \cdots, L-1$ are the cyclic shifted versions of $\mathbf{\Psi}_L^0$, where $\oplus$ denotes addition modulo-L. We define *ordered* Cartesian product as $\mathbf{V}_L \otimes \mathbf{\Psi}_L^i = \{(V_0, V_{m_0\oplus i}), \cdots, (V_{L-1}, V_{m_{L-1}\oplus i})\}$, that is, a concatenation of the corresponding elements in signal set $\mathbf{V}_L$ and $\mathbf{\Psi}_L^i$. Therefore, $\mathbf{V}_L \otimes \mathbf{\Psi}_L^i, 0 \leq i \leq L-1$ partition the set $\mathbf{V}_L \times \mathbf{V}_L$ into L subsets. This method guarantees that there are no identical signal blocks between any two 2-tuples in one subset.

There are $L!$ permutations to form $\mathbf{\Psi}_L^0$. For simplicity, we adopt the method in [5] to form $\mathbf{\Psi}_L^0$ systematically by $\mathbf{\Psi}_L^0 = \{V_{nj}, j = 0, \cdots, L-1\}$, where $n \in \{1, 3, \cdots, L-1\}$. As for each subset $\mathbf{V}_L \otimes \mathbf{\Psi}_L^i, 0 \leq i \leq L-1$, the effective diversity products between any two 2-tuples is

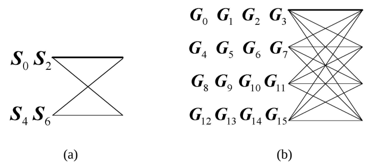

Fig. 2. Trellis diagram for MTC-DUSTM. (a)$M = 2, 2$ states, $k = 2$, (b)$M = 3, 4$ states, $k = 2$.

$$\xi = \zeta_{j,l}\zeta_{nj\oplus i,nl\oplus i} = \zeta(\varDelta_{j,l})\zeta(\varDelta_{nj,nl}), 0 \leq l \neq j \leq L-1 \qquad (14)$$

which is not a function of i. Therefore ξ_{min} for each subset is the same and Criterion 3 is satisfied.

Now it is sufficient to only consider $\mathbf{V}_L \otimes \mathbf{\Psi}_L^0$. To satisfy Criterion 2, the optimal odd integer n should be selected such that ξ_{min} in $\mathbf{V}_L \otimes \mathbf{\Psi}_L^0$ is maximized. Therefore the searching method is:

$$n_{opt} = \arg \max_{n=1,3,\cdots,L-1} \min_{0\leq j\neq l\leq L-1} \zeta_{j,l}\zeta_{nj,nl} \qquad (15)$$

To further partition each subset into two subsets, we let $\mathbf{V}_{L/2}^0 = \{V_{2j}, j = 0, \cdots, L/2-1\}$, $\mathbf{V}_{L/2}^1 = \{V_{2j+1}, j = 0, \cdots, L/2-1\}$ and correspondingly, $\mathbf{\Psi}_{L/2}^{0,i} = \{V_{2j\cdot n\oplus i}, j = 0, \cdots, L/2-1\}$ and $\mathbf{\Psi}_{L/2}^{1,i} = \{V_{(2j+1)\cdot n\oplus i}, j = 0, \cdots, L/2-1\}$, where $n \in \{1, 3, \cdots, L/2-1\}, 0 \leq i \leq L-1$. Thus the ordered Cartesian products $\mathbf{V}_{L/2}^0 \otimes \mathbf{V}_{L/2}^{0,i}$ and $\mathbf{V}_{L/2}^1 \otimes \mathbf{V}_{L/2}^{1,i}$ partition the size-L subset $\mathbf{V}_L \otimes \mathbf{\Psi}_L^i$ into two subsets. For the resulting size-$L/2$ subsets, it is found that $\xi = \zeta(\varDelta_{2j,2l})\zeta(\varDelta_{2jn,2ln}), 0 \leq l \neq j \leq L/2-1$. Therefore Criterion 3 is satisfied. In order to satisfy Criterion 2, (15) can be employed (replacing L with $L/2$) to find the optimal n. Similarly, we can further partition each subset into two "smaller" subsets successively.

The above set partitioning can be extended to the general case easily when $k \geq 2$.

5. EXAMPLES

We examine several MTC-DUSTM examples and evaluate the BEP by computer simulations. We assume a single receiver antenna ($N = 1$) and the information rate is $R = 1$ bit per channel use. The normalized Doppler frequency shift is 0.0025. To make a fair BEP comparison, we employ a size-2500 block-interleaver for all the examples.

First we consider the case where $M = 2$ and $k = 2$. Therefore signal set $\mathbf{V}_8$ is employed. According to (15),

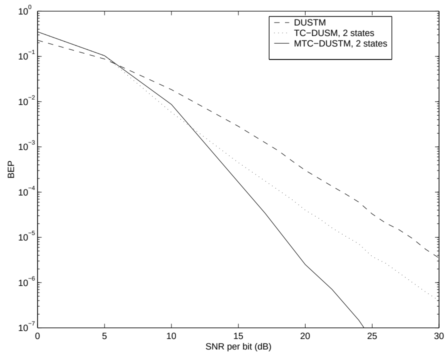

Fig. 3. BEP comparison between DUSTM, TC-DUSTM and MTC-DUSTM ($k = 2$), $M = 2$.

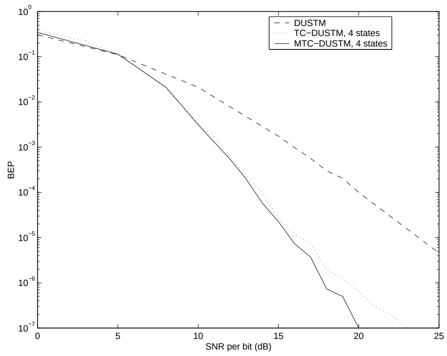

Fig. 4. BEP comparison between DUSTM, TC-DUSTM and MTC-DUSTM ($k = 2$), $M = 3$.

$n_{opt} = 1$. Thus, $\mathbf{\Psi}_8^0 = \mathbf{V}_8$ and the 2-fold Cartesian products are

$$\begin{aligned} \mathbf{S}_0 = \mathbf{V}_8 \otimes \mathbf{\Psi}_8^0 &= \{(V_0, V_0), \cdots, (V_7, V_7)\} \\ &\vdots \\ \mathbf{S}_7 = \mathbf{V}_8 \otimes \mathbf{\Psi}_8^7 &= \{(V_0, V_7), \cdots, (V_7, V_0)\}. \quad (16) \end{aligned}$$

As there are $2^{bk} = 16$ branches emanating from each state, a simple 2-state trellis can be used to let $\ell_{min} = 2$ and any four of the subsets in (16) can be employed. In Fig. 2(a), subsets $\mathbf{S}_{2i}, i = 0, 1, 2, 3$ are used. It is shown in Fig. 3 that MTC-DUSTM with $k = 2$ has significant coding gain over DUSTM and TC-DUSTM with the same number of states (2).

We also present another example where $M = 3$, $k = 2$ and signal set $\mathbf{V}_{16}$ is employed. According to (15), $n_{opt} = 7$ and the resulting subsets are

$$\begin{aligned} \mathbf{G}_0 = \mathbf{V}_{16} \otimes \mathbf{\Psi}_{16}^0 &= \{(V_0, V_0), \cdots, (V_{15}, V_9)\} \\ &\vdots \\ \mathbf{G}_{15} = \mathbf{V}_{16} \otimes \mathbf{\Psi}_{16}^{15} &= \{(V_0, V_{15}), \cdots, (V_{15}, V_8)\}. \end{aligned} \quad (17)$$

As there are $2^{kb} = 64$ branches emanating from each state and each subset in (17) comprises 16 elements, the trellis structure should have 4 states to let $\ell_{min} = 2$. We employ all the subsets $G_i, i = 0, \cdots, 15$, as shown in Fig. 2(b). We show in Fig. 4 that from around 15dB, the MTC-DUSTM begins to outperform the DUSTM and TC-DUSTM with the same number of states (4).

6. CONCLUSION

We have presented the derivation of the PEP and BEP formulae for the MTC-DUSTM scheme and found the design criteria and a systematic code design method. The proposed MTC-DUSTM has a better BEP performance over the uncoded DUSTM as well as the TC-DUSTM, which has the same trellis state number as that of the MTC-DUSTM.

7. REFERENCES

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