ON OPTIMAL CONSTELLATIONS FOR QUASI-ORTHOGONAL SPACE-TIME CODES

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ABSTRACT

Space-Time Block Codes (STBC) exploit multiple-input multiple-output (MIMO) communication systems in order to obtain diversity for high link reliability. Unfortunately, it is not possible to construct STBC with transmission rate equal one for more than two transmit antennas. A way to achieve higher transmission rates is to use quasiorthogonal STBC (QSTBC). We can improve the performance of QSTBC through a special constellation rotation. In this work, we analytically derive the optimal rotation. Numerical simulations show that our analytical approach outperforms the regular QSTBC as well as approaches, where a simulative method is used in order to obtain better performance in comparison to regular QSTBC. In addition to this, we study the impact of the rotation on the achievable portion of the outage capacity.

1. INTRODUCTION

Recent information theoretic results have demonstrated that the capacity of a system in the presence of Rayleigh fading improves significantly with the use of multiple transmit and receive antennas [1]. Space-Time Trellis Codes [2] and Space-Time Block Codes [3] exploit multiple antennas at both the transmitter and receiver in order to obtain transmit and receive diversity and therefore increase the reliability of the system. Space-Time Block Codes (STBC) provide full diversity and we use a very simple maximum-likelihood decoding algorithm at the receiver. Unfortunately, we can not construct an orthogonal space-time code with transmission rate equal one for more than two transmit antennas. Therefore, [4, 5] designed a quasi-orthogonal space-time block code (QSTBC) with transmission rate one for more than two transmit antennas. The disadvantage of QSTBC is the reduction of the transmit diversity, i.e. the slope of the bit error rate (BER) curve is not as steep as in the orthogonal case. Furthermore, the decoder of these codes works with pairs of transmitted symbols instead of single symbols as in the orthogonal case. In order to improve the BER performance, [6] proposed a rotation-based method, that aims at maximizing the minimum euclidian distance in the used signal constellation. However, in doing this, [6] used a computer based search, which is not optimal. In this work, we analytically maximize the minimum euclidian distance.

The rest of this paper is organized as follows. In Section 2 we introduce the system model and establish notation. The analytical results on the optimum rotation angle and the impact of the rotation on the achievable portion of the outage capacity C_{out} is described in Section 3. Section 4 provides simulation results, followed by some concluding remarks in Section 5.

This work was supported in part by the German ministry of education and research (BMBF) under grant 01BU150.

2. SYSTEM MODEL

We consider a system with n_T transmit and n_R receive antennas. Our system model is defined by

$$\mathbf{Y} = \mathbf{G}\mathbf{H} + \mathbf{N} \tag{1}$$

where $\mathbf{G}$ is the $(T \times n_T)$ transmit matrix, $\mathbf{Y}$ is the $(T \times n_R)$ receive matrix, $\mathbf{H}$ is the $(n_T \times n_R)$ complex uncorrelated Rayleigh channel matrix, and $\mathbf{N}$ is the complex $(T \times n_R)$ additive white Gaussian noise (AWGN) matrix, where an entry $\{n_i\}$ of $\mathbf{N}$ $(1 \leq i \leq n_R)$ denotes the complex noise at the ith receiver for a given time t $(1 \leq t \leq T)$ (for clarity we dropped the time index). The real and imaginary parts of n_i are independent and $\mathcal{N}(0, n_T/(2\text{SNR}))$ distributed. An entry of the channel matrix is denoted by $\{h_{ji}\}$. This represents the complex gain of the channel between the jth transmitter $(1 \leq j \leq n_T)$ and the ith receiver $(1 \leq i \leq n_R)$, where the real and imaginary parts of the channel gains are independent and normal distributed random variables with $\mathcal{N}(0,1/2)$ per dimension. The channel matrix is assumed to be constant for a block of T symbols and changes independently from block to block. The average energy of the symbols transmitted from each antenna is normalized to be one, so that the average power of the received signal at each receive antenna is n_T and the signal-to-noise ratio is SNR. It is further assumed that the transmitter has no channel state information (CSI) and the receiver has perfect CSI.

A space time block code is defined by a transmit matrix $\mathbf{G}$. In this work, we consider the performance of the following rate one quasiorthogonal space time block code [4]

$$\begin{aligned} \mathbf{G}(\mathbf{x}) &= \mathbf{G}(x_1, x_2, x_3, x_4) \\ &= \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ x_2^* & -x_1^* & x_4^* & -x_3^* \\ x_3 & -x_4 & -x_1 & x_2 \\ x_4^* & x_3^* & -x_2^* & -x_1^* \end{bmatrix}, \end{aligned} \tag{2}$$

where $x_1, \ldots, x_4$ are taken from a given constellation. The matrix $\mathbf{G}$ can be decomposed in two parts $\mathbf{G}_1$ and $\mathbf{G}_2$ as follows

$$\mathbf{G} = \mathbf{G}_1 + \mathbf{G}_2 , \tag{3}$$

where

$$\begin{aligned} \mathbf{G}_1 &= \mathbf{G}(x_1, 0, x_3, 0) \\ &= \begin{bmatrix} x_1 & 0 & x_3 & 0 \\ 0 & -x_1^* & 0 & -x_3^* \\ x_3 & 0 & -x_1 & 0 \\ 0 & x_3^* & 0 & -x_1^* \end{bmatrix} \end{aligned} \tag{4}$$

and $\mathbf{G}_2 = \mathbf{G}(0, x_2, 0, x_4)$ is defined similarly. An important property of the quasi-orthogonal space-time codes is

$$\mathbf{G}_1^H \cdot \mathbf{G}_2 + \mathbf{G}_2^H \cdot \mathbf{G}_1 = 0 \quad \forall \mathbf{x} , \tag{5}$$

which is crucial, because this enables a simple maximum-likelihood decoding algorithm. Assuming perfect channel estimation is available, the receiver computes the following decision metric over all possible transmit matrices and decides in favor of the transmit matrix that minimizes the following decision metric.

$$\begin{aligned} &||\mathbf{Y} - \mathbf{G}(\mathbf{x}) \cdot \mathbf{H}||_F^2 \\ &\quad = \operatorname{tr}\{(\mathbf{Y} - \mathbf{G}(\mathbf{x}) \cdot \mathbf{H})^H (\mathbf{Y} - \mathbf{G}(\mathbf{x}) \cdot \mathbf{H})\} \qquad (6) \\ &\quad = \operatorname{tr}\{\mathbf{Y}^H \mathbf{Y} - \mathbf{Y}^H \mathbf{G}(\mathbf{x}) \mathbf{H} - \\ &\qquad -(\mathbf{Y}^H \mathbf{G}(\mathbf{x}) \mathbf{H})^H + \mathbf{H}^H \mathbf{G}(\mathbf{x})^H \mathbf{G}(\mathbf{x}) \mathbf{H}\} . \end{aligned}$$

After some manipulations, we arrive at

$$\begin{aligned} &\operatorname{tr}\{\mathbf{Y}_1^H \mathbf{Y}_1 + \mathbf{Y}_1^H \mathbf{G}_1 \mathbf{H} + \mathbf{H}^H \mathbf{G}_1^H \mathbf{Y}_1 + \mathbf{H}^H \mathbf{G}_1^H \mathbf{G}_1 \mathbf{H} + \\ &\quad \mathbf{Y}_2^H \mathbf{Y}_2 + \mathbf{Y}_2^H \mathbf{G}_2 \mathbf{H} + \mathbf{H}^H \mathbf{G}_2^H \mathbf{Y}_2 + \mathbf{H}^H \mathbf{G}_2^H \mathbf{G}_2 \mathbf{H}\} , \end{aligned}$$

where $\mathbf{Y}_1$ and $\mathbf{Y}_2$ are given as

$$\begin{aligned} \mathbf{Y}_1 &= \mathbf{G}_1 \mathbf{H} + \mathbf{N} \\ \mathbf{Y}_2 &= \mathbf{G}_2 \mathbf{H} + \mathbf{N} . \end{aligned}$$

The above decision metric can be decomposed into two parts, one of which

$$\operatorname{tr}\{\mathbf{Y}_1^H \mathbf{Y}_1 + \mathbf{Y}_1^H \mathbf{G}_1 \mathbf{H} + \mathbf{H}^H \mathbf{G}_1^H \mathbf{Y}_1 + \mathbf{H}^H \mathbf{G}_1^H \mathbf{G}_1 \mathbf{H}\}$$

is only a function of $\mathbf{G}_1$, and the other one

$$\operatorname{tr}\{\mathbf{Y}_2^H \mathbf{Y}_2 + \mathbf{Y}_2^H \mathbf{G}_2 \mathbf{H} + \mathbf{H}^H \mathbf{G}_2^H \mathbf{Y}_2 + \mathbf{H}^H \mathbf{G}_2^H \mathbf{G}_2 \mathbf{H}\} ,$$

is only a function of $\mathbf{G}_2$. Thus the minimization of (6) is equivalent to minimizing this two parts separately. Since the two parts can be considered separately, it is possible to write an equivalent system model for each of them. The equivalent system model for $\mathbf{G}_1$ (and similarly for $\mathbf{G}_2$) for $n_R = 1$ receive antennas can be written as

$$\mathbf{Y}_1 = \widetilde{\mathbf{H}} \begin{bmatrix} x_1 \\ x_3 \end{bmatrix} + \mathbf{N} \tag{7}$$

with

$$\widetilde{\mathbf{H}} = \begin{bmatrix} h_1 & h_3 \\ -h_2^* & -h_4^* \\ -h_3 & h_1 \\ -h_4^* & h_2^* \end{bmatrix} .$$

By multiplying $\widetilde{\mathbf{H}}^H$ from left to (7)

$$\widetilde{\mathbf{Y}}_1 = \widetilde{\mathbf{H}}^H \widetilde{\mathbf{H}} \begin{bmatrix} x_1 \\ x_3 \end{bmatrix} + \underbrace{\widetilde{\mathbf{H}}^H \mathbf{N}}_{\widetilde{\mathbf{N}}} \tag{8}$$

where $\widetilde{\mathbf{N}}$ is no more AWGN. To compute the pre-whitening filter, we need the singular value decomposition (SVD) of $\widetilde{\mathbf{H}}$, which is given as $\widetilde{\mathbf{H}} = \mathbf{U}\mathbf{S}\mathbf{V}^H$. Therefore the pre-whitening filter is given as $\mathbf{F}_{\mathrm{PW}} = \mathbf{S}^{-1}\mathbf{V}^H$. By multiplying $\mathbf{F}_{\mathrm{PW}}$ to (8) and some manipulations we arrive at

$$\begin{aligned} \widehat{\mathbf{Y}}_1 &= \underbrace{\begin{bmatrix} \beta & i\beta \\ \epsilon & -i\epsilon \end{bmatrix}}_{\widehat{\mathbf{H}}} \begin{bmatrix} x_1 \\ x_3 \end{bmatrix} + \mathbf{W} \\ &= \begin{bmatrix} \beta(x_1 + ix_3) \\ \epsilon(x_1 - ix_3) \end{bmatrix} + \mathbf{W} , \end{aligned} \tag{9}$$

where $\mathbf{W}$ is white noise again and

$$\beta = \sqrt{\frac{\gamma + \frac{\alpha}{i}}{2}} \qquad \epsilon = \sqrt{\frac{\gamma - \frac{\alpha}{i}}{2}} ,$$

where γ and α are given as

$$\begin{aligned} \gamma &= \sum_{j=1}^{n_T} |h_j|^2 \\ \alpha &= 2i \cdot \operatorname{Im}(h_1^* h_3 + h_4^* h_2) . \end{aligned}$$

3. ANALYTICAL RESULTS ON OPTIMUM ROTATION ANGLE

Let us denote the constellation of x_1 from (9) as $\mathcal{A}$ (e.g. QPSK) and for x_3 as $\mathcal{B}$, where $\mathcal{B} = \mathcal{A}\exp(i\phi)$. Furthermore, let us denote the constellation of the "supersymbol"

$(x_1 + ix_3)$ (cf. (9)) as $\mathcal{C} = \mathcal{A} + i\mathcal{B}$ and for $(x_1 - ix_3)$ as $\mathcal{D}$. If $\phi = 0$, then $\mathcal{B}$ is equal to $\mathcal{A}$. Assume now, that we use QPSK modulation. In order to improve the BER performance, we rotate the constellation of x_3 with ϕ as depicted in Fig. 1. To illustrate the impact of this rotation, we take the

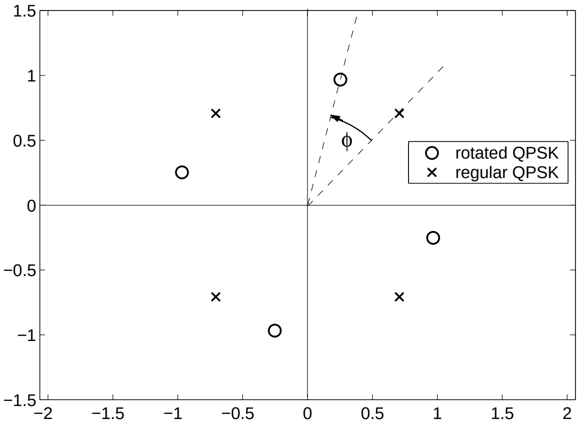

Fig. 1. Rotation of the QPSK constellation of x_3 with an angle ϕ.

constellation $\mathcal{C}$ and increase stepwise, as depicted in Fig 2, the angle ϕ from 0.0 rad ($*$) to 0.8 rad (□)in steps of 0.1 rad. Without rotation, there are some constellation points

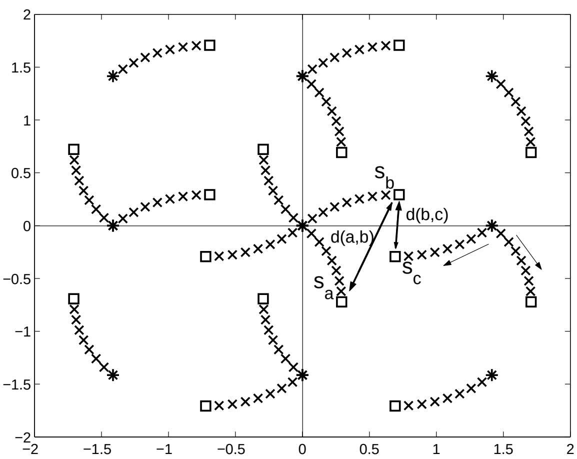

Fig. 2. Constellation of $\mathcal{C}$ for QPSK. The arrows show in direction of increasing angles ϕ. ϕ is increasing from 0.0 rad ($*$) to 0.8 rad (□)in steps of 0.1 rad.

dwell at the same position. By rotating, these constellation points change their position and the euclidian distances between them grow. But if the angle of rotation is too big, the euclidian distances between two neighboring constellation points get smaller again . Therefore, it exists an angle ϕ_{opt}, at which the distance between the nearest constellation points is maximal. To obtain the angle which maximizes this distance, we have to solve the following equation

$$\arg\max_{\phi} d(s_1, s_2), \text{ s.t. } d(s_1, s_2) \geq d(s_1, s_i) \; \forall i \neq 1, 2 \tag{10}$$

where $s_i \in \mathcal{C}$. As can be seen from Fig.2, there is a high symmetry in the constellation $\mathcal{C}$. Therefore, solving (10) is equal to solving

$$d(s_a, s_b) \stackrel{!}{=} d(s_b, s_c) \tag{11}$$

where the distances $d(s_a, s_b)$ and $d(s_b, s_c)$ are given (cf. Fig. 2) as

$$\begin{aligned} d(s_a, s_b) &= 2\frac{1}{\sqrt{2}} \left|-1 + \exp(-i\phi)\right| && (12) \\ d(s_b, s_c) &= 2\frac{1}{\sqrt{2}} \left|-i + i(1+i)\exp(-i\phi)\right| && (13) \end{aligned}$$

At $\phi = 0$, the distance $d(s_a, s_b)$ is equal to zero and $d(s_b, s_c)$ is equal to $\sqrt{2}$. After substituting (12) and (13) in (11) and some manipulations we get the optimum angle, which is given as

$$\phi_{\text{opt}} = \frac{\pi}{6} \; . \tag{14}$$

All constellation points are on different places for $\phi_{\text{opt}} = \pi/6$. Therefore we expect, that the performance of the constellation with rotation is better than without rotation. In Table 1, we show ϕ_{opt} for different modulation schemes.

	QPSK	8PSK	16QAM	64QAM
$\frac{\phi_{\text{opt}}}{[rad]}$	$\pi/6$	0.483	$\pi/6$	0.257

Table 1. ϕ_{opt} for different modulation schemes.

3.1. Capacity considerations

Now, we are interested in the impact of the constellation rotation on the outage capacity C_{out}. The instantaneous capacity of a MIMO system with $n_T = 4$ transmit and $n_R = 1$ receive antennas is as follows

$$C = \log_2 \det(\mathbf{I}_R + \frac{\rho}{n_T} \mathbf{H}^H \mathbf{H}) \; . \tag{15}$$

We use (15) to compute C_{out} based on monte-carlo simulations. The capacity of the quasiorthogonal scheme is

$$C_{qstbc} = \frac{2}{4} \log_2 \det(\mathbf{I}_R + \frac{\rho}{n_T} \bar{\mathbf{H}}^H \bar{\mathbf{H}}) \; , \tag{16}$$

where $\bar{\mathbf{H}}$ is given as follows in the case of not rotated constellation

$$\bar{\mathbf{H}} = \widehat{\mathbf{H}} \; ,$$

and $\widehat{\mathbf{H}}$ is given in (9). In the case of rotated constellation, $\bar{\mathbf{H}}$ is given as follows

$$\bar{\mathbf{H}} = \widehat{\mathbf{H}} \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & \exp(-i\phi) \end{bmatrix}}_{\mathbf{\Theta}}, \quad (17)$$

where $\mathbf{\Theta}$ is a unitary matrix and therefore does not deteriorate the capacity in comparison to the nonrotated case as depicted in Fig. 3.

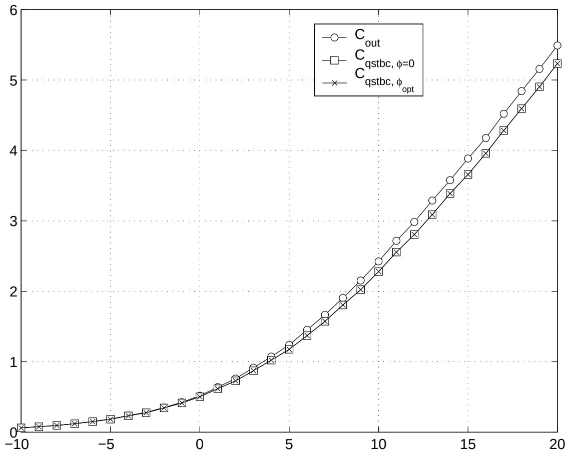

Fig. 3. 10% Outage Capacities of a MIMO system C_{out} and the QSTBC scheme C_{qstbc} with $n_T = 4$ transmit and $n_R = 1$ receive antennas.

4. NUMERICAL SIMULATIONS

In Fig. 4, the BER of the quasiorthogonal scheme from Jafarkhani [4] (without rotation,i.e. $\phi = 0$), from [6] (with rotation, determination of ϕ through simulations) denoted as "ShaPa" in Fig. 4 and the BER for ϕ_{opt} for a system with $n_T = 4$ transmit and $n_R = 1$ receive antennas and QPSK modulation are depicted. As can be seen from Fig. 4, the slope of the curve with ϕ_{opt} is steeper as that of [4] and [6]. The scheme with ϕ_{opt} outperforms both other schemes for higher SNR values.

5. CONCLUSION

In this paper, we analytically derived the optimal rotation angle ϕ_{opt} for different constellation sizes, which significantly improves the performance of QSTBC. We compared the BER performance of the scheme taking advantage of ϕ_{opt} to the regular QSTBC from [4]. In addition, we compared our scheme with the one from [6].In [6], a simulative approach is used to derive the optimal rotation. We showed that our scheme outperforms the schemes of [4, 6] for higher SNR values. Finally, we showed that there is no reduction on the achievable portion of the outage capacity through constellation rotation.

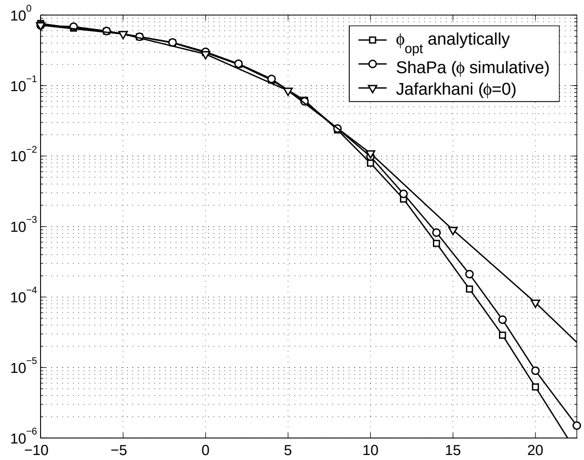

Fig. 4. BER of the quasiorthogonal scheme ($n_T = 4, n_R = 1$) with and without constellation rotation, uncoded QPSK modulation.

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