# AN OPTIMAL RECEIVER FOR TRANSMISSION DIVERSITY OVER UNCERTAIN CHANNELS

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$Abstract$– In this paper, a receiver for a Dual Transmit Diversity system is designed that takes into account the channel estimation error, assuming the unknown channel to have a given complex bivariate Gaussian probability density function (pdf) (*i.e.*, a Ricean pdf). This criterion of the receiver, which is based on the Maximum *A Posteriori* (MAP), is expressed in a quadratic form, and in extreme cases, represents either a linear detector or a non-coherent-non-linear detector. Simulations of the Symbol Error Probability (SEP) of the receiver and analysis of an Upper Bound (UB) and a Lower Bound (LB) confirm that the proposed detector achieves robust performance against channel imperfections.

## I. INTRODUCTION

To improve data communication quality (*e.g.*, by reducing the effective error rate) in a multipath fading environment, it is crucial to successfully reduce the effect of fading at both the mobile units and the base stations. In most scattering environments, antenna diversity is a practical and efficient technique for reducing the effect of multipath fading [1]. These schemes employ pre-coding, namely Space-Time Coding (STC), which is appropriate for multiple transmit antenna systems. STC leads to a considerable increase in bandwidth efficiency and system capacity [2].

Although it has been proved in [5] that in the presence of small errors in the channel state information, STCs still result in an improved bandwidth efficiency over classical transmitting schemes, a considerable degradation is observed when the channel estimation error increases. This could be improved by sending more pilot symbols (training symbols) during the transmission at the cost of losing some bandwidth efficiency, especially in the case of fast time-varying channels [2, 5]. Hence, robust detection for these methods is needed for good operation when the Channel State Information (CSI) is not exactly known.

In this paper we derive a new MAP data detection algorithm that takes into account the channel estimation errors. In the proposed method, the channel error is assumed to be a complex Gaussian random vector with a known mean (based on a previous estimate [9], a guess, or initialized at zero) and a covariance matrix (as a measure of the deviation from the estimated value). Here, for simplicity, we consider a system with two transmit antennas and one receive antenna as in [1]. The results can be extended to a general case of multiple transmitters and multiple receivers.

The paper is organized as follows: In Section II the system and receiver structure are provided. In Section II-A, we also simplify the receiver for an important and simple case. In section II-B the performance of the receiver, *i.e.*, the Symbol Error Probability (SEP), is analyzed for a simple case by simulations and by derivation of an Upper Bound (UB) and a Lower Bound (LB). Finally, some concluding remarks are discussed in Section III.

## II. SYSTEM MODEL AND RECEIVER STRUCTURE

For simplicity in this paper, the Dual Transmit Diversity (DTD) technique is considered that is proposed by Alamouti [1]. This scheme can be described as follows:

$$R \triangleq \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} = SH + N. \quad (1)$$

where for $i = 1, 2$, the vectors $r_i \in \mathbb{C}^L$, and $n_i \in \mathbb{C}^L$ are the received signals and the Additive White Gaussian Noise (AWGN), respectively. Transmitted symbols, $s_1$ and $s_2$ both take their values randomly from $\mathcal{C} = \left\{c_i \in \mathbb{C}^L\right\}_{i=1}^K$, where $K$ is the number of constellation points and $L$ is the dimension of the transmitted signal space. The channel gains $h_1$ and $h_2$ are complex random variables. The notations $(.)^*$, $(.)^T$ and $(.)^H$ stand for complex conjugate, transposition and Hermitian, respectively.

*Remark 1:* It is easy to see that the signal $R$ remains invariant by transforming the quadruplet $(s_1, s_2, h_1, h_2)$ into $(e^{j\phi}s_1, e^{-j\phi}s_2, e^{-j\phi}h_1, e^{j\phi}h_2)$. Assuming that elements of this quadruplet are unknown, this property leaves ambiguity, which we shall call Phase Ambiguity (PA), in determination of the phase $e^{j\phi}$ when only $R$ is observed in order to estimate this quadruplet. The PA depends on the set of alphabets $\mathcal{C}$. For example, using a 4QAM modulation scheme, we have $e^{j\phi} \in \{\pm 1 \pm j\}$; in this case the receiver must know $e^{j\phi}$, which is the equivalent of two bits or one symbol. To resolve the PA, the direction of one of the components of this quadruplet must be determined. One way to resolve the PA is to control the set of alphabets $\mathcal{C}$ for $s_1$ and $s_2$, *e.g.*, using one pair of training symbols. If the phase varies slowly enough with time, it can be tracked accurately after the first initialization [9]. However, a sudden rotation of the channel coefficients results in a dual rotation of the detected data after that event. Another alternative to overcome the PA is to use differentially coded modulation schemes (*e.g.*, see [4] and references therein for more details). In these schemes the information is embedded into in the transmitted sequence in a such a way that after decoding the effect of $e^{j\phi}$ is cancelled, usually at the expense of about 3dB noise augmentation.

The following theorem, which is the basis of the proposed receiver, assumes a Ricean pdf for the channel $H \sim \mathcal{N}(\widetilde{H}, \widetilde{\Sigma})$ at a particular moment. The noise $N$ is zero-mean, white and Gaussian and is assumed to be independent of the channel coefficients. In this and the next sections, $\widetilde{(.)}$ and $\widehat{(.)}$ denote *a priori* and *a posteriori* values, respectively. The receiver is provided with the received signal $R$ and inaccurate *a priori* channel information $\widetilde{H}$ where the matrix $\widetilde{\Sigma}$ represents *a priori* covariance of the channel errors.

*Theorem 1:* If the *a priori* pdf of the channel vector $H$ is Gaussian and is provided with the mean $\widetilde{H}$ and the covariance matrix $\widetilde{\Sigma}$, *i.e.*, $H \sim \mathcal{N}(\widetilde{H}, \widetilde{\Sigma})$, and the additive noise is a zero-mean white stationary Gaussian vector, *i.e.*, $N \sim \mathcal{N}(0, \sigma^2 I_{2L})$, then the optimal receiver should maximize the conditional pdf of the received vector signal given by $f(R|S) = \frac{|\widehat{\Sigma}| \exp(-B)}{\pi^{2L}\sigma^{2L}|\widetilde{\Sigma}|}$ where

$$\widehat{\Sigma}^{-1} = \frac{1}{\sigma^2} S^H S + \widetilde{\Sigma}^{-1} = \alpha I_2 + \widetilde{\Sigma}^{-1}, \quad \text{(2a)}$$

$$B = -\widehat{H}^H \widehat{\Sigma}^{-1} \widehat{H} + \frac{1}{\sigma^2} R^H R + \widetilde{H}^H \widetilde{\Sigma}^{-1} \widetilde{H}, \quad \text{(2b)}$$

$$\widehat{H} = \widetilde{H} + \frac{1}{\sigma^2} \widehat{\Sigma} S^H \left( R - S\widetilde{H} \right), \quad \text{(2c)}$$

where $\alpha \triangleq \frac{\|s_1\|^2 + \|s_2\|^2}{\sigma^2} = \frac{|S|}{\sigma^2}$ and $|A|$ stands for the magnitude of the determinant of the matrix $A$ (or for $\sqrt{\det(A^H A)}$ if $A$ is not a square matrix) and $||.||$ is the Euclidian distance. Furthermore, if $S$ is the true transmitted value or if the error probability is small enough, then the *a posteriori* pdf of the channel after observation of $R$ and detection of $S$ is also Gaussian, with $\widehat{H}$ as its mean and with $\widehat{\Sigma}$ as its covariance matrix, *i.e.*, $f(H|R, S) = \mathcal{N}(\widehat{H}, \widehat{\Sigma})$. For proof See [9]. ■

*Remark 2:* Since the *a posteriori* and the *a priori* pdfs of the channel have a same form, the assumption of this theorem is justified if an iterative channel estimation is employed.

By virtue of this theorem, an optimal receiver can maximize the log–likelihood function, *i.e.*, $\log(f(R|S))$, as the decision rule; therefore, the following metric $M(C_{p,q}, R)$

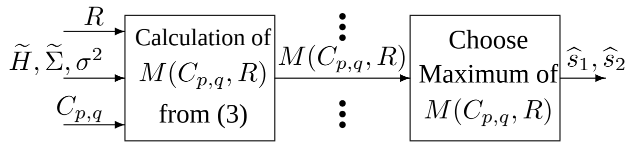


Fig. 1. The block diagram of the optimal receiver. The detector receives $R$ and detects the transmitted pairs by receiving some statistical information of the channel, $(\widetilde{H}, \widetilde{\Sigma})$, and the noise variance, $\sigma^2$.

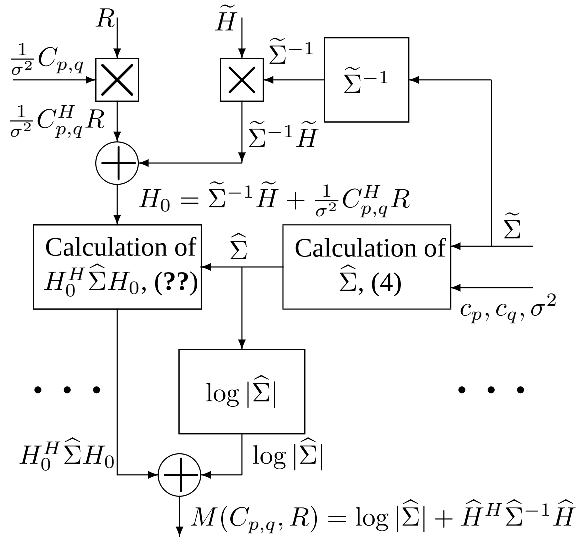


Fig. 2. The structure of the MAP metric calculator for an unknown channel $H$ with a given Gaussian pdf $\mathcal{N}(\widetilde{H}, \widetilde{\Sigma})$.

should be maximized,

$$\widehat{S} = \arg \max_{c_p, c_q \in \mathcal{C}} M(C_{p,q}, R) \quad \text{(3)}$$

where $M(C_{p,q}, R) = \log|\widehat{\Sigma}| + \widehat{H}^H \widehat{\Sigma}^{-1} \widehat{H}$ and $C_{p,q} \triangleq \begin{bmatrix} c_p & c_q \\ -c_q^* & c_p^* \end{bmatrix}$. The block diagram of this receiver in Figure 1 shows how the receiver uses some statistical information about the channel and the noise. Figure 2 shows the details of the metric calculator in the receiver. It demonstrates that each path in the receiver is simple as to the computational complexity, and its digital implementation is both feasible and cheap. In this paper, we consider $\widetilde{\Sigma} \triangleq \varsigma^2 \begin{bmatrix} 1 & \rho \\ \rho^* & 1 \end{bmatrix}$ as the error covariance matrix of the channel vector. This assumption results in

$$\widehat{\Sigma} = \frac{1}{(\alpha + \beta)^2 - \beta^2 |\rho|^2} \begin{bmatrix} \alpha + \beta & \beta\rho \\ \beta\rho^* & \alpha + \beta \end{bmatrix}, \quad \text{(4)}$$

where $\beta \triangleq \frac{\varsigma^{-2}}{1 - |\rho|^2}$, $\varsigma^2$ and $\rho$ are the variance and the correlation of the *a priori* channel estimation error, respectively, and $|\widehat{\Sigma}| = \frac{1}{(\alpha+\beta)^2 - \beta^2|\rho|^2}$.

*Remark 3:* The value $\widehat{\Sigma}$, that represents the *a posteriori* covariance of the channel coefficients when the error probability is very small, satisfies $\mathrm{Cov}(H|R, S = \widehat{S}) = \widehat{\Sigma} \leqslant \widetilde{\Sigma}$.

Theorem 1 and this inequality show that $\widehat{H}$ can be considered as the *a posteriori* estimate of the channel when the Symbol Error Probability (SEP) is low. This implies that $\widehat{H}$ can be used as the output of an iterative algorithm to estimate the channel coefficients. See [10] for more details.

*Remark 4:* We can see that if $\widetilde{\Sigma} \rightarrow 0$, *i.e.*, when the channel is known and the additive noise is white and Gaussian, then the receiver simplifies to a linear receiver as proposed in [1].

### A. Simplified Receiver Structure

In this section, the receiver can be simplified for the special case where $||s_1||^2 + ||s_2||^2 = |S|$ is taken to be constant at the transmitter. This constraint means that the total energy used for transmission of one pair of symbols, $s_1$ and $s_2$, is constant. This condition is less strict compared to the case of equal energy signals. It can be seen from Theorem 1 that this constraint makes $\widehat{\Sigma}$ independent of $s_1$ and $s_2$; therefore, the first term of the metric $M$ in (3) plays no role in optimization, and the simplified metric $H_0^H \widehat{\Sigma} H_0$ is to be maximized. The receiver structure in this case will be the same as depicted in Figure 1, except for the calculation of $\log|\widehat{\Sigma}|$ that is no longer required. So, imposing such a simple constraint on the energy of signals the transmitter results in a more computationally efficient receiver.

### B. Symbol Error Probability (SEP)

In the following theorem, an upper bound (UB) and a lower bound (LB) are given for the SEP of the above simplified receiver, when $\rho = 0$. The exact SEP is then evaluated by simulations and is compared with the UB and the LB as a function of $\varsigma^2$, $\rho$ and $||\widetilde{H}||$.

*Theorem 2:* For a 4QAM modulation scheme and the case of $\rho = 0$, the SEP is bounded as follows (See [9] for proof):

$$Q\left(\frac{d(C_{3,2}, C_{1,1})}{\sigma_{\widetilde{N}}}\right) \leqslant P_s \leqslant \sum_{(p,q) \neq (1,1)} Q\left(\frac{d(C_{p,q}, C_{1,1})}{\sigma_{\widetilde{N}}}\right), \quad (5)$$

where $\Sigma_{\widetilde{N}} \triangleq \sigma^2 I_2 + C_{1,1}\widetilde{\Sigma}C_{1,1}^H$,

$$d(C_{p,q}, C_{1,1}) \triangleq \frac{\sigma^2 \widetilde{H}^H \mathrm{Re}\{(C_{1,1} - C_{p,q})^H C_{1,1}\}\widetilde{H}}{\varsigma^2 |C_{1,1}|},$$

$$\sigma_{\widetilde{N}}^2 \triangleq \frac{\sigma^4 \widetilde{H}^H \left(C_{p,q} - C_{1,1}\right)^H \Sigma_{\widetilde{N}} \left(C_{p,q} - C_{1,1}\right) \widetilde{H}}{\varsigma^4 |C_{1,1}|^2}. \quad \blacksquare$$

In this theorem, when $C_{1,1}$ is transmitted, $d\left(C_{p,q}, C_{1,1}\right)$ measures an approximate distance between the transmitted symbol, $C_{1,1}$ and a tentative detection outcome, $C_{p,q}$. And $\sigma_{\widetilde{N}}^2$ is the equivalent noise variance. This equivalent noise variance includes the effects of both additive noise and channel uncertainties. For example, if the channel is exactly known, *i.e.*, $\varsigma^2 = 0$, the function $d\left(C_{p,q}, C_{1,1}\right)$ becomes the Euclidian distance for the received constellations and $\sigma_{\widetilde{N}}^2 = \sigma^2$ represents the variance of the additive noise.

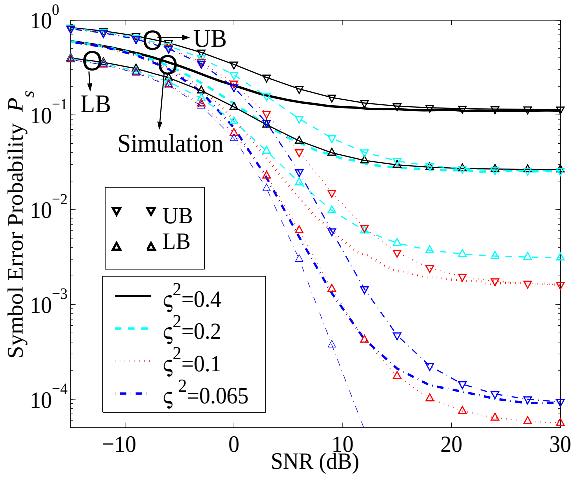


Fig. 3. The experimental SEP evaluation and the error bounds (5) for a 4QAM modulation for different values of the variance of the channel estimation error, $\varsigma^2$, when $\widetilde{H} = [1; 1]$.

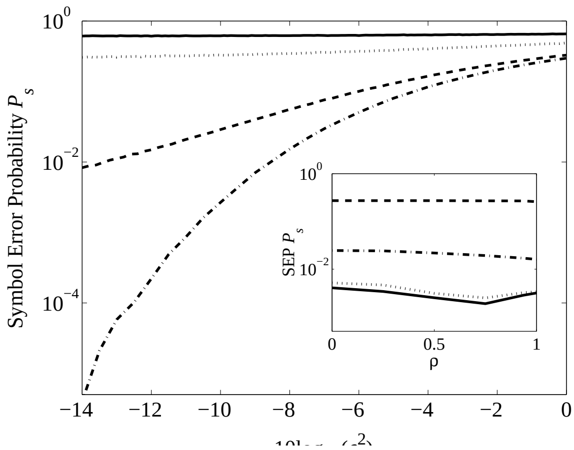


Fig. 4. The effect of $\varsigma^2$ on the SEP for different SNRs, when $\widetilde{H} = [1; 1]$ and $\rho = 0$; Solid: SNR = -10dB; Dotted: SNR = 0dB; Dashed: SNR = 10dB; Dashed-Dotted: SNR = 20dB. This also shows the effect of $\rho$ on SEP, when $\varsigma^2 = 0.12$ and $\widetilde{H} = [1; 1]$; Dashed: SNR = -5dB; Dashed-Dotted: SNR = 5dB; Dotted: SNR = 15dB; Solid: SNR = 25dB (small picture).

The function $d(C_{p,q}, C_{1,1})$ is a non-negative quadratic form of the *a priori* channel estimate $\widetilde{H}$ (because the matrix $\mathrm{Re}\{(C_{1,1} - C_{p,q})^H C_{1,1}\}$ is non-negative and $\frac{\sigma^2}{\varsigma^2 |C_{p,q}|} \geqslant 0$), *i.e.*, it represents an energy-type function of the known parts of the channel. This simplification means that the SEP reduces when the energy of the known parts of the channel increases or when the energy of the unknown parts reduces.

As the channel variation is modeled by a Gaussian pdf, provided that $S = C_{1,1}$, $R$ is the summation of two independent Gaussian vectors and therefore is Gaussian with a mean of $C_{1,1}\widetilde{H}$ and a covariance matrix $\sigma^2 I_2 + C_{1,1}\widetilde{\Sigma}C_{1,1}^H$. This covariance matrix includes the uncertainties caused by both channel imperfections and additive noise. To achieve a better understanding of the SEP and the above bounds, simulation results are illustrated in Figures 3 and 4, and the effects of different parameters are discussed in the following cases:

- $\varsigma^2 \to 0$: In this case, the channel is known, *i.e.*, $H = \widetilde{H}$ and the SEP bounds become identical with those of a known channel with AWGN, as expected (See Figure 3).
- $\varsigma^2 \to \infty$: This condition implies that no *a priori* information is provided about the channel coefficients, and the receiver detects randomly. This shows that either an initialization is required or a small subset of constellation points could be used to allow channel identification (See Figure 3).
- The SEP has a floor which is a function of $\varsigma^2$, because when the channel uncertainties diminish, the effect of the additive noise becomes dominant at certain points, as is evident in Figure 3. In other words, in low SNRs the additive noise limits the system performance, while in high SNRs the channel uncertainties mainly limit the performance (SNR$\stackrel{\Delta}{=} \frac{|S|(2\varsigma^2+||\widetilde{H}||^2)}{2\sigma^2}$). Therefore, it is very important to use efficient channel estimators. Figure 4 depicts the effect of the channel estimation error, $\varsigma^2$, on the SEP. It also verifies that the effect of $\varsigma^2$ is considerable in high SNRs while its effect is neglectable in low SNRs. This figure also shows that $\rho$ has a minor effect on SEP. To study the variation modes of $\sigma^2_{\widetilde{N}}$, which reflects the effect of additive noise and channel uncertainties, we consider the eigenvalues of $\Sigma_{\widetilde{N}}$ as follows:

$$\lambda_{1,2} = \sigma^2 + \varsigma^2|S|\left(1 \pm |\rho|\right). \tag{6}$$

  Looking at the larger eigenvalue, we see the effect of uncertainties in the worst case. Variations of $\lambda_{1,2}$ in the worst case of $|\rho| = 1$ show that the eigenvalues lie between $\sigma^2$ and $\sigma^2 + 2\varsigma^2\,|S|$. This intuitively means that the additive noise is dominant in small SNRs and the channel uncertainties are dominant in large SNRs.
- $\widetilde{H} \to 0$: This condition implies a Rayleigh fading channel. For a QAM scheme, the detection performance of the receiver will be very poor. In this case for an orthogonal modulation scheme, *i.e.*, $c_p^H c_q = \delta_{p,q}$ as in FSK, this receiver simplifies to a kind of non-coherent receiver. This case is formulated and studied in [8]. The system is not bandwidth-efficient in this scheme. However, this scheme (which involves reducing the size of the set of the alphabets) could be used as an important alternative to the training mode, allowing lower data rate communication during the training mode.
- $||\widetilde{H}|| \to \infty$: This condition implies a very strong Line of Sight (LOS) gain in the transmission. In this case, the bounds converge to zero and hence $P_s \to 0$, as is obviously expected.
- If $||\widetilde{H}||$ increases, the SEP reduces and the error floor of SEP occurs at higher SNRs. The bounds are also tighter when $||\widetilde{H}||$ is higher.
- Figure 4 also illustrates the effect of the correlation between the channel estimate coefficients, $\rho$, on the SEP. It is observed that in low SNRs, the SEP does not depend on $\rho$. This is easily justified in this case by considering (6) as $\lambda_{1,2} \simeq \sigma^2$ (See Figure 4 for SNR<0dB). In high SNRs, from (6) it is observed that $\lambda_{1,2} \simeq \varsigma^2|S|\left(1 \pm |\rho|\right)$. In this case, the SEP bounds do not vary greatly with variations of SNR (See Figure 4 for SNR>15dB). In this case, the SEP is determined by the channel uncertainties that are characterized by $\lambda_{1,2}$ and the average effect of $\rho$ is less than the impact of 3dB variations of $\varsigma^2$.

## III. Conclusions

In this paper an optimal MAP receiver for Transmission Diversity is proposed that takes channel uncertainties into account. The channel estimation error is assumed to be Gaussian. The performance of this detection algorithm is analyzed using bounds of Symbol Error Probability. Both the SEP curves obtained by simulations and the bounds indicate that the proposed algorithm is robust and simple. Such a receiver results in a considerable performance improvement in the presence of channel imperfections and is very suitable for use in fast fading environments in combination with an adaptive channel tracking algorithm [9, 10].

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