QPSK ORTHOGONAL SPACE-TIME CODING SCHEME WITH FULL-RATE AND FULL-DIVERSITY FOR SYSTEM WITH FOUR TRANSMIT ANTENNAS

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ABSTRACT

In this work, we present a novel space-time orthogonal coding scheme with full-rate and full-diversity on QPSK constellation for communication systems with four transmit antennas. We also study the performance of the newly proposed coding scheme in comparison with that of half rate full diversity orthogonal codes as well as full rate half diversity quasi-orthogonal codes. We demonstrate that the proposed coding scheme not only offers full rate but also outperforms the other two schemes when SNR increases.

1. INTRODUCTION

Diversity techniques are effective ways of combating channel fading and providing reliable system performance in wireless communications. To achieve spatial diversity, antenna arrays can be deployed at the transmitter and/or the receiver. However, considering the fact that receivers of mobile users are typically required to be small, it may not be practical to use multiple receiving antennas. Therefore, transmit diversity technique becomes a promising approach to achieve diversity in wireless communications. The first space-time block code, which is also the first full rate full diversity orthogonal code, was proposed by Alamouti for a system with two transmit antennas [1]. In [2], Tarokh *et al.* developed the construction criteria, and studied trade-offs between constellation size, data rates, diversity advantage and complexity. The general design of full diversity orthogonal space-time code was presented in [3]. The orthogonal codes proposed in [3] have a normalized rate of 1 symbol/s over real constellations; over complex constellations, codes achieving 1/2 and 3/4 rates are proposed for systems with three and four transmit antennas. Another obvious advantage of those codes proposed in [3] is that given the knowledge of channel, the maximum-likelihood (ML) decoding of the information can be achieved with a linear complexity. Unfortunately, it is proved that an orthogonal space-time code that can provide full-diversity and full transmission rate, and can be decoded by simple *linear decoder* does not exist for systems with more than two transmit antennas over complex constellation. Therefore, other researchers made trade-offs among orthogonality, rate, diversity and decoding complexity in their subsequent studies. Trading the code orthogonality and diversity for higher coding rates, a full-rate and half diversity code, the so called quasi-orthogonal codes, was proposed in [4], and similar code construction can be found in [5],[6]. To date there has no full-rate full-diversity orthogonal design been reported, that can be used in a system with more than two transmit antennas over complex constellation. In this work, we present a novel full-rate full-diversity orthogonal space-time coding scheme for systems employing four transmit antennas using QPSK constellation. The full rate full diversity potential is achieved at the cost of increased decoding complexity. Since the proposed code is not a linear processing orthogonal codes, there is no linear decoder for it. The ML detection has to be performed by exhaustive search over four dimensional QPSK vectors.

We also report the results on performance of the new code and compare it to those codes with full diversity and partial rates or full rates but partial diversity. Simulation results of the one receiver antenna system show that the performance of the new code is very close to that of the quasi-orthogonal code and is obviously better than that of the half rate orthogonal code when signal to noise ratio (SNR) is low (SNR$<$15 dB). However, as the SNR is high, full diversity codes work better and benefit more from SNR increasing than partial diversity code. The new code shows a performance gain about 2 dB at the bit error rate (BER) of 10^{-3}, compared to the half rate orthogonal code. When 2 transmit antennas are used, the novel code outperforms the quasi-orthogonal code at a even lower SNR (7.5 dB), and a 4 dB performance gain achieved at the bit error rate (BER) of 10^{-3}, compared to the half rate orthogonal code.

The organization of the paper is as follows. Section II provides a brief summary of space-time block codes. Section III proposes a new structure to design full-rate full-

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diversity orthogonal codes for QPSK signals transmitted by 4 transmit antennas. Simulation results and conclusions are presented in Section IV.

2. SPACE-TIME BLOCK CODES

Space-time block coding [1],[7], provides transmit diversity for a system with multiple transmit antennas in wireless communications. In general , a complex space-time block code is given by a $T \times N$ transmission matrix $\mathbf{C} \in \mathcal{G}$. Here, T represents the number of time slots for transmitting one block of symbols and N represents the number of transmit antennas.

2.1. Encoding

At first, the information bits are mapped to constellation symbols by digital modulation. Let $\mathcal{A}$ denote a signal constellation of cardinality 2^b. At each block, $b \times K$ bits are input into a digital modulator, hence mapped into K constellation symbols s_1, s_2, ... , s_K. Then, the block of K constellation symbols is mapped to one transmission matrix $\mathbf{C}$. Usually each element of $\mathbf{C}$ still belongs to $\mathcal{A}$. The nth element of the tth row of $\mathbf{C}$, c_{tn}, represents the signal transmitted by the nth transmit antenna at the tth time slot. Therefore, all the transmit antennas transmit simultaneously and all the transmitted symbols have the same time duration. The coding rate is therefore defined as $R = K/T$. If $K = T$, the code is termed full rate or rate 1 code.

2.2. Channel Model and Data Formulation

A wireless communication system with N transmit antennas at the base station and M receiving antennas at the mobile host is considered. The wireless channel is assumed to be quasi-static so that the path gains are constant over a frame, and vary from one frame to another (block fading channel). Within each block, the path gain coefficients from transmit antenna n to receive antenna m, $h_{n,m}$'s, are modeled as a normalized samples of independent complex Gaussian random variables, $h_{n,m} \sim \mathcal{CN}(0, 1)$.

At time slot t, the received signal at antenna m, $y_{t,m}$, is given by

$$y_{t,m} = \sum_{n=1}^{N} h_{n,m} c_{t,n} + v_{t,m}, \quad (1)$$

where the noise samples $v_{t,m}$ are spatially and temporally independent samples from a zero mean complex Gaussian random family, i.e. $v_{t,m} \sim \mathcal{CN}(0, 1/\text{SNR})$. Note that the average energy of the symbols transmitted from each antenna is normalized to be $1/N$. The average power of the received signal at each receiver antenna is normalized, so that the signal to noise ratio SNR=σ^{-2} is presented as the *effective* noise variance at each receiving antenna.

We can recast this model in an equivalent matrix form

$$\mathbf{y}_m = \mathbf{C}\,\mathbf{h}_m + \mathbf{v}_m, \quad (2)$$

where $\mathbf{y}_m$ and $\mathbf{v}_m$ are $T \times 1$ vectors obtained by stacking $r_{t,m}$ and $v_{t,m}$ during processing time slots of dimension T, respectively. $\mathbf{h}_m = \begin{bmatrix} h_{1,m} & \cdots & h_{t,m} \end{bmatrix}^T$ is the $N \times 1$ fading channel vector , where "T" denotes the transpose operator.

If all the M receiving antenna are considered the system model is

$$\mathbf{Y} = \mathbf{C}\,\mathbf{H} + \mathbf{V} \quad (3)$$

where $\mathbf{Y} = \{\ \mathbf{y}_1 \quad \mathbf{y}_2 \quad \cdots \quad \mathbf{y}_m\ \}$ is the $T \times M$ receive matrix, $\mathbf{H} = \{\ \mathbf{h}_1 \quad \mathbf{h}_2 \quad \cdots \quad \mathbf{h}_m\ \}$ is the $N \times M$ fading channel matrix, and $\mathbf{V}$ =$\{\ \mathbf{v}_1 \quad \mathbf{v}_2 \quad \cdots \quad \mathbf{v}_m\ \}$ is the $T \times M$ noise matrix.

2.3. Coherent Detection

Most work on space-time coding has assumed that perfect channel state information (CSI) is available at the receiver. It means that the receiver (but not the transmitter) knows the fading channel matrix $\mathbf{H}$. When $\mathbf{H}$ is known at the receiver, the pdf of the received data given that $\mathbf{C} \in \mathcal{G}$ was transmitted is,

$$p(\mathbf{Y}|\mathbf{H}, \mathbf{C}) = \frac{\exp(-\sigma^{-2}\text{tr}\{(\mathbf{Y} - \mathbf{C}\,\mathbf{H})(\mathbf{Y} - \mathbf{C}\,\mathbf{H})^H\})}{(\pi\sigma^2)^{TM}} \quad (4)$$

where "tr" is the trace, "H" is conjugate transpose.

Maximum-likelihood (ML) receiver reduces to the minimum Euclidean distance detector, i.e.

$$\begin{aligned} \widehat{\mathbf{C}} &= \arg\max_{\mathbf{C}}\ p(\mathbf{Y}|\mathbf{H}, \mathbf{C}) \quad (5) \\ &= \arg\max_{\mathbf{C}}\ \text{tr}\{(\mathbf{Y} - \mathbf{C}\,\mathbf{H})(\mathbf{Y} - \mathbf{C}\,\mathbf{H})^H\} \end{aligned}$$

For unit-energy design, $\mathbf{C}^H\mathbf{C} = \mathbf{I}_{N \times N}$, Eq. (5) can be simplified as,

$$\widehat{\mathbf{C}} = \arg\max_{\mathbf{C}} \mathcal{R}e\left\{\sum_{m=1}^{M} \mathbf{y}_m^H \mathbf{C}\,\mathbf{h}_m\right\}. \quad (6)$$

3. NEW SCHEME OF FULL RATE FULL DIVERSITY ORTHOGONAL SPACE-TIME BLOCK CODE

3.1. Alamouti Code and Quasi-Orthogonal Code

Alamouti scheme is an example of full-rate full-diversity complex space-time block code. The scheme can be defined by the following transmission matrix, so called Alamouti matrix[1]:

$$\mathbf{C} = \begin{pmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{pmatrix} \quad (7)$$

It is the only case for complex linear processing orthogonal design[3]. It means that, if we try to design a complex orthogonal transmission matrix $\mathbf{C}_{N\times N}$ with entries chosen from $\pm s_1$, $\pm s_2$, ..., $\pm s_N$, and their conjugates $\pm s_1^*$, $\pm s_2*$, ..., $\pm s_N*$, or multiples of these by $\pm j$ where $j = \sqrt{-1}$, the design exists if and only if $N = 2$. In [3], the full-rate full-diversity real linear processing orthogonal design and partial-rate full-diversity complex linear processing orthogonal design for 4 and 8 transmit antenna scenario are also provided.

As an example of full-rate partially diversity scheme, Jafarkhani proposed a so called quasi-orthogonal space-time block code for a system with four transmit antenna system in [4]. The scheme was defined by the following transmission matrix,

$$\mathbf{C} = \begin{pmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ -s_3^* & -s_4^* & s_1^* & s_2^* \\ s_4 & -s_3 & -s_2 & s_1 \end{pmatrix} = \begin{pmatrix} \mathbf{C}_1 & \mathbf{C}_2 \\ -\mathbf{C}_2^* & \mathbf{C}_1^* \end{pmatrix} \tag{8}$$

This kind of code achieves a diversity of $2M$, instead of full diversity $4M$, while the rate of the code is one.

3.2. Full-Rate Full-Diversity Orthogonal Code for QPSK Signal

It has been proved in [3] that the maximum diversity of $4M$ for a rate one complex *linear* processing orthogonal code is impossible in a 4 transmit antenna system. However, the possibility of constructing orthogonal space-time block code by *non-linear* processing has not been studied. Without the linear processing constrains, the full-rate full-diversity orthogonal codes may be find for complex symbols of some special constellations.

In this work, we present the design of the QPSK symbols transmitted through a system with 4 transmit antennas. The goal in our design is to achieve full rate and full diversity, while remaining the orthogonality of code. Now, let us consider the following space-time block code for the choice of system parameters: $N = T = K = 4$. We construct a novel transmission matrix as follows

$$\begin{aligned} \mathbf{C} &= \frac{1}{\sqrt{2}} \begin{pmatrix} \mathbf{C}_1 & \mathbf{C}_2 \\ -\mathbf{C}_1^H \mathbf{C}_2^H \mathbf{C}_1 & \mathbf{C}_1^H \end{pmatrix} \\ &= \frac{1}{\sqrt{2}} \begin{pmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ -x_1^* & x_2 & s_1^* & -s_2 \\ -x_2^* & -x_1 & s_2^* & s_1 \end{pmatrix}, \end{aligned} \tag{9}$$

where s_i belongs to the signal constellation $\mathcal{A}$, defined as $\mathcal{A} = \left\{\frac{e^{j(k\pi/2+\pi/4)}}{\sqrt{2}}\right\}_{k=0}^{3}$, and $x_1 = \mathcal{R}e\{s_3\} - j\mathcal{I}\text{m}\{2s_1s_2s_4^*\}$, $x_2 = s_1^{*2}s_4 + s_2^2 s_4^* + s_1^* s_2 s_3 - s_1^* s_2 s_3^*$.

If $s_i \in \mathcal{A}$, obviously $x_1 \in \mathcal{A}$. Then we prove $x_2 \in \mathcal{A}$. Generally, let $s_i = e^{j(\frac{k_i}{2}\pi)} s_1$, where $k_i = 1, 2, 3, 4$ and $k_1 = 0$. Simple manipulation provides following formulas for x_2:

$$\begin{aligned} x_2 &= |s_1|^2 (e^{j\frac{k_4}{2}\pi} s_1^* + e^{j\frac{2k_2-k_4}{2}\pi} s_1 + e^{j\frac{k_2+k_3}{2}\pi} s_1 \\ &\quad - e^{j\frac{k_2-k_3}{2}\pi} s_1^*) \\ &= \frac{1}{2} e^{j\frac{k_2}{2}\pi} (e^{j\frac{k_4-k_2}{2}\pi} s_1^* + e^{j\frac{k_2-k_4}{2}\pi} s_1 + e^{j\frac{k_3}{2}\pi} s_1 \\ &\quad - e^{j\frac{-k_3}{2}\pi} s_1^*) \\ &= e^{j\frac{k_2}{2}\pi} \left(\text{Re}\left\{ e^{j\frac{k_2-k_4}{2}\pi} s_1 \right\} + j\,\text{Im}\left\{ e^{j\frac{k_3}{2}\pi} s_1 \right\} \right) \end{aligned} \tag{10}$$

From (10), it is clear $x_2 \in \mathcal{A}$. Therefore, our coding process does not expand the QPSK constellation.

Since x_1 =Re$\{s_3\} - j$Im$\{2s_1s_2s_4^*\}$, and s_3 is independent to $s_1s_2s_4^*$, to avoid the expansion of constellation, signal formed by any combination of possible real part and imaginary part of symbols from certain constellation should still belong to that constellation. Hence, we conclude that QPSK signal is the only possible complex MPSK signal, that has the full-rate orthogonal design by our scheme. Note that for BPSK constellation, using our design scheme, we have $s_i = \pm\sqrt{2}/2$, $x_1 = s_3$, $x_2 = s_4$, which simply yields the rate one orthogonal code with *linear processing* developed in [3].

The diversity property of the new code can be examined by testing the rank conditions of all the possible matrices $\mathbf{C}(s_1 - \tilde{s}_1, s_2 - \tilde{s}_2, s_3 - \tilde{s}_3, s_4 - \tilde{s}_4)$. The non-singularity of all possible $\mathbf{C}(s_1 - \tilde{s}_1, s_2 - \tilde{s}_2, s_3 - \tilde{s}_3, s_4 - \tilde{s}_4)$ confirms that the new proposed code indeed provides full transmit diversity over a quasi-static fading channel [2].

Since the new scheme is also a unit-energy design, to reduce decoding complexity, we can use Eq.(6) to realize ML detection instead of using Eq.(5).

4. SIMULATION RESULTS AND CONCLUSIONS

In this section, we provide simulation results for the proposed new space-time code in Eq. (9) and compare it to the results for the half-rate orthogonal codes [3] and quasi-orthogonal codes presented in [4], respectively. Fig.1 provides simulation results for the transmission of 2/bits/s/Hz by 4 transmit antennas and 1 receiving antenna using QPSK modulated full-rate full-diversity code and full-rate quasi-orthogonal code, and 16 QAM modulated half rate full-diversity code.

Simulation results from Fig.1show that the performance of the new code is close to that of the quasi-orthogonal code and is better that of the half rate orthogonal code when signal to noise ratio (SNR) is low. However, as the SNR increases, full diversity codes work better and benefit more from SNR increase than partial diversity code. The new

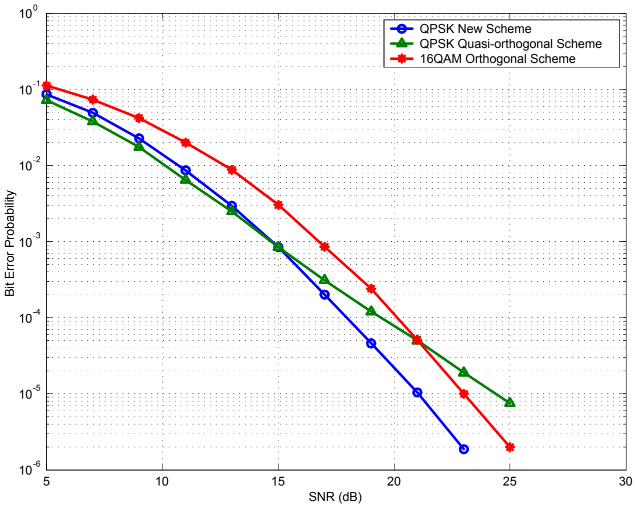

Fig. 1. Bit-error probability versus SNR for space-time block codes at 2 bits/s/Hz. 4 transmit antennas and 1 receive antenna are used.

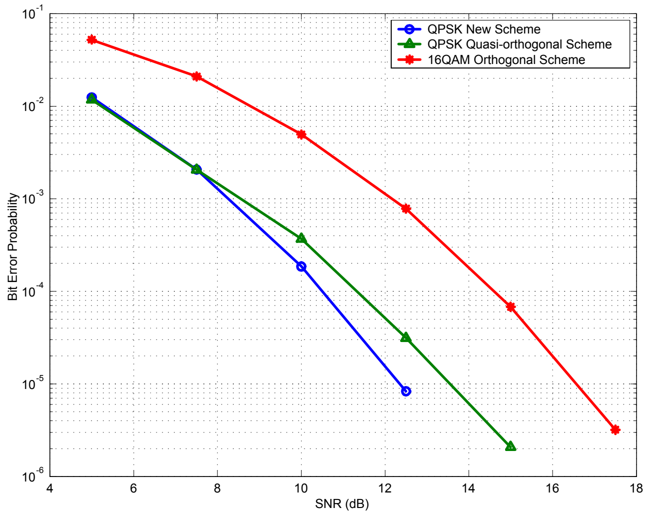

Fig. 2. Bit-error probability versus SNR for space-time block codes at 2 bits/s/Hz. 4 transmit antennas and 2 receive antennas are used.

space-time coding scheme demonstrates a performance gain about 2 dB at the bit error rate (BER) as 10^{-3}, compared to the half rate orthogonal code. Since the degree of diversity is reflected by the slope of the BER-SNR curve at high SNR, the similar slopes of the BER-SNR curve for new code and the full-diversity half-rate code at high SNR also prove the full diversity of new proposed code.

Fig.2 provides simulation results for the transmission of 2/bits/s/Hz by 4 transmit antennas and two receiving antenna using the proposed QPSK modulated full-rate full-diversity code and full-rate quasi-orthogonal code, and 16 QAM modulated half rate full-diversity code. When two transmit antennas are used, the novel code outperforms the quasi-orthogonal code at a even lower SNR (7.5 dB), and a 4 dB performance gain is achieved at the bit error rate (BER) of 10^{-3}, compared to the half rate orthogonal code.

This work demonstrate the existence of a QPSK full-rate full-diversity space-time orthogonal coding for a system with four transmit antennas.

It should be pointed out that the receiver of the half-rate full-diversity codes can decode the symbols one by one, and that of the full-rate half-diversity quasi-orthogonal codes can decode the symbols pair by pair [4]. This means that the full rate full diversity potential of the proposed space-time coding scheme is achieved at the cost of increased decoding complexity. For QPSK the decoding complexity of new orthogonal codes is nearly 8 times of that of the quasi-orthogonal codes. The encoding complexity of the proposed orthogonal codes is a little higher than that of the other two, although all the codes have very low encoding complexity.

5. REFERENCES

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