CONCATENATED SPACE-TIME CODING WITH OPTIMAL TRADE-OFF BETWEEN DIVERSITY AND CODING GAINS

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ABSTRACT

We propose a flexible method for designing space-time block codes capable of achieving the desired trade-off between diversity and coding gain. The proposed system is valid for frequency selective, block fading channels and refers to a block coding scheme capable of achieving full rate transmission, for any number of transmit antennas. We derive a closed form expression for the pairwise error probability and the maximum diversity and coding gain. These expressions are instrumental to design the coding strategy able to yield the required trade-off between coding and diversity gain, in order to reach the desired average BER with the smallest SNR. Finally, we check our theoretical derivations with simulations and compare our approach with alternative ones.

1. INTRODUCTION

Since the introduction of space-time coding methods, many efforts have been devoted to finding out coding strategies able to achieve the maximum diversity gain and then consider the coding gain as a by-product of the design. This approach is certainly meaningful, as the diversity gain controls the slope of the average bit error rate (BER) curves, at high SNR. However, especially when the maximum potential diversity gain is high, the maximum slope is often observed at high SNR, where the average BER can be unnecessarily low. In principle, transmitting over a frequency selective channel with $L+1$ multiple rays, using T transmit and R receive antennas, the diversity gain can be made equal to the product $G_d = TR(L+1)$. Clearly, this number can be quite high, even for small values of T and R. However, since the ultimate performance parameter, at the physical layer level, is BER, in many situations, it would be better to have a lower diversity gain, but a higher coding gain, so that the desired average BER could be achieved with a lower SNR. The aim of this work is precisely to propose a coding strategy able to achieve the desired balance between coding and diversity gain.

We start with the space-time coding scheme proposed in [2], as a method able to reach the maximum diversity gain, with full rate for any number of transmit antennas, with affordable receiver complexity. However, differently from [2], where the coding method is linear, we use a nonlinear encoding strategy to increase the coding gain. We also borrow some ideas of [1] about the optimal coding strategy over Rayleigh flat fading channels. In [1] it was proved that, as the number of receive antennas increases, the optimal space-time coding matrix can be built by encoding the information bits using off-the-shelf error correction codes (ECC) and arranging the resulting coded stream into a matrix whose size is dictated by the number of transmit elements. Building on [2] and [1], we propose a novel coding strategy that concatenates the symbols transmitted over successive blocks in order to achieve the desired trade-off between coding and diversity gain. We derive a closed form expression for the pairwise error probability, coding and diversity gains, which will provide the guideline for the design of our coding strategy.

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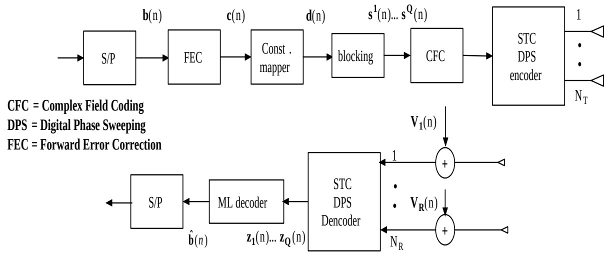

Fig. 1. Block coding scheme.

2. SYSTEM MODEL

We consider a block transmission system with T transmit and R receive antennas and we make the following assumptions: (a1) each discrete-time channel is described by an FIR filter of maximum order L, whose coefficients are zero-mean complex Gaussian random variables (Rayleigh fading model), with $L = \lceil \frac{\tau_{max}}{T_s} \rceil$, where τ_{max} is the maximum delay spread of the $T\,R$ channels; T_s is the symbol sampling period and $\lceil\,\rceil$ stands for integer-ceiling; (a2) all channels are time-invariant within each transmitted block; (a3) transmitters and receivers are synchronous and channel state information (CSI) is available only at the receiver; (a4) the received data are degraded by zero-mean complex additive white Gaussian noise (AWGN). The discrete-time baseband equivalent model of our Concatenated-Space-Time-Coded (CSTC) system is depicted in Fig.1. The input information bit stream is parsed in blocks of L_b bits $\boldsymbol{b}(n) := [b_1(n), \ldots, b_{L_b}(n)]$, where n is the block index. Each block is then mapped onto a sequence of L_c coded bits $\boldsymbol{c}(n) := [c_1(n), \ldots, c_{L_c}(n)]$. The sequence $\boldsymbol{c}(n)$ feeds then a constellation mapper which associates to each set of $\log_2 \Omega$ bits a complex QAM symbol belonging to a finite alphabet $\mathscr{A}$ with cardinality Ω. The result of this mapping is a sequence of $N_s := L_c/\log_2 \Omega$ complex symbols $\boldsymbol{d}(n) \in \mathscr{D}$, where $\mathscr{D}$ is the set of all possible vectors $\boldsymbol{d}(n)$. The elements of $\boldsymbol{d}(n)$ are then distributed among a set of Q vectors $\{\boldsymbol{s}^1(n), \ldots, \boldsymbol{s}^Q(n)\}$, each one of length $M := N_s/Q$ with $M > L$. Next, we introduce the matrices $\boldsymbol{S}(n) := [\boldsymbol{s}^1(n), \ldots, \boldsymbol{s}^Q(n)]$ and denote by $\mathscr{S}$ the set of all possible matrices $\boldsymbol{S}(n)$[1]. Then we apply a linear complex-field (CF) precoding to $\boldsymbol{S}(n)$, multiplying each vector $\boldsymbol{s}^q(n)$ by the $N \times M$ matrix $\boldsymbol{F}^q$, with $N \geq M$ and $q = 1, 2, \dots Q$.

The key idea underlying our coding strategy is that the non-linear encoder concatenates the blocks $\boldsymbol{s}^q(n)$ in a way that yields the desired trade-off between coding and diversity gain. But the concatenation law and the choice of the matrices $\boldsymbol{F}^q$ will be made clear only after we will have derived, in Section 3, a closed form expression for the pairwise error probability.

Each block $\boldsymbol{s}^q(n)$ passes through a space-time encoder implementing the space-time digital phase sweeping (ST-DPS) encoder proposed in [2], which we briefly recall here. As opposed to the orthogonal space-time encoder, ST-DPS exists for any number of transmit antennas and does not incur in any rate loss when $T > 2$ [2]. In ST-DPS, each vector $\boldsymbol{F}^q \boldsymbol{s}^q(n)$ is sent to all T transmit antennas after multiplication by proper precoding matrices. More specifically, the n-th block transmitted by the t-th antenna is $\boldsymbol{x}_t^q(n) := OFDM\{\frac{1}{\sqrt{T}}\boldsymbol{P}_t \boldsymbol{F}^q \frac{\boldsymbol{s}^q(n)}{\sqrt{\gamma}}\}$, where γ is a normalization factor used to enforce the desired transmit power, $\boldsymbol{P}_t = diag\left(1, e^{j\Phi_t}, \dots, \ e^{j\Phi_t(N-1)}\right)$, with $\Phi_t = -2\pi(t-1)(L+1)/N \ \forall t \in [1,\ T]$, and the symbol $OFDM\{\boldsymbol{x}\}$ indicates an Orthogonal Frequency Division Multiplexing (OFDM) processor which computes the IFFT of $\boldsymbol{x}$ and appends a cyclic prefix (CP) at the beginning of the resulting vector. We recall from [2] that the above choice of the matrices $\{\boldsymbol{P}_t\}_{t=1}^T$ is used to map the TR frequency selective $(L+1)$-length channels into R equivalent ones with length $T(L+1)$.

At the receiver, after discarding the cyclic prefix and applying an FFT, using (a1)...(a4), the block $\boldsymbol{y}_r^q$ from the r-th antenna in the q-th slot is given by [2]

$$\boldsymbol{y}_r^q = \frac{1}{\sqrt{T\gamma}} \sum_{t=1}^{T} \boldsymbol{\Lambda}_{r\,t}^q \boldsymbol{P}_t \boldsymbol{F}^q \boldsymbol{s}^q + \boldsymbol{v}_r^q, \quad r = 1, 2, \dots R. \tag{1}$$

where: $\boldsymbol{\Lambda}_{r\,t}^q$ is the $N \times N$ diagonal matrix whose diagonal entries are the samples of the channel transfer function between the t-th transmit and the r-th receive antenna in the q-th slot, i.e. $\{\boldsymbol{\Lambda}_{r\,t}^q\}_{kk} = \sum_{l=0}^{L} h_{r\,t}^q(l) \exp(-j\,2\pi kl/N)$; $\{h_{r\,t}^q(l)\}_{l=0}^L$ denotes the impulse response of the above channel in the q-th slot; $\boldsymbol{v}_r^q$ is a N-length additive gaussian noise vector, with zero mean and covariance matrix $\boldsymbol{C}_v = \sigma_v^2 \boldsymbol{I}_N$; $\boldsymbol{I}_N$ is the $N \times N$ identity matrix. Denoting by $\boldsymbol{h}_r^q := \left[\boldsymbol{h}_{r\,1}^{q^T}, \dots, \boldsymbol{h}_{r\,T}^{q^T}\right]^T$ the equivalent $T(L+1)$-length channel vector corresponding to the r-th receive antenna and by $\boldsymbol{W}^H$ the $N \times T(L+1)$ FFT matrix with elements $\{\boldsymbol{W}^H\}_{kk} := \exp(-j2\pi kl/N)\,/\sqrt{N}$, it is straightforward to check [2] that, for $r \in [1,\ R]$[3], $\sum_{t=1}^{T} \boldsymbol{\Lambda}_{r\,t}^q\ \boldsymbol{P}_t = diag(\boldsymbol{W}^H \boldsymbol{h}_r^q) := \boldsymbol{\Lambda}_r^q$, where $\boldsymbol{\Lambda}_r^q$ is the $N \times N$ diagonal matrix whose diagonal entries are $\{\boldsymbol{\Lambda}_r^q\}_{kk} := \sum_{l=0}^{L} h_r^q(l)\ \exp(-j\,2\pi kl/N)$. The input-output relationship (1) between the block $\boldsymbol{s}_q$ transmitted in the q-th slot and the received one from the r-th antenna may thus be rewritten as

$$\boldsymbol{y}_r^q = \frac{1}{\sqrt{T\gamma}} \boldsymbol{\Lambda}_r^q \boldsymbol{F}^q \boldsymbol{s}^q + \boldsymbol{v}_r^q, \quad r = 1, 2, \dots R. \tag{2}$$

Since the entries of $\boldsymbol{v}_r^q$ are i.i.d random gaussian variables $\forall r \in [1,\ R]$, we can apply a maximal ratio combiner (MRC) on $\boldsymbol{y}_r^q$, without loosing any information about $\boldsymbol{s}_q$, so as to get:

$$\boldsymbol{z}^q := \sum_{r=1}^{R} \boldsymbol{\Lambda}_r^{q^*} \boldsymbol{y}_r^q = \frac{1}{\sqrt{T\gamma}} \boldsymbol{D}_h^q \boldsymbol{F}^q \boldsymbol{s}^q + \widetilde{\boldsymbol{v}}^q, \quad q = 1, 2, \dots Q. \tag{3}$$

where $\boldsymbol{D}_h^q := \sum_{r=1}^{R} |\boldsymbol{\Lambda}_r^q|^2$ and $\widetilde{\boldsymbol{v}}^q := \sum_{r=1}^{R} \boldsymbol{\Lambda}_r^{q^*} \boldsymbol{v}_r^q$. To recover the information bits in the n-th block, we need to stack the Q blocks $\boldsymbol{z}^q$. Introducing the vector $\boldsymbol{z} := [\boldsymbol{z}^{1^T}, \dots,\ \boldsymbol{z}^{Q^T}]^T$ and the block diagonal matrices $\widetilde{\boldsymbol{\mathcal{F}}} := diag\left(\mathbf{F}^1, .., \mathbf{F}^Q\right)$ of dimension $Q\,N \times N_s$; $\boldsymbol{D}_h := diag\left(\mathbf{D}_h^1, .., \mathbf{D}_h^Q\right)$ of dimension $Q\,N \times Q\,N$; $\boldsymbol{S} := (\boldsymbol{s}^1, \dots, \boldsymbol{s}^Q)$; and $\boldsymbol{V} := (\widetilde{\boldsymbol{v}}^1, \dots,\ \widetilde{\boldsymbol{v}}^Q)$, we can rewrite (3) as

$$\mathbf{z} = \frac{1}{\sqrt{T\gamma}} \boldsymbol{D}_h \widetilde{\boldsymbol{\mathcal{F}}} vec\,(\mathbf{S}) + vec\,(\mathbf{V})\,. \tag{4}$$

To collect the full diversity gain, ML decoding is needed. Denoting with $\boldsymbol{S} = \mathcal{E}(\boldsymbol{b})$ the nonlinear mapping relating the information bits with the transmitted blocks, the decoder chooses the vector $\hat{\boldsymbol{b}}$, composed of L_b bits, such that

$$\hat{\boldsymbol{b}} = \mathrm{argmin} \|\boldsymbol{z} - \frac{1}{\sqrt{T\gamma}} \boldsymbol{D}_h \widetilde{\boldsymbol{\mathcal{F}}} vec\left(\mathcal{E}(\hat{\boldsymbol{b}})\right) \|_{\boldsymbol{C}_V^{-1}}^2, \tag{5}$$

where the symbol $\|\boldsymbol{x}\|_{\boldsymbol{A}}^2$ denotes $\boldsymbol{x}^H \boldsymbol{A} \boldsymbol{x}$ and $\boldsymbol{C}_V = \sigma_v^2 \boldsymbol{D}_h$ is the covariance matrix of $vec(\boldsymbol{V})$.

3. PERFORMANCE AND DESIGN CRITERIA

We characterize the performance of our transmission system by deriving a closed form expression for the maximum achievable diversity and coding gains and an upper bound for the average pairwise error probability (PEP) $\mathscr{P}\left(\mathbf{S} \to \mathbf{S}'\right)$ that the matrix $\mathbf{S}$ is transmitted but erroneously decoded as $\mathbf{S}' \neq \mathbf{S}$. We preventively introduce the following notation: $\mathscr{S}_E := \{\boldsymbol{E} := \boldsymbol{S} - \boldsymbol{S}^{'} | \boldsymbol{S} \neq \boldsymbol{S}^{'}, \boldsymbol{S}, \boldsymbol{S}^{'} \in \mathscr{S}\}$ is the set of all possible error events; $\rho(\boldsymbol{d}, \overline{\boldsymbol{d}})$ is the *Hamming distance* between the vectors $\boldsymbol{d}$ and $\overline{\boldsymbol{d}}$; $d_H := \min\{\rho(\boldsymbol{d}, \overline{\boldsymbol{d}}) | \boldsymbol{d} \neq \overline{\boldsymbol{d}} \in \mathscr{D}\}$[4] is the minimum Hamming distance; $d_{min} := \min\{|d_k - d_j| \,|d_k \neq d_j \in \mathscr{A}\}$ is the minimum *Euclidean distance* between two symbols of the constellation with alphabet $\mathscr{A}$; $\boldsymbol{h}^q := \left[\boldsymbol{h}_1^{qT}, \dots \boldsymbol{h}_R^{qT},\right]^T$ is the $T\,R(L+1)$-length vector containing the R *elongated* channel impulse responses. We use also the eigenvalue decomposition of the channel autocorrelation matrix $\boldsymbol{R}_h^q := E\{\boldsymbol{h}^q \boldsymbol{h}^{qH}\} := \boldsymbol{U}_h^q \boldsymbol{\Psi}_h^q \boldsymbol{U}_h^{qH}$, where $\boldsymbol{U}_h^q$ is a $TR(L+1) \times r_h^q$ para-unitary matrix, $\boldsymbol{\Psi}_h^q$ is a $r_h^q \times r_h^q$ diagonal matrix whose diagonal entries are the non-zero eigenvalues of $\boldsymbol{R}_h^q$ arranged in non-increasing order and r_h^q is the rank of $\boldsymbol{R}_h^q$. Finally, we define the $TR(L+1) \times r_e^q$ error matrix $\boldsymbol{A}_e^q := (\boldsymbol{I}_R \otimes \boldsymbol{D}_e^q \boldsymbol{V}) \boldsymbol{U}_h^q \boldsymbol{\Psi}_h^{q\frac{1}{2}}$, where $\boldsymbol{D}_e^q := diag(\boldsymbol{F}^q \boldsymbol{e}^q)$, $\boldsymbol{e}_q$ is the q-th column of the error matrix $\boldsymbol{E}$, $\boldsymbol{V}$ is a $N \times T(L+1)$ Vandermonde matrix with $[\boldsymbol{V}]_{ik} = \exp(-j2\pi ik/N)$, and $\otimes$ denotes the Kronecker product.

3.1. Pairwise error probability

Assuming perfect CSI at the receiver and high SNR (i.e. high values of $\frac{1}{\gamma T \sigma_v^2}$), we have computed the upper bound of the PEP, using

[1] We assume here, for simplicity, that N_s/Q is even integer. In case this assumption does not hold true, we can group together a set of s matrices, with s such that sN_s/Q is even integer, and proceed on the new matrix as we do here with $\boldsymbol{S}(n)$.

[2] We drop the index n for simplicity of notation, since all processing refers to the same information block.

[3] To avoid the leakage effect, it has to be $N \geq T(L+1)$.

[4] Clearly d_H is less or equal to the minimum Hamming distance of the adopted GF code.

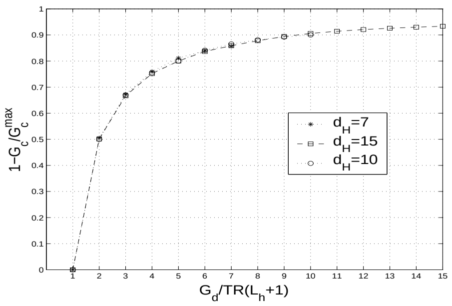

Fig. 2. G_c loss vs. $G_d^{max}/(TR(L+1)) = Q$.

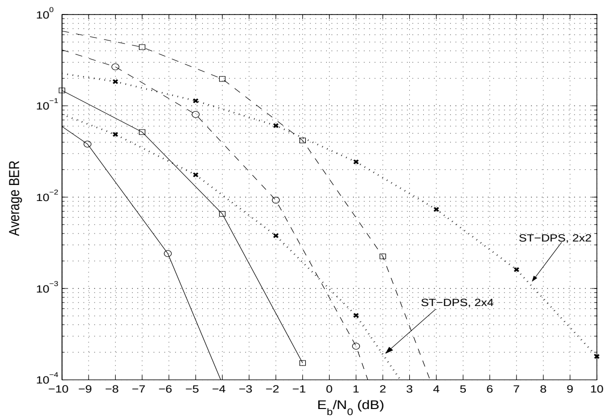

Fig. 3. Average BER of our C-STC method (dashed lines) vs. ST-DSP (dotted line); PEP curves (solid line) vs. E_b/N_0 (dB).

the same approach as in [3], and the result is

$$\wp\left(\mathbf{S}\to\mathbf{S}'\right)\leq\left(\frac{1}{\gamma\sigma_v^2}\right)^{-G_{d,e}}\left(\prod_{q=1}^{Q}\prod_{l=0}^{r_e^q-1}\lambda_e^q(l)/T\right)^{-1}=\left(\frac{1}{\gamma\sigma_v^2}G_{c,e}\right)^{-G_{d,e}}$$

where $\{\lambda_e^q(l)\}_{l=1}^{r_e^q}$ are the non increasing eigenvalues of the matrix ${\boldsymbol{A}_e^q}^H\boldsymbol{A}_e^q$; r_e^q is the rank of $\boldsymbol{A}_e^q$; $G_{d,e} := \prod_{q=1}^Q r_e^q$ is the diversity gain and $G_{c,e} := 1/T\left(\prod_{q=1}^Q \prod_{l=0}^{r_e^q-1}\lambda_e^q(l)\right)^{1/G_{d,e}}$ denotes the coding gain for the symbol error event $\boldsymbol{E}$. Since $G_{d,e}$ and $G_{c,e}$ depend on the choice of $\boldsymbol{E}$, we define the diversity and coding gains of our system as $G_d = \min_{\boldsymbol{E}\in\mathscr{S}_e} G_{d,e}$ and $G_c = \min_{\boldsymbol{E}\in\mathscr{S}_e} G_{c,e}$, respectively. The maximum achievable diversity order is $G_d^{max} = QTR(L+1)$ and it is reached iff the channel correlation matrices $\{\boldsymbol{R}_h^q\}_{q=1}^Q$ and the error matrices $\{\boldsymbol{A}_e^q\}_{q=1}^Q$ are full rank. It is interesting to observe that the maximum value of the diversity gain is given by the product of three factors: i) the space diversity, given by the product of the number of transmit and receive antennas; ii) the multipath diversity, equal to the number of independent paths $L+1$; iii) the time diversity, given by the number of independent channel fading blocks Q over which we span our n-th coded block. Under the maximum diversity gain, the maximum coding gain is (see the appendix): [5]

$$G_c^{max} = |\boldsymbol{R}_h|^{1/TR(L+1)} f_{d_H}(Q)\frac{d_{min}^2}{T}, \quad (6)$$

where $f_{d_H}(Q)=\left(\prod_{q=0}^Q\left\lceil\frac{d_H-q}{Q}\right\rceil\right)^{1/Q}\in[1,\,d_H]$ and $Q\in[1,\,d_H]$. Note that $f_{d_H}(d_H)=1$ and $f_{d_H}(1)=d_H$. From G_d^{max} and G_c^{max} expressions, we infer that: i) the upper bound of the coding gain depends on both the Hamming distance (d_H) of the chosen ECC as well as on the minimum Euclidean distance (d_{min}) depending on the chosen constellation; ii) for a given CF and ECC coding strategy, there exists a tradeoff between the maximum achievable coding and diversity gains. To better understand the relationship between them, in Fig. 2, we plot the G_c loss, defined as $(G_c^{max}-G_c)/G_c^{max}$, as a function of $G_d^{max}/(TR(L+1)) = Q$ for $d_H = 7, 10, 15$. Interestingly, we can see that the G_c loss is almost exactly inversely proportional to G_d^{max}.

3.2. Design Criteria

We have several degrees of freedom for designing our system: i) the number of blocks Q; ii) the ECC coding; iii) the strategy for mapping the elements of the ECC coded vector $\boldsymbol{d}(n)$ within the blocks $\boldsymbol{s}^q(n)$; iv) the constellation structure, and v) the precoding matrices $\{\boldsymbol{F}^q\}_{q=1}^Q$. We will adopt the PEP bound[6] and the closed form expression of the coding gain as the guidelines for choosing the system's parameters in order to satisfy the requirements. Specifically our design criterion is the following.

1. We start with a certain d_H and choose the number $Q\in[1,d_H]$ of blocks over which we want to spread our concatenated code. Then, we insure the maximum diversity gain $G_d = QTR(L+1)$. A sufficient, although not necessary, criterion for guaranteeing maximum diversity is that, for each Q, the matrices $\{\boldsymbol{A}_e^q\}_{q=1}^Q$ must be full rank. Since $rank\{\boldsymbol{A}_e^q\} = rank\{\boldsymbol{D}_e^q\}$, this implies that, for every error event $\boldsymbol{E}\in\mathscr{S}_e$, the vectors $\{\boldsymbol{F}^q\boldsymbol{e}^q\}_{q=1}^Q$ must have all non-zero entries. To insure that, it is sufficient that: i) for each given ECC strategy, we accept, among the different alternatives for distributing the symbols $\boldsymbol{d}(n)$ among the blocks $\{\boldsymbol{s}^q\}_{q=1}^Q$, only those strategies such that each error matrix $\boldsymbol{E}\in\mathscr{S}_e$ has all columns with at least one nonzero entry[7]; ii) the CF coding matrices $\{\boldsymbol{F}^q\}_{q=1}^Q$ should be chosen such that the vector $\boldsymbol{F}^q\boldsymbol{e}^q$ contains nonzero entries for all $\boldsymbol{e}^q\neq 0$, $\forall q\in[1,Q]$. This problem has been already solved in [5], where a class of redundant constellation-independent precoders is given, and in [4] where a class of non-redundant, but constellation-dependent precoders is suggested. Using a Vandermonde redundant CF precoding, the maximum diversity is guaranteed and the achievable coding gain belongs to the following interval (see Appendix):

$$|\boldsymbol{R}_h|^{1/TR(L+1)}\frac{d_{min}^2}{T}\leq G_c\leq|\boldsymbol{R}_h|^{1/TR(L+1)} f_{d_H}(Q)\frac{d_{min}^2}{T}, \quad (7)$$

Note that, if $Q=d_H$, $f_{d_H}(d_H) = 1$ and then G_c must coincide with the inferior extreme of (7). However, if we take $d_H\geq Q$, the interval (7) increases and higher values of G_c can be reached by properly choosing the ECC.

2. To achieve the required coding gain, we choose the ECC parameter (d_H), the modulation order (d_{min}) and the number of blocks Q, according to (7) and to the tradeoff between spectral efficiency

[5]We have assumed that the channels have the same statistical properties during the transmission of Q blocks, that is $\boldsymbol{R}_h^q = \boldsymbol{R}_h, \forall q\in[1,Q]$.

[6]In the next section we will prove, via simulation, that the PEP curve follows quite closely the simulation results and thus it is indicative of the average BER.

[7]In [3] it has been shown that, for each ECC, such a mapping strategy always exists.

and diversity gain. Then, the specific ECC can be chosen according to the hierarchy defined by the PEP curve versus SNR, as will be clarified with the simulation results illustrated in the next section.

4. SIMULATION RESULTS

In Fig. 3, we compare the PEP (solid line) and the average BER of our proposed C-STC method (dashed line) versus E_b/N_0, where E_b is the average bit energy at the transmitter and N_0 is the power spectral density of the AWGN at the receiver. We have adopted the following system parameters: $T = R = 2$; $L_h = 2$; BCH coding with code-rate=$1/3$; CF non-redundant coding [5]; $L_b = 5$, $L_c = 15$, $d_H = 7$. We assumed $Q = 2$ (circles) or $Q = 4$ (squares). The difference between the PEP and BER curves is due to the fact that the PEP curve refers to the worst case, i.e. $\max_{\boldsymbol{E} \in \mathscr{S}_E} \mathscr{P}(\boldsymbol{E})$, whereas the BER curve is the average bit-error rate. In spite of such a difference, from all our simulations we have always observed that the hierarchy established by the PEP formula is the same as the one given by the BER simulations, so that the PEP formula is a useful tool to rank different coding strategies. Interestingly, for meaningful average BER values, the curve with $Q = 2$ gives a lower SNR than the curve with $Q = 4$, even though in the latter case the diversity gain is higher. This happens because of the higher coding gain, achieved with $Q = 2$. In Fig. 3, we also compare our method with the ST-DPS method [2], using $M = 5$, BCH coding with code rate=$1/3$ and Vandermonde non-redundant precoding with $N = 16$. We implemented the ST-DPS method as in [2], for time-invariant channels. However, in such a case, our method would benefit of the time diversity, thanks to the code concatenation over Q independent blocks. Hence, to compare the two methods with the same diversity gain in both cases, we adopt, in the ST-DPS scheme, 2 and 4 receive antennas. Note that, thanks to the optimal concatenated strategy, our method outperforms [2] also for the same available diversity gain. This advantage is obtained at the price of a higher decoding delay (because of the interleaving over Q independent blocks) and a slight increase of complexity. Further investigations should address the problem of optimal interleaving and the fitness of the independent block fading model with experimental channel measurements.

5. APPENDIX

We derive, under the maximum diversity gain assumption (MDGA), the closed form expression for the upper bound of the coding gain, and we will show how , using Vandermonde redundant CF precoding, the available coding gain is given by (7).
From (MDGA), it follows that $G_d{=}QTR(L{+}1)$, with $Q{\in}[1,\ d_H]$ and $rank\{\boldsymbol{R}_h^q\}{=}rank\{\boldsymbol{A}_e^{qH}\boldsymbol{A}_e^q\}{=}TR(L+1)\ \forall q{\in}[1,\ Q]$. The coding gain for the generic error event $\boldsymbol{E}{\in}\mathscr{S}_e$ becomes $G_{c,e}{=}1/T \prod_{q=1}^{Q}\left|\boldsymbol{A}_e^{qH}\boldsymbol{A}_e^q\right|^{1/G_d}{=}\ 1/T|\boldsymbol{R}_h|^{1/TR(L+1)}\ \prod_{q=1}^{Q}\ \left|\boldsymbol{V}^H\boldsymbol{D}_e^{q*}\boldsymbol{D}_e^q \boldsymbol{V}\right|^{1/QT(L+1)}$, where we have assumed $\boldsymbol{R}_h^q = \boldsymbol{R}_h\ \forall q \in [1,\ Q]$. In order to find the upper bound of the coding gain, we need to maximize $\left|\boldsymbol{V}^H\boldsymbol{D}_e^{q*}\boldsymbol{D}_e^q\boldsymbol{V}\right|$ for all $q \in [1,\ Q]$. It is straightforward to check that $\boldsymbol{V}^H\boldsymbol{D}_e^{q*}\boldsymbol{D}_e^q\boldsymbol{V}$ is a $T(L+1) \times T(L+1)$ matrix, whose diagonal entries are all equal to $trace\{\boldsymbol{D}_e^{q*}\ \boldsymbol{D}_e^q\}$ $= \|\boldsymbol{F}^q\boldsymbol{e}^q\|^2$. Since $\boldsymbol{D}_e^q\boldsymbol{V}$ is full column rank (because $\boldsymbol{A}_e^{qH}\boldsymbol{A}_e^q$ is full rank), the matrix $\boldsymbol{V}^H\boldsymbol{D}_e^{q*}\boldsymbol{D}_e^q\boldsymbol{V}$ is positive definite, thus we may apply Hadamard's inequality and write $\left|\boldsymbol{V}^H\boldsymbol{D}_e^{q*}\ \ \boldsymbol{D}_e^q\boldsymbol{V}\right|\leq \|\boldsymbol{F}^q\mathbf{e}^q\|^{2T(L+1)}$. Introducing the above expression in the definition of coding gain G_c, we get

$$G_c \leq 1/T\,|\boldsymbol{R}_h|^{1/TR(L+1)} \min_{\boldsymbol{E}\in\mathscr{S}_e} (\prod_{q=1}^{Q} \|\boldsymbol{F}^q\mathbf{e}^q\|^2)^{1/Q}.$$

Since $\{\boldsymbol{F}^q\}_{q=1}^Q$ are para-unitary matrices, under the MDGA (there exists at least one non zero entry in each vector $\boldsymbol{e}^q$), the minimum value of the upper bound of the coding gain depends only on the error events $\boldsymbol{E}$, having d_H non-zero entries. In such a case $\min_{\boldsymbol{E}\in\mathscr{S}_e} \left(\prod_{q=1}^{Q}\|\boldsymbol{F}^q\mathbf{e}^q\|^2\right) = \min_{\boldsymbol{E}\in\mathscr{S}_e} \left(\prod_{q=1}^{Q}\|\mathbf{e}^q\|^2\right) = \prod_{q=0}^{Q}\left\lceil\frac{d_H - q}{Q}\right\rceil$. This proves (6). Now we show that, using an $N{\times}M$ Vandermonde CF coding matrix $\overline{\boldsymbol{V}}$, with $N{\geq}M+T(L+1)$, the available coding gain is lower bounded by $|\boldsymbol{R}_h|^{1/TR(L+1)} d_{min}^2/T$. We need to show that $\min_{\boldsymbol{E}\in\mathscr{S}_e}\prod_{q=1}^{Q}\left|\boldsymbol{V}^H\boldsymbol{D}_e^{q*}\boldsymbol{D}_e^q\boldsymbol{V}\right|^{1/Q} \geq d_{min}^{2T(L+1)}$ when $\{\boldsymbol{F}^q\}_{q=1}^Q{=}\overline{\boldsymbol{V}}$. We focus on $\left|\boldsymbol{V}^H\boldsymbol{D}_e^{q*}\boldsymbol{D}_e^q\boldsymbol{V}\right|$. Since an $N{\times}M$ Vandermonde matrix is formed by the first M columns of a $N \times N$ FFT matrix $\boldsymbol{W}_N^H$, $\boldsymbol{F}^q\boldsymbol{e}^q$ is the N-point FFT of $\boldsymbol{e}^q$. Hence, we can write $\boldsymbol{D}_e^q\boldsymbol{V}{=}\boldsymbol{W}_N^H\widetilde{\boldsymbol{C}}_e^q\widetilde{\boldsymbol{I}}{=}\boldsymbol{W}_N^H\boldsymbol{C}_e^q$, where $\widetilde{\boldsymbol{C}}_e^q$ is an $N \times N$ circulant Toeplitz matrix with first column $[\boldsymbol{e}^{qT}, 0, \ldots, 0]^T$ and first row $[e^q(1), 0, \ldots, e^q(M), \ldots, e^q(2)]$; $\widetilde{\boldsymbol{I}}$ is the $N{\times}T(L+1)$ matrix containing the first $T(L+1)$ columns of a $N{\times}N$ identity matrix and $\boldsymbol{C}_e^q{:=}\widetilde{\boldsymbol{C}}_e^q\widetilde{\boldsymbol{I}}$ is the $N{\times}T(L{+}1)$ Toeplitz matrix with first column $\boldsymbol{e}^q$ and first row $[e^q(1), 0, \ldots, 0]$. We decompose $\boldsymbol{C}_e^q$ as $[\boldsymbol{C}_{e1}^{qT}, \boldsymbol{C}_{e2}^{qT}, \boldsymbol{C}_{e3}^{qT}]^T$, where $\boldsymbol{C}_{e2}^q$ is a $T(L+1){\times}T(L+1)$ lower triangular matrix whose diagonal entries are all equal to the first non-zero component of $\boldsymbol{e}^q$, whereas $\boldsymbol{C}_{e1}$ and $\boldsymbol{C}_{e3}$ come out as a consequence of the partitioning. Using this setup, we may write[8] $\left|\boldsymbol{V}^H\boldsymbol{D}_e^{q*}\boldsymbol{D}_e^q\boldsymbol{V}\right| = \left|\boldsymbol{C}_{e1}^{qH}\boldsymbol{C}_{e1}^q{+}\boldsymbol{C}_{e2}^{qH}\boldsymbol{C}_{e2}^q{+}\boldsymbol{C}_{e3}^{qH}\boldsymbol{C}_{e3}^q\right| \geq |\boldsymbol{C}_{e2}^q|^2$. Hence $\min_{\boldsymbol{E}\in\mathscr{S}_e}\prod_{q=1}^{Q}\left|\boldsymbol{V}^H\boldsymbol{D}_e^{q*}\boldsymbol{D}_e^q\boldsymbol{V}\right|^{1/Q} \geq \min_{\boldsymbol{E}\in\mathscr{S}_e}\prod_{q=1}^{Q} |\boldsymbol{C}_{e2}^q|^{2/Q} = d_{min}^{2T(L+1)}$.

6. REFERENCES

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[8] The inequality follows from $|\boldsymbol{A}+\boldsymbol{B}| \geq |\boldsymbol{A}|$, when $\boldsymbol{A}$ is positive definite and $\boldsymbol{B}$ is positive semi-definite. Note that $\boldsymbol{C}^q{:=}\ \boldsymbol{C}_{e1}^{qH}\boldsymbol{C}_{e1}^q + \boldsymbol{C}_{e3}^{qH}\boldsymbol{C}_{e3}^q$ is positive semi-definite because $[\boldsymbol{C}^q]_{ii}[\boldsymbol{C}^q]_{jj}{\geq}|[\boldsymbol{C}^q]_{ij}|^2$ for $i, j{=}1, 2, \ldots, T(L+1)$.