MAXIMUM LIKELIHOOD METHOD FOR BLIND IDENTIFICATION OF MULTIPLE AUTOREGRESSIVE CHANNELS

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ABSTRACT

Abstract- We present a two-step maximum likelihood (TSML) algorithm for blind identification of single-input-multiple-output (SIMO) channels modeled as autoregressive (AR) system. The AR-TSML algorithm provides a new and useful alternative to a previously developed TSML algorithm for moving-average (MA) system. The AR-TSML algorithm is shown to be more robust than the MA-TSML algorithm if the channel impulse responses have long tails.

1. INTRODUCTION

In wireless communications, the radio energy of transmitted signal is distributed in space. Utilizing the spatially distributed energy can yield a higher channel capacity or enhanced signal reception. Spatially distributed antennas are common for exploiting the spatial diversity. For accoustic or speech communications, an array of microphones can be deployed to achieve a similar purpose. For applications like the above, the signals received at the sensors can be modelled as the output of a single-input-multiple-output (SIMO) system where the input represents the desired signal. The impulse response of the SIMO system accounts for the distortion of the desired signal. For many situations, one also needs to retrieve the desired signal from the system output without the knowledge of the channel response. This is known as blind identification. Extensive surveys on blind identification of SIMO systems are available in [1] and [2]. Among many available techniques, there is one method known as two-step maximum likelihood (TSML) [3]. The TSML algorithm approaches the (optimal) performance of the exact maximum likelihood at a moderate signal-to-noise ratio (SNR) but only needs to compute the solutions of two quadratic minimizations. The two-channel special cases of the TSML algorithm are also available in [4] and [5].

In this paper, we further develop the TSML concept. While the original TSML algorithm applies to finite-impulse-response (FIR) or moving-average (MA) channels, the new TSML algorithm we develop here is based on the model of autoregressive (AR) channels. We will refer to them as MA-TSML and AR-TSML, respectively. For certain applications such as accoustic reverberation inside a room, the AR model may be more appropriate than the MA model. This seems also true if the impulse response of the system has long tails. A simulation example will support this obseravtion. We note that a more general model would be an ARMA system. But for an ARMA system, some spectral information of the input is needed or otherwise the system is not identifiable. The ARMA system will not be further addressed here.

The complexity of the TSML algorithm differs significantly from the two-channel case to the more-than-two-channel case [3]. To illustrate the AR-TSML concept in a simple way, we only consider the two-channel case here. The rest of this paper is as follows.

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2. PROBLEM FORMULATION

Consider the following discrete two-channel AR system:

$$x_i(n) = \frac{1}{A_i(z)} s(n) \quad (1)$$

$$y_i(n) = x_i(n) + w_i(n) \quad (2)$$

where $i = 1, 2$, $n = 0, 1, \cdots, N-1$, $x_i(n)$ is the noiseless output, $s(n)$ is the input signal, $y_i(n)$ denotes the noise-corrupted measurement of $x_i(n)$, $w_i(n)$ is noise, and $A_i(z)$ is defined by $A_i(z) = \sum_{l=0}^{L} a_i(l) z^{-l}$.

The following assumptions are made on the model (1)-(2):

A1. All zeros of $A_i(z)$ are strictly inside the unit circle, which guarantees that the system is stable.

A2. $A_1(z)$ and $A_2(z)$ have no common factor, which ensures the identifiability of the system up to a complex scalar in the absence of noise.

A3. $\sum_{i=1}^{2} \sum_{l=0}^{L} \|a_i(l)\|^2 = 1$, which removes a real-valued scaling ambiguity of the system.

A4. $w_i(n)$ is white Guassian noise.

3. BLIND CHANNEL IDENTIFICATION

We now develop a maximum likelihood algorithm for estimating the channel coefficient vector $\boldsymbol{a}$:

$$\boldsymbol{a} = [\boldsymbol{a}_1^T \quad \boldsymbol{a}_2^T]^T \tag{3}$$

where $\boldsymbol{a}_i = [a_i(0), a_i(1), \cdots, a_i(L)]^T$.

The system (2) can also be expressed as $\boldsymbol{y} = \boldsymbol{x} + \boldsymbol{w}$ where $\boldsymbol{y}$ is the measurement vector

$$\boldsymbol{y} = [\boldsymbol{y}_1^T \quad \boldsymbol{y}_2^T]^T \tag{4}$$

with $\boldsymbol{y}_i = [y_i(N-1), y_i(N-2), \cdots, y_i(0)]^T$, and the vectors $\boldsymbol{x}$ and $\boldsymbol{w}$ are similarly defined.

We can regard the AR channels as MA channels of infinitely long impulse responses, i.e.,

$$x_i(n) = \sum_{l=0}^{\infty} h_i(l)s(n-l) \tag{5}$$

where $h_i(l) \cong 0$ for $l > M >> L$.

Define $H_i(z) = \sum_{l=0}^{\infty} h_i(l)z^{-l}$, then

$$A_1(z)H_1(z) = A_2(z)H_2(z) = 1 \tag{6}$$

The equation (5) can be rewritten as $\boldsymbol{x} = \boldsymbol{H}\boldsymbol{s}$ with $\boldsymbol{H} = [\boldsymbol{H}_1^T \quad \boldsymbol{H}_2^T]^T$, where $\boldsymbol{H}_i$ is a matrix of $N \times (M+N)$ defined as $\boldsymbol{H}_i =$

$$\begin{bmatrix} h_i(0) & \cdots & \cdots & \cdots & h_i(M+N-1) \\ & h_i(0) & \cdots & \cdots & h_i(M+N-2) \\ & \ddots & \ddots & \ddots & \vdots \\ & & h_i(0) & \cdots & h_i(M) \end{bmatrix} \tag{7}$$

and $\boldsymbol{s} = [s(N-1), s(N-2), \cdots, s(0), \cdots, s(-M)]^T$ Note that in theory, M can be arbitrarily large.

The maximum likelihood (ML) estimation of the channel coefficient vector $\boldsymbol{a}$ is obtained by (e.g., see [6]) $\max_{\boldsymbol{a},\boldsymbol{s}} f(\boldsymbol{y})$ or equivalently

$$\min_{\boldsymbol{a},\boldsymbol{s}} J = \min_{\boldsymbol{a},\boldsymbol{s}} \|\boldsymbol{y} - \boldsymbol{H}\boldsymbol{s}\|^2 \tag{8}$$

where $f(\boldsymbol{y})$ is the probability density function (PDF) of $\boldsymbol{y}$.

It is well known that the optimal input vector is given by $\hat{\boldsymbol{s}} = \boldsymbol{H}^+\boldsymbol{y}$ where $\boldsymbol{H}^+$ is the pseudoinverse of $\boldsymbol{H}$, and hence the cost function becomes

$$J = \|\boldsymbol{y} - \boldsymbol{H}\boldsymbol{H}^+\boldsymbol{y}\|^2 = \|(\boldsymbol{I} - \boldsymbol{P}_H)\boldsymbol{y}\|^2 \tag{9}$$

where $\boldsymbol{P}_H$ is the orthogonal projector onto the range space of $\boldsymbol{H}$. Note that $\boldsymbol{H}$ does not have a full rank in either row or column in general.

Now we develop a relationship between $h_i(n)$ and $a_i(n)$, where the latter are considered to be the free (i.e., unconstrained) unknowns. From (6), we can obtain

$$\sum_{l=0}^{L} h_1(n-l)a_1(l) = \sum_{l=0}^{L} h_2(n-l)a_2(l) = 1 \tag{10}$$

and hence

$$\boldsymbol{H}_1^T\boldsymbol{A}_1^T = \boldsymbol{H}_2^T\boldsymbol{A}_2^T = \begin{bmatrix} \boldsymbol{I} \\ \boldsymbol{0} \end{bmatrix} \tag{11}$$

where $\boldsymbol{H}_i$ is defined in (7), and $\boldsymbol{A}_i$ is a matrix of $(N-L) \times N$, defined as $\boldsymbol{A}_i =$

$$\begin{bmatrix} a_i(0) & \cdots & \cdots & a_i(L) & & & \\ & a_i(0) & \cdots & \cdots & a_i(L) & & \\ & & \ddots & \ddots & \ddots & \ddots & \\ & & & a_i(0) & \cdots & \cdots & a_i(L) \end{bmatrix} \tag{12}$$

The equation (11) implies

$$\begin{bmatrix} \boldsymbol{H}_1^T & \boldsymbol{H}_2^T \end{bmatrix} \begin{bmatrix} \boldsymbol{A}_1^T \\ -\boldsymbol{A}_2^T \end{bmatrix} = 0 \tag{13}$$

which is equal to

$$\boldsymbol{H}^H\boldsymbol{A} = 0 \tag{14}$$

where $\boldsymbol{A}$ is defined by

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}_1^H \\ -\boldsymbol{A}_2^H \end{bmatrix} \tag{15}$$

In the appendix, we will prove that:

$$\mathcal{R}(\boldsymbol{A}) = \mathcal{N}(\boldsymbol{H}) \tag{16}$$

where $\mathcal{R}(\cdot)$ denotes the range space and $\mathcal{N}(\cdot)$ denotes the null space. Making use of (16), we can write

$$\boldsymbol{P}_A = \boldsymbol{P}_H^{\perp} = \boldsymbol{I} - \boldsymbol{P}_H \tag{17}$$

Inserting (17) into (9) yields the following cost function:

$$J = \|\boldsymbol{P}_A\boldsymbol{y}\|^2 = \boldsymbol{y}^H\boldsymbol{A}(\boldsymbol{A}^H\boldsymbol{A})^{-1}\boldsymbol{A}^H\boldsymbol{y} \tag{18}$$

Next we can observe

$$\boldsymbol{A}^H\boldsymbol{y} = \begin{bmatrix} \boldsymbol{A}_1 & -\boldsymbol{A}_2 \end{bmatrix} \boldsymbol{y} = \boldsymbol{A}_1\boldsymbol{y}_1 - \boldsymbol{A}_2\boldsymbol{y}_2 \tag{19}$$

It is easy to show that the following structural relation holds:

$$\boldsymbol{A}_i\boldsymbol{y}_i = \boldsymbol{Y}_i\boldsymbol{a}_i \tag{20}$$

where $\boldsymbol{Y}_i =$

$$\begin{bmatrix} y_i(N-1) & y_i(N-2) & \cdots & y_i(N-L-1) \\ \vdots & \vdots & \vdots & \vdots \\ y_i(L+1) & y_i(L) & \cdots & y_i(1) \\ y_i(L) & y_i(L-1) & \cdots & y_i(0) \end{bmatrix} \tag{21}$$

Inserting (20) into (19) yields the following expression:

$$A^H y = \left[\begin{array}{cc} Y_1 & -Y_2 \end{array}\right] \left[\begin{array}{c} a_1 \\ a_2 \end{array}\right] = Ya \tag{22}$$

By inserting (22) into (18), we can obtain

$$J = a^H Y^H W_A^{-1} Y a \tag{23}$$

where

$$W_A = A^H A = A_1 A_1^H + A_2 A_2^H \tag{24}$$

Hence, the ML estimate of a can be obtained by minimizing J shown above. This expression of J suggests the following iterative two-step estimation procedure, i.e., AR-TSML:

Step1: Minimize $a^H Y^H Y a$ with $\|a\| = 1$ to yield a_c.

Step2: Minimize $a^H Y^H \hat{W}_A^{-1} Y a$ with $\|a\| = 1$ to yield a_e, where $\hat{W}_A^{-1}$ is constructed from a_c according to (12) and (24).

It can be shown that the step 1 is consistent, i.e., it yields the exact result when the noise is absent.

To estimate $s(k)$, we first need to estimate $h_i(l)$ from a. Note that (10) implies

$$\overline{A}_i^T h_i = e \tag{25}$$

with $e = [1, 0, 0, \cdots, 0]^T$, $h_i = [h_i(0), h_i(1), \cdots, h_i(M-1)]^T$, and $\overline{A}_i$ is an $M \times (M+L)$ matrix defined as $\overline{A}_i =$

$$\left[\begin{array}{ccccccc} a_i(0) & \cdots & \cdots & a_i(L) & & & \\ & a_i(0) & \cdots & \cdots & a_i(L) & & \\ & \ddots & \ddots & \ddots & \ddots & & \\ & & a_i(0) & \cdots & \cdots & a_i(L) \end{array}\right] \tag{26}$$

Thus we can obtain the estimate of h_i by:

$$\hat{h}_i = (\overline{A}_i^T)^+ e \tag{27}$$

This estimate is consistent as M becomes large.

Now, the estimation of s is straightforward by using the expression following (8). If we know that $s(k) = 0$ for $k < 0$, then we define $\overline{H} = [\overline{H}_1^T \quad \overline{H}_2^T]^T$ where

$$\overline{H}_i = \left[\begin{array}{cccc} h_i(0) & \cdots & \cdots & h_i(N-1) \\ & h_i(0) & \cdots & h_i(N-2) \\ & & \ddots & \vdots \\ & & & h_i(0) \end{array}\right] \tag{28}$$

and then compute $\hat{s} = \overline{H}^+ y$. $\overline{H}$ has a full column rank in general.

4. SIMULATION

We considered a simple two-channel system, where each channel is a second-order AR filter with the channel coefficients defined as follows:

$$\begin{aligned} A_1(z) &= 1.0000 + 1.4000z^{-1} + 0.9800z^{-2} \\ A_2(z) &= 0.9490 - 1.3097z^{-1} + 0.9037z^{-2} \end{aligned} \tag{29}$$

The system was driven by a sample sequence of speech signal. The noise was additive white Gaussian. After the channel coefficient vector a was estimated, two estimates of the channel impulse response were obtained using two different values of M. Using the first N samples of each estimated channel impulse response, the input was then estimated. Fig. 1 shows the performance of the AR-TSML method versus SNR. We can see that the larger M is, the more accurate is the estimate of the input. Note that the estimate of the channel coefficient vector a is not affected by M.

We also considered a two-channel MA system of order 15. The impulse response of the channels was generated by an i.i.d random sequence of values chosen within [-1, 1] and then multiplied by an envelope 0.5^n. This generated long tail impulse responses. We applied both AR-TSML (assuming AR order $L = 2$) and MA-TSML (assuming MA order $L = 15$) to this MA system for 100 independent runs at each SNR. For each run, independent channel impulse responses and independent additive noise were generated. One realization of the channel impulse responses is shown in Fig. 2. The mean-squared-errors (MSE) of the estimated input signals using the two methods are shown in Fig. 3. It is clear that the AR-TSML method yielded much better performance than the MA-TSML method. In fact, the results from the MA-TSML algorithm were too poor to be useful in this case.

5. CONCLUSION

We have developed the AR-TSML algorithm based on a two-channel SIMO AR system. The AR-TSML algorithm provides a new and useful alternative to the previously developed MA-TSML algorithm. For channels with long tails, the AR-TSML algorithm appears more robust than the MA-TSML algorithm.

6. APPENDIX

Proof of (16): Making use of (14), we can obtain $\mathcal{R}(A) \subset \mathcal{N}(H)$. But we also need to show $\mathcal{N}(H) \subset \mathcal{R}(A)$.

To show the above equation, we note that for any vector $c \in \mathcal{R}^{2N\times 1}$, which satisfies $H^T c = 0$ or equivalently

$$H_1^T c_1 = -H_2^T c_2 = g_0 \tag{30}$$

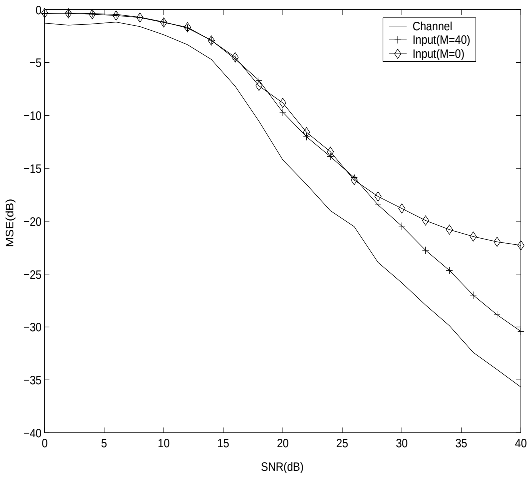

Fig. 1. Performance of the AR-TSML method

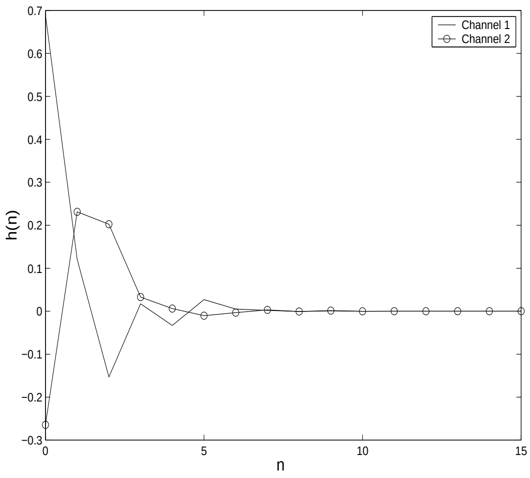

Fig. 2. Channel Impulse Response

with $\boldsymbol{g}_0^T = [g_0(0), g_0(1), \cdots, g_0(N+M-1)]$ and $\boldsymbol{c} = \begin{bmatrix} \boldsymbol{c}_1^T & \boldsymbol{c}_2^T \end{bmatrix}^T$, where $c_i^T = [c_i(0), c_i(1), \cdots, c_i(N-1)]$.

Hence we have

$$h_1(n) * c_1(n) = -h_2(n) * c_2(n) = g_0(n) \tag{31}$$

or equivalently (assuming M is arbitrarily large)

$$H_1(z)C_1(z) = -H_2(z)C_2(z) = G_0(z) \tag{32}$$

where $G_0(z) = \sum_{l=0}^{N+M-1} g_0(l)z^{-l}$. Making use of (6) yields:

$$\frac{1}{A_1(z)}C_1(z) = -\frac{1}{A_2(z)}C_2(z) = G_0(z) \tag{33}$$

Hence,

$$C_1(z) = A_1(z)G_0(z) \qquad C_2(z) = -A_2(z)G_0(z) \tag{34}$$

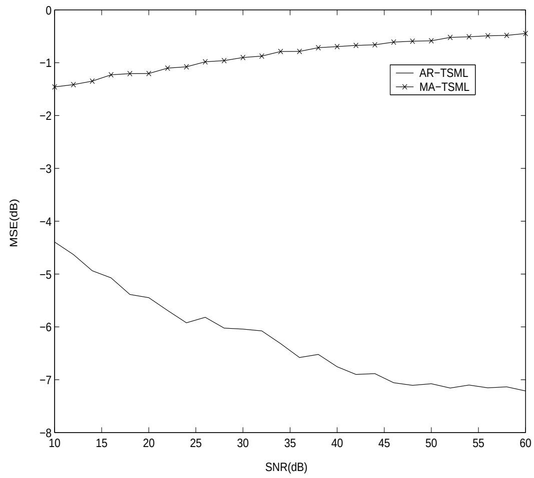

Fig. 3. Performance of AR-TSML and MA-TSML

Since $deg(C_i(z)) = N-1$ and $deg(A_i(z)) = L$, then $deg(G_0(z)) = N-L-1$, where $deg(\cdot)$ denotes degree of polynomial. Hence, only the first $N-L$ elements of $\boldsymbol{g}_0$ are nonzero, i.e. $\boldsymbol{g}_0^T = \left[\boldsymbol{g}_0^{'\,T}, 0, 0, \cdots, 0\right]$ with $\boldsymbol{g}_0^{'\,T} = [g_0(0), g_0(1), \cdots, g_0(N-L-1)]$. Hence, the equation (34) can be rewritten into matrix form: $\boldsymbol{c}_1 = \boldsymbol{A}_1^T\boldsymbol{g}_0^{'}$ and $\boldsymbol{c}_2 = -\boldsymbol{A}_2^T\boldsymbol{g}_0^{'}$. Henceforth, $\boldsymbol{c}^* = \boldsymbol{A}\boldsymbol{g}_0^{'\,*}$, which means $\boldsymbol{c}^* \in \mathcal{R}(\boldsymbol{A})$. Here, $(\cdot)^*$ denotes conjugation. The above implies that $\mathcal{N}(\boldsymbol{H}) \subset \mathcal{R}(\boldsymbol{A})$. Therefore, the proof is completed.

7. REFERENCES

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