A SUBSPACE ALGORITHM FOR GUARD INTERVAL BASED CHANNEL IDENTIFICATION AND SOURCE RECOVERY REQUIRING JUST TWO RECEIVED BLOCKS

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ABSTRACT

Blind channel identification techniques usually exploit a known property of the source symbols such as a statistical or finite alphabet property. Recently, a purely algebraic approach that relies on guard intervals (sequences of zeros equal or longer in length than the channel memory) inserted between transmitted blocks has been considered. It was proved that only two received blocks suffice for channel identification and source recovery. In this paper, the channel identification problem is approached from a z-domain perspective. It is shown that, in the z-domain, the channel is a common factor in all received blocks. (This fundamental property appears to have gone unnoticed in the literature.) This allows a subspace method for computing the greatest common divisor (GCD) to be applied to the channel estimation problem. The algorithm achieves the theoretical limit in that only two received blocks are required before the channel can be identified, but of course, the more blocks that are used, the better the performance in the presence of noise.

1. INTRODUCTION

Techniques for estimating the channel and the source symbols must use either a training sequence or exploit some other property of the source symbols. Training sequences, however, consume transmission bandwidth, especially when the channel is time varying and a training sequence must be sent frequently. Therefore, blind techniques exploiting a known property of the source have become popular [1, 2, 3, 4]. These techniques usually exploit the (second or higher order) statistics [5, 6, 7] or the finite alphabet property of the source symbols [8, 9] to identify the channel. The statistical methods normally require many symbols to be received before the channel can be identified accurately. Deterministic methods relying only on the finite alphabet property of the source symbols [8, 9] can sometimes offer an advantage of requiring less data, however, there is no theoretical upper bound on the number of received symbols before identification is feasible[1]. This paper proposes a different method that relies only on the presence of guard intervals, which are introduced between transmitted blocks to prevent inter-block interference by a number of communication systems [10, 11, 12, 13]. Guard intervals are used in time division multiple access (TDMA) systems such as the mobile communication system GSM and can also be used in orthogonal frequency division multiplexing (OFDM) systems [10, 14, 15]. (It is possible to exploit both the finite alphabet property and guard intervals for channel identification; see [16] for theoretical details and [17] for a practical algorithm.)

In [18] (see also Section 2) it was proved that it is theoretically possible to identify the channel and recover the source symbols from just two received blocks simply by exploiting the presence of guard intervals. The proof was based in the time domain and used techniques from algebraic geometry.

The key contribution of this paper is the derivation of a practical algorithm which meets the theoretical limit of being able to identify the channel using just two received blocks. This is considerably less than the method in [19] which requires as many received blocks as there are symbols in each block. Moreover, when the algorithm presented here is generalised to use the same number of blocks as the algorithm in [19], simulations show the performance of the proposed algorithm is superior to that in [19].

The proposed algorithm is based on the simple observation, first made in [16], that in the z-domain, each received block $Y(z)$ is given by $Y(z) = S(z)H(z)$ where $S(z)$ is the z-transform of the transmitted symbols in the block and $H(z)$ the z-transform of the channel impulse response. This means that, given two received blocks $Y_1(z) = S_1(z)H(z)$ and $Y_2(z) = S_2(z)H(z)$, the channel can be estimated (up to a scaling factor) by computing the greatest common divisor (GCD) of $Y_1(z)$ and $Y_2(z)$, provided of course that the transmitted blocks $S_1(z)$ and $S_2(z)$ are co-prime. It is proposed here to use the subspace algorithm in [20] to find the GCD.

It is instructive to compare a guard interval based Single Input Single Output (SISO) system with a conventional Single Input Multiple Output (SIMO) system which has been studied extensively in the past [7, 21]. Indeed, it is a standard result that the output of the mth channel in a SIMO system is given by $Y_m(z) = H_m(z)S(z)$, and thus blind channel identification algorithms for SIMO systems are essentially algorithms for finding the GCD of the outputs $Y_1(z), \cdots, Y_M(z)$ of the M channels. The only difference is that in SIMO systems, the GCD is the transmitted sequence $S(z)$ whereas in a SISO system with guard intervals, the GCD is the channel transfer function $H(z)$. This suggests that algorithms developed for SIMO channel identification can be adapted for guard interval based channel identification.

The precise communication system considered in this paper

[1]For example, if the same source symbol is transmitted continuously then there is not sufficient persistence of excitation to allow the channel to be identified.

is now stated formally[2]. The system breaks the source sequence into blocks of fixed length p. To avoid inter-block interference the system flushes the channel memory by appending $L-1$ zeros to each block before transmitting them over an FIR channel of length at most L, denoted by the vector $\mathbf{h} = [h_0, \ldots, h_{L-1}]^T \in \mathbb{C}^L$. The channel remains constant over the duration of M blocks. If $\mathbf{s}_m = [s_m(1), \ldots, s_m(p)]^T \in \mathbb{C}^p$ is the mth block of source symbols then $[s_m(1), \ldots, s_m(p), 0, \ldots, 0]^T$ is the transmitted block, which is the block $\mathbf{s}_m$ appended with $L-1$ zeros. Due to the previous transmitted block ending in $L-1$ zeros, the mth received blocks $\mathbf{y}_m = [y_m(1), \ldots, y_m(p+L-1)]^T \in \mathbb{C}^{p+L-1}$ is related to the mth transmitted block and the channel as

$$\mathbf{y}_m = \mathbf{S}_m \mathbf{h}, \quad \text{for } m = 1, \ldots, M \tag{1}$$

where $\mathbf{S}_m \in \mathbb{C}^{(p+L-1)\times L}$ is the lower triangular Toeplitz matrix having the vector $[s_m(1), \ldots, s_m(p), 0, \ldots, 0]^T$ as the first column, namely

$$\mathbf{S}_m = \begin{bmatrix} s_m(1) & & \\ \vdots & \ddots & s_m(1) \\ s_m(p) & & \vdots \\ & \ddots & s_m(p) \end{bmatrix}. \tag{2}$$

This signal model allows to covert the system of equations (1) to the z-domain as

$$Y_m(z) = S_m(z)H(z), \quad \text{for } m = 1, \ldots, M. \tag{3}$$

When $S_1(z), \ldots, S_M(z)$ are co-prime, then $H(z)$ can be found as the GCD of the polynomials $Y_1(z), \ldots, Y_M(z)$. It is also observed that $S_m(z)H(z) = (\lambda S_m(z))(\lambda^{-1}H(z))$ for any non-zero $\lambda \in \mathbb{C}$, therefore, usually the best that can be done is to determine $H(z)$ and $S_m(z)$ up to an unknown scaling factor; this is called scale ambiguity. Furthermore, for some cases there may be extra ambiguity. Take $S_m(z) = z-1$, $\tilde{S}_m(z) = z+1$, $H(z) = z+1$ and $\tilde{H}(z) = z-1$; then, $S_m(z)H(z) = \tilde{S}_m(z)\tilde{H}(z) = z^2 + 0z - 1$, but $\tilde{S}_m(z) \neq \lambda S_m(z)$ and $\tilde{H}(z) \neq \lambda^{-1}H(z)$ for any λ. Therefore, we are motivated to define the feasibility of channel identification and source recovery as in the next section.

This paper is organized as follows. Section 2 summaries some known identifiability results. Section 3 presents a subspace algorithm for identifying the channel. Simulation results appear in Section 4 while Section 5 concludes the paper.

2. IDENTIFIABILITY RESULTS

This section summarizes the main identifiability results in [18]. Although it is clear from the z-domain interpretation in Section 1 that only two received blocks are required to identify the channel in the noise free case, the proof in [18] is considerably more complicated because it deals with the case when noise is present too.

The system of equations (1) can be written in the following form.

$$\mathbf{y}^{(M)} = \mathbf{S}^{(M)}\mathbf{h} \tag{4}$$

where

$$\mathbf{y}^{(M)} = [\mathbf{y}_1^T, \ldots, \mathbf{y}_M^T]^T \in \mathbb{C}^{(p+L-1)M},$$
$$\mathbf{s}^{(M)} = [\mathbf{s}_1^T, \ldots, \mathbf{s}_M^T]^T \in \mathbb{C}^{pM},$$
$$\mathbf{S}^{(M)} = [\mathbf{S}_1^T, \ldots, \mathbf{S}_M^T]^T \in \mathbb{C}^{(p+L-1)M\times L},$$

and $\mathbf{S}_1, \ldots, \mathbf{S}_M$ are constructed from $\mathbf{s}_1, \ldots, \mathbf{s}_M$ respectively according to (2). In the presence of noise, (4) becomes

$$\mathbf{y}^{(M)} = \mathbf{S}^{(M)}\mathbf{h} + \mathbf{n}^{(M)} \tag{5}$$

where $\mathbf{n}^{(M)} \in \mathbb{C}^{(p+L-1)M}$ denotes additive Gaussian noise. For a given received vector $\mathbf{y}^{(M)}$, the channel identification and source recovery problem is to solve the system of equations (4) or (5) for $\mathbf{h}$ and $\mathbf{s}^{(M)}$ (up to a scaling factor). The following defines channel identification as being feasible in the case of noise free and the case of noise presence.

Definition 1 (Noise Free Identifiability) The system (4) is identifiable (up to a scaling ambiguity) if there exists a non-zero polynomial $g : \mathbb{C}^{pM} \to \mathbb{C}^L$ such that, for any $\mathbf{s}^{(M)}$ and $\mathbf{h}$ satisfying $g(\mathbf{s}^{(M)}, \mathbf{h}) \neq 0$,

$$\begin{aligned} &\forall \tilde{\mathbf{s}}^{(M)} \in \mathbb{C}^{pM}, \forall \tilde{\mathbf{h}} \in \mathbb{C}^L, \mathbf{S}^{(M)}\mathbf{h} = \tilde{\mathbf{S}}^{(M)}\tilde{\mathbf{h}} \text{ implies} \\ &\exists \lambda \in \mathbb{C}, \lambda \neq 0, \tilde{\mathbf{s}}^{(M)} = \lambda \mathbf{s}^{(M)}, \tilde{\mathbf{h}} = \lambda^{-1}\mathbf{h}. \end{aligned} \tag{6}$$

Definition 2 (Identifiability in Noise) Let $\mathbf{n}^{(M)} \in \mathbb{C}^{(p+L-1)M}$ be a random vector whose probability measure is absolutely continuous with respect to Lebesgue measure. For any triple $\mathbf{s}^{(M)}, \mathbf{h}, \mathbf{n}^{(M)}$, defined $\Omega_{\mathbf{s}^{(M)},\mathbf{h},\mathbf{n}^{(M)}}$ to be the set of all global minima of $\|\mathbf{S}^{(M)}\mathbf{h} + \mathbf{n}^{(M)} - \tilde{\mathbf{S}}^{(M)}\tilde{\mathbf{h}}\|^2$. The system (5) is identifiable (up to a scaling ambiguity) if there exists a non-zero polynomial $g : \mathbf{C}^{pM} \times \mathbb{C}^L \to \mathbb{C}$ such that for any $\mathbf{s}^{(M)}$ and $\mathbf{h}$ satisfying $g(\mathbf{s}^{(M)}, \mathbf{h}) \neq 0$,

$$\begin{aligned} &(\mathbf{s}^{(M)}, \mathbf{h}) \text{ and } (\tilde{\mathbf{s}}^{(M)}, \tilde{\mathbf{h}}) \in \Omega_{\mathbf{s}^{(M)},\mathbf{h},\mathbf{n}^{(M)}} \text{ implies} \\ &\exists \lambda \in \mathbb{C}, \lambda \neq 0, \tilde{\mathbf{s}}^{(M)} = \lambda \mathbf{s}^{(M)}, \tilde{\mathbf{h}} = \lambda^{-1}\mathbf{h} \end{aligned} \tag{7}$$

holds almost surely.

The conditions (6) and (7) are necessary, since either $\mathbf{h} = \mathbf{0}$ or $\mathbf{s}^{(M)} = \mathbf{0}$ then $\mathbf{y}^{(M)} = \mathbf{0}$ and the channel is unidentifiable. The following states the identifiability theorem from [18].

Theorem 1 For any $p \geq 1$ and $L \geq 2$, the systems (4) and (5) are identifiable, according to Def.1 and Def.2 respectively, using two blocks. Moreover, they are identifiable using just one block if and only if $p = 1$.

In reality, the block length is much longer than one symbol, the minimum number of received symbols needed for the system to be identifiable is $(p+L-1) \times 2$. This requirement is different from [19], where it proves that for identifiability the number of blocks must not less than the block length, in other words, the number of received symbols required for the system to be identifiable is at least $(p+L-1) \times p$.

3. CHANNEL IDENTIFICATION ALGORITHM

This section presents an algorithm for identifying the channel. The algorithm is adapted from the subspace method for computation of the GCD of polynomials in [20] and can be explained as follows.

[2]Throughout this paper, superscript T and H denote transpose and Hermitian transpose respectively.

Equations (1) can be written in a new form as

$$\mathcal{Y}_m = \mathcal{S}_m \mathcal{H} \qquad m = 1, \ldots, M \tag{8}$$

where $\mathcal{Y}_m \in \mathbb{C}^{(p+L-1)\times(2p+2L-3)}$, $\mathcal{S}_m \in \mathbb{C}^{(p+L-1)\times(2p+L-2)}$, $\mathcal{H} \in \mathbb{C}^{(2p+L-2)\times(2p+2L-3)}$, and

$$\mathcal{S}_m = \begin{bmatrix} s_m(1) & \ldots & s_m(p) & & \\ & \ddots & & \ddots & \\ & & s_m(1) & \ldots & s_m(p) \end{bmatrix} \tag{9}$$

$$\mathcal{H} = \begin{bmatrix} h(0) & \ldots & h(L-1) & & \\ & \ddots & & \ddots & \\ & & h(0) & \ldots & h(L-1) \end{bmatrix} \tag{10}$$

$$\mathcal{Y}_m = \begin{bmatrix} y_m(1) & \ldots & y_m(p+L-1) & & \\ & \ddots & & \ddots & \\ & & y_m(1) & \ldots & y_m(p+L-1) \end{bmatrix} \tag{11}$$

By defining

$$\mathcal{S}^{(M)} = [\mathcal{S}_1^T, \ldots, \mathcal{S}_M^T]^T, \;\; \mathcal{Y}^{(M)} = [\mathbf{Y}_1^T, \ldots, \mathbf{Y}_M^T]^T, \tag{12}$$

Equations (8) can be rewritten as

$$\mathcal{Y}^{(M)} = \mathcal{S}^{(M)} \mathcal{H}. \tag{13}$$

Taking singular-value decomposition (SVD) of ${\mathcal{Y}^{(M)}}^T$ we have

$${\mathcal{Y}^{(M)}}^T = [\mathbf{U}_r \mathbf{U}_0] \begin{bmatrix} \Sigma_r & \\ & \mathbf{0} \end{bmatrix} [\mathbf{V}_r \mathbf{V}_0]^H \tag{14}$$

where $r = rank(\mathcal{Y}^{(M)}) = 2p + L - 2$, Σ_r contains r non-zero eigenvalues, $\mathbf{U}_r$ contains r left principal eigenvectors and $\mathbf{U}_0$ contains $L-1$ left null eigenvectors. Due to the columns of $\mathcal{H}$ and the columns of $\mathbf{U}_r$ span the same subspace and $\mathbf{U}_r$ and $\mathbf{U}_0$ are orthogonal, it follows that

$$\mathcal{H}\mathbf{U}_0 = \mathbf{0}. \tag{15}$$

This equation has a unique solution for $\mathbf{h}$ up to a scalar [20, 22] and can be solved using the least square approach. Let $\mathbf{u}_l$ denote the lth column of the matrix $\mathbf{U}_0$, define $\mathcal{U}_l$ as

$$\mathcal{U}_l = \begin{bmatrix} \mathbf{u}_l(1) & \mathbf{u}_l(2) & \ldots & \mathbf{u}_l(L) \\ \mathbf{u}_l(2) & \mathbf{u}_l(3) & \ldots & \mathbf{u}_l(L+1) \\ \ldots & \ldots & \ldots & \ldots \\ \mathbf{u}_l(r) & \mathbf{u}_l(r+1) & \ldots & \mathbf{u}_1(2p+2L-3) \end{bmatrix} \tag{16}$$

then according to (15)

$$\sum_{l=1}^{L-1} \|\mathcal{H}\mathbf{u}_l\|^2 = 0, \;\; \text{implies} \;\; \sum_{l=1}^{L-1} \|\mathcal{U}_l \mathbf{h}\|^2 = 0 \tag{17}$$

Equations (17) can also be written as

$$\sum_{l=1}^{L-1} \mathbf{h}^H \mathcal{U}_l^H \mathcal{U}_l \mathbf{h} = \mathbf{h}^H (\sum_{l=1}^{L-1} \mathcal{U}_l^H \mathcal{U}_l) \mathbf{h} = \mathbf{h}^H \mathbf{R} \mathbf{h} = 0 \tag{18}$$

where $\mathbf{R}$ is defined as

$$\mathbf{R} = \sum_{l=1}^{L-1} \mathcal{U}_l^H \mathcal{U}_l. \tag{19}$$

This shows that the solution $\mathbf{h}$ is the eigenvector of $\mathbf{R}$ associated with the zeros eigenvalue. When noise is present, $\mathbf{h}$ is the eigenvector of $\mathbf{R}$ associated with the smallest eigenvalue. The necessary steps for implementing the algorithm is as follows.

Step-1: Compute the matrix $\mathcal{Y}^{(M)}$ according to (11) and (12).

Step-2: Compute $\mathbf{U}_0$ and $\mathbf{R}$ according to (14) and (19).

Step-3: Eigen-decompose $\mathbf{R}$ and find $\mathbf{h}$.

Once the channel has been identified, the source symbols can be recovered by a number of equalization techniques such as zero forcing or minimum mean-squared error equalization.

4. SIMULATION RESULTS

In this section, our proposed method is tested and compared with the existing method in [19], which we will denote as SGB method. A Rayleigh fading channel of length $L = 4$ is used. The transmitter breaks the source sequence into blocks of $p = 7$ symbols each and appends $L - 1 = 3$ zeros to each block. Each received block has $(p + L - 1) = 10$ symbols. The normalized least squared channel error (NLSCE), denoted as E_{ch}, is used as the figure of merit for channel identification and is defined as follows.

$$E_{ch} = \|\hat{\mathbf{h}} - \mathbf{h}\|^2 / \|\mathbf{h}\|^2. \tag{20}$$

where $\hat{\mathbf{h}}$ and $\mathbf{h}$ are the estimated and the true channel vectors respectively. The estimated channel is used for recovery of the source symbols using a zero forcing equalizer. Bit Error Rate (BER) is the figure of merit for source recovery. The simulated NLSEC is shown in Figure 1 and the corresponding BER is presented in Figure 2. The results show that using only two blocks to estimated the channel and recover the channel leads to an acceptable BER. Figures 1 and 2 also show an improvement of the NLSCE and BER when using 7 blocks (70 received symbols). This result demonstrates that with the same number of received symbols our method outperforms the algorithm SGB in [19].

5. CONCLUSION

Guard intervals are often inserted between blocks of symbols to prevent inter-block interference by a number of communication systems. Exploiting the guard interval property in each block, this paper proposes a new method that solves the problem of channel identification in the z-domain. Simulation results verified the proposed method and corroborated the theoretical results.

6. REFERENCES

[1] A. Benveniste, M. Goursat, and G. Ruget, "Robust identification of a non-minimum phase system: Blind adjustment of a linear equalizer in data communication," *IEEE Transactions on Automatic Control*, vol. 25, no. 3, pp. 385–399, June 1980.

[2] O. Shalvi and E. Weinstein, "New criteria for blind deconvolution of nonminimum phase systems (channels)," *IEEE Transactions on Information Theory*, vol. 36, no. 2, pp. 312–321, Mar. 1990.

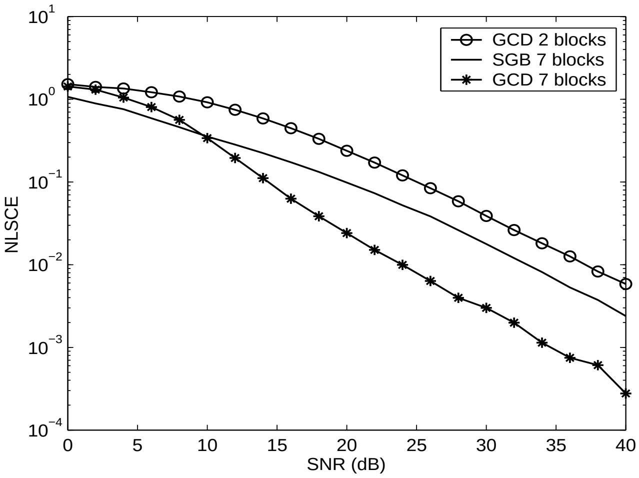

Fig. 1. Norm. least squared channel error: GCD vs. SGB method.

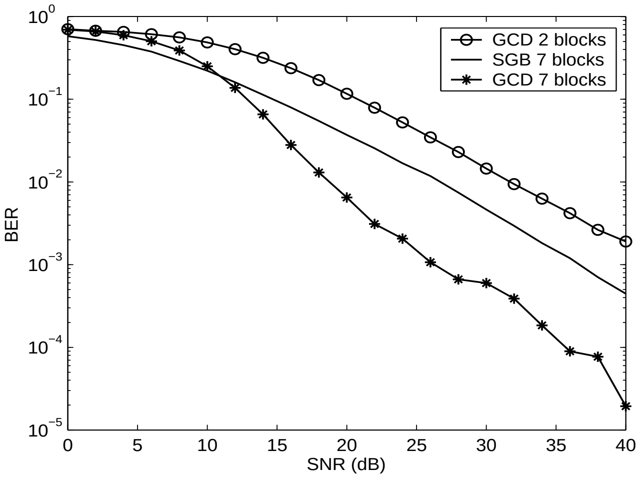

Fig. 2. Bit error rate : GCD vs. SGB method.

[3] C. R. Johnson, "Admissibility in blind adaptive channel equalization," *IEEE Control and Systems Magazine*, pp. 3–15, Jan. 1991.

[4] J. A. Cadzow, "Blind deconvolution via cumulant extrema," *IEEE Signal Processing Magazine*, vol. 13, no. 3, pp. 24–42, May 1996.

[5] A. Benveniste and M. Goursat, "Blind equalizers," *IEEE Transactions on Communications*, vol. 32, pp. 871–883, 1984.

[6] B. Porat and B. Friedlander, "Blind equalization of digital communication channels using high-order moments," *IEEE Transactions on Signal Processing*, vol. 39, no. 2, pp. 522–526, Feb. 1991.

[7] L. Tong, G. Xu, and T. Kailath, "Blind identification and equalization based on second-order statistics: A time domain approach," *IEEE Transactions on Information Theory*, vol. 40, no. 2, pp. 340–349, Mar. 1994.

[8] D. Yenllin and B. Porat, "Blind identification of FIR systems excited by discrete-alphabet inputs," *IEEE Transactions on Signal Processing*, vol. 41, no. 3, pp. 1331–1339, Mar. 1993.

[9] F. Gustafsson and B. Wahlberg, "Blind equalization by direct examination of the input sequences," *IEEE Transactions on Communications*, vol. 43, no. 7, pp. 2213–2222, July 1995.

[10] J. G. Proakis, *Digital Communications, 3nd Edition*, Mc Graw Hill, 1995.

[11] S. Benedetto, E. Biglieri, and V. Castellani, *Digital Transmission Theory*, Prentice-Hall, Inc., 1987.

[12] H. Meyr, M. Moeneclaey, and S.A. Fechtel, *Digital Communication Receivers: Synchronization, Channel Estimation, and Signal Processing*, John Wiley and Sons, Inc., 1997.

[13] W. T. Webb and L. Hanzo, *Modern Quadrature Amplitude Modulation: Principles and Applications for Fixed and Wireless Channels*, IEEE Press, 1994.

[14] A. Peled and A. Ruiz, "Frequency domain data transmission using reduced computational complexity algorithms," in *IEEE International Conference on Acoustic Speech and Signal Processing*, 1980, pp. 964–967.

[15] A. Ruiz, J. M. Cioffi, and S. Kasturia, "Discrete multiple tone modulation with coset coding for the spectrally shaped channel," *IEEE Transactions on Communications*, vol. 40, pp. 1012–1019, 1992.

[16] J. H. Manton, "Finite alphabet source recovery in polynomial systems," *Systems and Control Letters*, 2001, Accepted.

[17] D. H. Pham and J. H. Manton, "A practical algorithm for optimal blind source recovery and channel identification," in preparation.

[18] J. H. Manton and W. D. Neumann, "Totally blind channel identification by exploiting guard intervals," *Systems and Control Letters*, 2002, To appear.

[19] A. Scaglione, G. B. Giannakis, and S. Barbarossa, "Redundant filterbanks precoders and equalizers, Part II: Blind channel estimation, synchronization and direct equalization," *IEEE Transactions on Signal Processing*, vol. 47, pp. 2007–2022, July 1999.

[20] W. Qiu, Y. Hua, and K. Abed-Meraim, "A subspace method for the computation of the GCD of polynomials," *Automatica*, vol. 33, no. 4, pp. 741–743, April 1997.

[21] E. Moulines, P. Duhamel, J. Cardoso, and S. Mayrargue, "Subspace method for the blind identification of multichannel FIR filters," *IEEE Transactions on Signal Processing*, vol. 43, pp. 516–525, February 1995.

[22] H. Liu and G. Xu, "A deterministic approach to blind symbol estimation," *IEEE Signal Processing Letters*, vol. 1, no. 12, pp. 205–207, December 1994.