BLIND CHANNEL IDENTIFICATION ROBUST TO ORDER OVERESTIMATION : A CONSTANT MODULUS APPROACH

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ABSTRACT

In this paper, the problem of Blind System Identification (BSI) of Single-Input Multiple-Output (SIMO) Finite Impulse Response (FIR) channel is addressed. A plethora of methods and techniques have been so far proposed in the literature for BSI including the subspace method. A difficulty with the subspace based identification methods is their sensitivity to mis-specification of the channel order. In this contribution, we propose an identification algorithm robust to channel order overestimation. This algorithm is based on the minimization of a Constant Modulus (CM) in conjunction with subspace orthogonality criteria. Numerical simulations and investigations are presented to demonstrate the potential of the proposed algorithms.

1. INTRODUCTION

Blind identification of communication channels stands for those signal processing techniques that estimate the channel impulse response using only its output statistics. For several years, many methods have been developed to blindly identify the Single-Input Multiple-Output (SIMO) systems from the Second-Order-Statistics (SOS) of data [1, 2]. An important class of blind SOS-based identification algorithms is based on subspace decomposition [3, 4].

One of the important advantages of subspace (SS) technique is its deterministic property. That is, the channel parameters can be recovered perfectly in absence of noise, using only a finite set of data samples, without any statistical assumption over input data. Therefore, subspace method is promising for applications where only a few number of output data are available, or the input data is arbitrary.

However, it is shown in [5] that, subspace method requires the exact prior knowledge or estimation of the channel order; otherwise it fails. When the system order is known, the channel can be estimated up to a constant scalar factor and if an overestimation of system order occurs, there is a linear space of possible but undesirable solutions.

The general approach, presented in this paper, consists of choosing the appropriate channel estimate (among the linear space of possible solutions of the SS criterion) which minimizes the Constant Modulus (CM) criterion.

Based on this idea and exploiting the relationship between Minimum Mean Square Error (MMSE) and CM equalizer (first observed in [6] and widely studied later), we propose a new algorithm for robust channel identification. Proposed algorithm allows blind identification of channel without the prior detection of the system order. It is shown by simulation that this algorithm ensures good performance at low/moderate value of signal to noise ratio even when the channel order is highly overestimated.

2. SUBSPACE ALGORITHM (OVERVIEW)

In this section, the behavior of subspace method when the system order is overestimated is presented based on the results given in [5]. Let $\mathbf{y}(n)$ be a q-variate discrete-time stationary time series given by

$$\begin{aligned}\mathbf{y}(n) &= \sum_{k=0}^{M} \boldsymbol{h}(k)s(n-k) + \mathbf{n}(n) \\ &= [\mathbf{h}(z)]s(n) + \mathbf{n}(n)\end{aligned} \tag{1}$$

where $\mathbf{h}(z) = \sum_{k=0}^{M} \boldsymbol{h}(k) z^{-k}$ is a $q \times 1$ polynomial transfer function modeling the channel and $\{\mathbf{n}(n)\}$ is a measurement noise. It is assumed that $\{\mathbf{n}(n)\}$ is white both temporally and spatially ($E(\mathbf{n}(n)\mathbf{n}^T(n)) = \sigma^2 \mathbf{I}_q$, where σ^2 is unknown), and is independent from the symbol sequence.

Under the hypothesis that $\mathbf{h}(z)$ is full-rank for each z

$$\mathbf{h}(z) \neq 0 \text{ for each } z \qquad \deg(\mathbf{h}(z)) = M \tag{2}$$

it has been shown in [1], that $\mathbf{h}(z)$ and σ^2 are identifiable from a finite number of auto covariance coefficients. A subspace based identification scheme is presented in [3]. This method is based on the covariance matrix of the spatio-temporal vector $\mathbf{Y}(n)$ expressed as

$$\begin{aligned}\mathbf{Y}(n) &= [\mathbf{y}^T(n), ..., \mathbf{y}^T(n-N+1)]^T \\ &= \mathcal{T}_N(\boldsymbol{h})\mathbf{S}(n) + \mathbf{N}(n)\end{aligned} \tag{3}$$

where $\mathbf{S}(n) = [s(n), ..., s(n-N-M+1)]^T$ and $\mathbf{N}(n) = [\mathbf{n}^T(n), ..., \mathbf{n}^T(n-N+1)]^T$. N is a chosen processing window length and $\mathcal{T}_N(\boldsymbol{h})$ is a $qN \times (N+M)$ block Sylvester matrix associated to $\boldsymbol{h} \triangleq [\boldsymbol{h}^T(0), ..., \boldsymbol{h}^T(M)]^T$. The covariance matrix of $\mathbf{Y}(n)$ may be expressed as

$$\boldsymbol{R}_N = \mathcal{T}_N(\boldsymbol{h})\mathcal{S}\mathcal{T}_N^T(\boldsymbol{h}) + \sigma^2 \boldsymbol{I}_{qN} \tag{4}$$

where $\mathcal{S} \triangleq E(\mathbf{S}(n)\mathbf{S}^T(n))$.

The first term in the right hand side of (4) is singular as soon as $qN > (N+M)$ (this condition is assumed to hold throughout). In this case, the noise variance σ^2 is the smallest eigenvalue of $\boldsymbol{R}_N$. The eigenspace associated to σ^2 is referred to as the *noise subspace*. $\mathcal{S}$ is assumed to be positive-definite and thus the noise subspace is the orthogonal complement of Range$(\mathcal{T}_N(\boldsymbol{h}))$, the *signal subspace*.

The eigen-decomposition of $\boldsymbol{R}_N$ allows to identify the noise subspace Range$(\mathcal{T}_N(\boldsymbol{h}))^{\perp}$. Denote by Π_N the orthogonal projection matrix onto Range$(\mathcal{T}_N(\boldsymbol{h}))^{\perp}$.

In order to estimate the channel, we have to characterize the set of all polynomials $\mathbf{h}'(z)$ of degree M', satisfying $\Pi_N \mathcal{T}_N(\boldsymbol{h}') = 0$. The following lemma holds:

Lemma 1 : Assume (2) holds and $N \geq M$. Denote by Π_N the orthogonal projection matrix onto the noise subspace of $\boldsymbol{R}_N$. The matrix equation

$$\Pi_N \mathcal{T}_N(\boldsymbol{h}') = 0 \tag{5}$$

does not admit any solution if $M' < M$ (where $M' = \deg(\mathbf{h}'(z))$. If $M' \geq M$, the solutions of (5) are of the form $\mathbf{h}'(z) = r(z)\mathbf{h}(z)$ where $r(z)$ is a random scalar polynomial of degree $M' - M$.

Matrix equation (5) is solved in a least squares sense. Since the Sylvester matrix depends linearly on its parameters, the least squares criterion can be rewritten as a quadratic form

$$\mathrm{Tr}(\mathcal{T}_N^T(\boldsymbol{h}')\Pi_N \mathcal{T}_N(\boldsymbol{h}')) = \boldsymbol{h}'^T \mathcal{Q}_N \boldsymbol{h}'$$

$\mathcal{Q}_N$ denotes a $q(M'+1) \times q(M'+1)$ symmetric matrix. When the degree M is known or correctly estimated, $\mathbf{h}'(z)$ can be written as $\mathbf{h}'(z) = \alpha \mathbf{h}(z)$ (where α is a real scalar) according to *lemma 1*. Therefore, the channel can be estimated up to a scale factor by minimizing in $\boldsymbol{h}'$ the above mentioned criterion under a suitable constraint.

On the other hand, when $M' > M$, there are many solutions of the form $\mathbf{h}'(z) = r(z)\mathbf{h}(z)$. In this case, we have the following result:

Proposition 1 : Under the above mentioned conditions, matrix $\mathcal{Q}_N$ is singular with a null space of dimension $d = M' - M + 1$.

Consequently, the linear space of solutions can be obtained using $L \geq d$ eigen-vectors, represented here by matrix $\mathbf{U_L}$, associated to the L least eigen-values of $\mathcal{Q}_N$. Mathematically, the desired solution can be expressed as

$$\boldsymbol{h}'_{\mathbf{v}} = \mathbf{U_L}\mathbf{v} \tag{6}$$

where $\mathbf{U_L}$ denotes a $q(M'+1) \times L$ matrix, $\mathbf{v}$ stands for an L-dimensional vector to be estimated and

$$\boldsymbol{h}'_{\mathbf{v}} = [\boldsymbol{h}'^T_{\mathbf{v}}(0), ..., \boldsymbol{h}'^T_{\mathbf{v}}(M')]^T.$$

3. PROPOSED ALGORITHM

In this section, we mention first a well known property concerning the CM criterion. Godard was the first who observed that the mean square error performance of CMA is close to that of the MMSE equalizer [6]. For several years, a lot of research efforts was made to confirm or prove that, under several conditions, the CM minima remain in the vicinity of MSE minima for different choices of delay and sign (see [11, 10] and [12]). Among other works, we can cite an approach used in [9]. It consists of plotting the contour of CM cost function in equalizer space and compare the location of CM minima and MMSE equalizer. This approach confirms the above mentioned result, under certain conditions.

As mentioned before, the main idea of this paper is to find the L-dimensional vector $\mathbf{v}$, minimizing the CM cost function under the CM assumption i.e. $\forall n \quad |s(n)| = C$. Where $C > 0$ is a given constant. More precisely, given the channel estimate in (6), we can express an MMSE equalizer vector as a function of $\mathbf{v}$ according to :

$$\begin{aligned} \mathbf{w} &= \hat{\mathbf{R}}_N^{-1} \begin{bmatrix} \boldsymbol{h}'_{\mathbf{v}}(M') \\ \vdots \\ \boldsymbol{h}'_{\mathbf{v}}(0) \\ \mathbf{0}_{q(N-M'-1),1} \end{bmatrix} \\ &= \mathbf{W}\mathbf{v} \end{aligned} \tag{7}$$

Where

$$\mathbf{W} \triangleq \hat{\boldsymbol{R}}_N^{\ -1} \begin{bmatrix} \boldsymbol{U}_{L,M'} \\ \vdots \\ \boldsymbol{U}_{L,0} \\ \mathbf{0}_{q(N-M'-1),L} \end{bmatrix}$$

$\boldsymbol{U}_{L,i}$ denotes the $q \times L$ matrix corresponding to the i_th block of matrix $\boldsymbol{U}_L$ (i.e. $\boldsymbol{U}_L = [\boldsymbol{U}_{L,0}^T, ..., \boldsymbol{U}_{L,M'}^T]^T$). The desired vector $\mathbf{v}$ associated to the desired channel estimate (the desired channel estimate corresponds to $\boldsymbol{h}'_{\mathbf{v}} = \alpha[\mathbf{0}^T_{qk,1},\ \boldsymbol{h}^T,\ \mathbf{0}^T_{q(M'-M-k),1}]^T$. for a given scalar constant α and a positive integer k) is obtained by minimizing the following CM criterion:

$$\begin{aligned} \min\ \mathcal{J}(\mathbf{w}) &= \min E(|\mathbf{w}^T\mathbf{Y}(n)|^2 - r)^2 \\ &= \min_{\mathbf{v}} E(\ |\mathbf{v}^T\boldsymbol{Z}(n)|^2 - r\ )^2 \end{aligned} \tag{8}$$

where $\boldsymbol{Z}(n) = \mathbf{W}^T\mathbf{Y}(n)$ and r represents the dispersion constant.

In order to minimize equation (8), we constrain $\mathbf{v}$ to be of unit norm and we use a parameterization based on the following result [7][1] :

Lemma 2 : Each unit norm row vector can be represented as the last row of an orthogonal matrix $\mathbf{P}$ given by:

$$\mathbf{P} = \prod_{1 \leq p \leq it\_nb} \left(\prod_{1 \leq i \leq L-1} \boldsymbol{P}(\theta_p^i) \right) \tag{9}$$

where θ_p^i are a set of rotation angles in $]-\pi/2, \pi/2]$ and

$$\boldsymbol{P}(\theta_p^i) = \begin{bmatrix} \boldsymbol{I}_{i-1} & & & \\ & \cos(\theta_p^i) & \dots & -\sin(\theta_p^i) \\ & \vdots & \boldsymbol{I}_{L-i-1} & \vdots \\ & \sin(\theta_p^i) & \dots & \cos(\theta_p^i) \end{bmatrix} \tag{10}$$

Consequently, we propose a recursive minimization algorithm, where at each step, the cost function (8) is written as a function of rotation angle θ_p^i :

$$\min_{\theta} \mathcal{J}(\theta_p^i) = E(\ |\mathbf{v}_0^T(\boldsymbol{P}(\theta_p^i))^T\boldsymbol{Z}(n)|^2 - r\ )^2 \tag{11}$$

$\mathbf{v}_0^T$ is a row vector of length L with all components equal to zero except the last one which is equal to one. This choice of $\mathbf{v}_0^T$ permits us to select the last row of orthogonal matrix $\boldsymbol{P}(\theta_p^i)$.

At each iteration, the angle θ_p^i that minimizes the cost function (11) is computed. The algorithm is stopped when $\boldsymbol{P}(\theta_p^i)$ are close to identity matrix for all $1 \leq i \leq L-1$. More precisely, we have the following iterative process :

[1] Note that constraining $\mathbf{v}$ to be of unit norm is equivalent to constrain $||\boldsymbol{h}'_{\mathbf{v}}|| = 1$ since $\mathbf{U_L}$ is unitary.

1. Initialization [2]:

$$\mathbf{v_0}^T = [\underbrace{0, ..., 0}_{L-1}, 1] \quad (12)$$

2. For $i = 1, 2, ..., L-1$ and the current iteration p, find the rotation which minimizes the cost function :

$$\theta_p^i = \text{argmin}\{\mathcal{J}(\theta_p^i)\} \quad (13)$$

The minimization details are discussed below.

3. Compute the new values of $\boldsymbol{Z}$ and $\mathbf{v}$:

$$\begin{aligned} \boldsymbol{Z} &:= \boldsymbol{P}(\theta_p^i)^T \boldsymbol{Z} \\ \mathbf{v} &:= \boldsymbol{P}(\theta_p^i)\, \mathbf{v} \end{aligned}$$

where $\boldsymbol{Z} \triangleq [\boldsymbol{Z}(N), ..., \boldsymbol{Z}(T)]$ (T being the sample size).

4. If θ_p^i for all $1 \leq i \leq L-1$ are close to zero, then stop. Else, $p = p+1$ and go to step 2.

Here, we describe how the cost function $\mathcal{J}(\theta_p^i)$ is minimized. This cost function can be written as (we replace the expectation by time averaging):

$$\begin{aligned} \mathcal{J}(\theta_p^i) &= \sum_n \left(|-\sin(\theta_p^i) Z^i(n) + \cos(\theta_p^i) Z^L(n)|^2 - r\right)^2 \\ &= \sum_n \left(\mathbf{u}^T \boldsymbol{y}^i(n) + \alpha^i(n)\right)^2 \quad (14) \\ &= \mathbf{u}^T \underbrace{\left(\sum_n \boldsymbol{y}^i(n) \boldsymbol{y}^i(n)^T\right)}_{\mathbf{G}} \mathbf{u} + 2 \underbrace{\left(\sum_n \alpha^i(n) \boldsymbol{y}^i(n)\right)}_{\mathbf{g}^T} \mathbf{u} \end{aligned}$$

[3]where

$$\mathbf{u} = \begin{bmatrix} \cos(2\,\theta_p^i) \\ \sin(2\,\theta_p^i) \end{bmatrix} \quad (15)$$

and

$$\begin{aligned} \boldsymbol{y}^i(n) &= \begin{bmatrix} \left(Z^i(n)^2 - Z^L(n)^2\right)/2 \\ -Z^i(n) Z^L(n) \end{bmatrix}, \\ \alpha^i(n) &= \frac{Z^i(n)^2 + Z^L(n)^2}{2} - r. \quad (16) \end{aligned}$$

where $Z^i(n)$ denotes the i-th entry of $\boldsymbol{Z}(n)$. Therefore, minimizing $\mathcal{J}(\theta_p^i)$ versus θ_p^i is equivalent to minimizing the following equation subject to $||\mathbf{u}||^2 = 1$.

$$\min_{||\mathbf{u}||^2=1} \tilde{\mathcal{J}}(\mathbf{u}) = \min_{||\mathbf{u}||^2=1} (\mathbf{u}^T \mathbf{G} \mathbf{u} + 2\mathbf{g}^T \mathbf{u}) \quad (17)$$

In order to minimize equation (17), we use the method of Lagrange multipliers [8]. By zeroing the gradient of (17), we obtain the following expression of $\mathbf{u}$:

$$\begin{aligned} \mathbf{u} &= -(\mathbf{G} + \lambda \boldsymbol{I})^{-1} \mathbf{g} \quad (18) \\ &= -\left[\frac{(\boldsymbol{u}_1^T \mathbf{g})}{(\lambda + \lambda_1)} \boldsymbol{u}_1 + \frac{(\boldsymbol{u}_2^T \mathbf{g})}{(\lambda + \lambda_2)} \boldsymbol{u}_2\right] \end{aligned}$$

[2]This initialization corresponds to choosing at first the channel estimate of the standard subspace algorithm that is given by the least eigenvector of $\mathcal{Q}_N$, i.e. the last column vector of $\mathbf{U_L}$.

[3]In (14), we omit a constant term independent from θ_p^i.

where $\mathbf{G} = \lambda_1 \boldsymbol{u}_1 \boldsymbol{u}_1^T + \lambda_2 \boldsymbol{u}_2 \boldsymbol{u}_2^T$ is the eigen decomposition of $\mathbf{G}$. Using the fact that $||\mathbf{u}||^2 = 1$, λ must satisfy:

$$\left[\frac{(\boldsymbol{u}_1^T \mathbf{g})}{(\lambda + \lambda_1)}\right]^2 + \left[\frac{(\boldsymbol{u}_2^T \mathbf{g})}{(\lambda + \lambda_2)}\right]^2 = 1 \quad (19)$$

This corresponds to a polynomial equation of degree 4. By solving this equation and choosing λ equal to its real root, we obtain $\mathbf{u}$ from equation (18). Consequently, the corresponding θ_p^i can be found. If there are more than one real root to the above equation then, the solution which minimizes the cost function (17) is selected as the desired one.

Remark: In the case where the source signal is not of constant modulus, the minima of MMSE and CM criteria do not coincide (or at least are not close) [9] and thus the proposed algorithm fails to provide a consistent channel estimate.

4. SIMULATION RESULTS

In this section, we present simulation results in order to assess the performance of our algorithm. We have considered a one-input two-output system. The input sequence is an i.i.d. zero mean unit variance BPSK process. We consider a normalized channel of degree 2. The channel coefficients are :

$$\begin{aligned} h_1(z) &= -0.2931 - (0.0151)z^{-1} - (0.1497)z^{-2} \\ h_2(z) &= 0.5029 + (0.7448)z^{-1} + (0.1505)z^{-2} \end{aligned}$$

The algorithm performance is measured in terms of distance between estimated and real channel, as in [13], by :

$$\begin{aligned} \text{MSE}(\hat{\boldsymbol{h}}) &\triangleq \min_{\alpha, k \geq 0} \left\| \alpha \hat{\boldsymbol{h}} - \begin{bmatrix} \mathbf{0}_{qk,1} \\ \boldsymbol{h} \\ \mathbf{0}_{q(M'-M-k),1} \end{bmatrix} \right\|^2 \\ &= \min_{k \geq 0} \left\| (\mathbf{I} - \hat{\boldsymbol{h}} \hat{\boldsymbol{h}}^{\#}) \begin{bmatrix} \mathbf{0}_{qk,1} \\ \boldsymbol{h} \\ \mathbf{0}_{q(M'-M-k),1} \end{bmatrix} \right\|^2 \end{aligned}$$

where $\hat{\boldsymbol{h}}^{\#} = \hat{\boldsymbol{h}}^T / ||\hat{\boldsymbol{h}}||^2$. Statistics are evaluated over 100 Monte-Carlo runs.

Figure 1 presents the MSE of the estimated channel for different values of SNR with a sample size $T = 1000$. For this simulation L is fixed to $M' - M + 1$. The performance of the algorithm when an overestimation of channel order occurs remains acceptable. For example, for a SNR = 25 dB, $M' = M+2$ and $L = 3$, the distance between estimated and real channel is -23 dB.

Table 1 provides simulation results for several values of M' and L. The SNR is fixed to 30 dB. It is shown that even for large values of M' and L we obtain relatively good performance.

	MSE (dB)		
M'	$L=M'-M+1$	$L=M'-M+2$	$L=M'-M+3$
3	-32.15	-20.37	-26.45
4	-24.10	-23.22	-21.35
5	-19.77	-17.84	-18.18
6	-16.17	-18.76	-18.35

Table 1. Channel estimates MSE using CMA approach

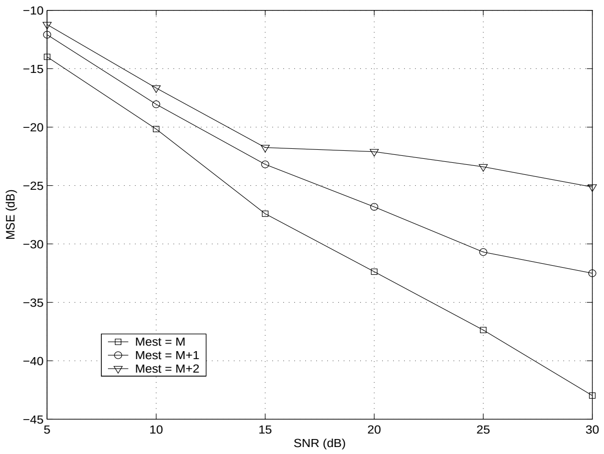

Fig. 1. MSE versus SNR

Table 2 demonstrates the performance of proposed algorithm for a SNR=30 dB. Different values of T and M' have been considered. The value of L is equal to $M' - M + 1$. The performance of the algorithm depends on the sample size. A sample size $T = 500$ seems to be sufficient to achieve acceptable MSE even for large values of the estimated order M'.

	MSE (dB)		
T	$M' = M + 1$	$M' = M + 2$	$M' = M + 3$
100	-15.23	-12.39	-7.65
250	-22.38	-18.96	-15.78
500	-28.42	-20.49	-18.27

Table 2. Channel estimates MSE using CMA approach

5. CONCLUSION

In this contribution, we propose an identification algorithm robust to channel order overestimation. This algorithm is based on the minimization of a CM cost function in conjunction with a subspace based criterion. It is shown by simulation that when constant modulus source signal is used, this algorithm achieves good performance in low/moderate signal to noise ratio.

APPENDIX
(COMPLEX CASE**)**

For simplicity, we have considered previously the case where the signals and the channels are real-valued. In the complex case, the proposed algorithm remains essentially the same except for the fact that the rotation matrices are function of two angle parameters θ and α (i.e., $-\sin(\theta)$ and $\sin(\theta)$ are replaced by $-\sin(\theta)e^{j\alpha}$ and $\sin(\theta)e^{-j\alpha}$, respectively).

In that case, optimizing the CM cost function versus these angle parameters leads to:

$$\min_{||\overline{\mathbf{u}}||^2=1} \overline{\mathcal{J}}(\overline{\mathbf{u}}) = \min_{||\overline{\mathbf{u}}||^2=1} (\overline{\mathbf{u}}^T \overline{\mathbf{G}}\overline{\mathbf{u}} + 2\overline{\mathbf{g}}^T \overline{\mathbf{u}}) \qquad (20)$$

where

$$\overline{\mathbf{G}} = \sum_n \overline{\boldsymbol{y}}^i(n)\overline{\boldsymbol{y}}^i(n)^T \quad \text{and} \quad \overline{\mathbf{g}} = \sum_n \overline{\alpha}^i(n)\overline{\boldsymbol{y}}^i(n)$$

$$\begin{aligned}
\overline{\mathbf{u}} &= [\cos(2\theta), \sin(2\theta)\cos(\alpha), \sin(2\theta)\sin(\alpha)]^T, \\
\overline{\boldsymbol{y}}^i(n) &= \begin{bmatrix} \left(|Z^i(n)|^2 - |Z^L(n)|^2\right)/2 \\ -\Re e(Z^i(n)Z^L(n)^*) \\ -\Im m(Z^i(n)Z^L(n)^*) \end{bmatrix}, \\
\overline{\alpha}^i(n) &= \frac{|Z^i(n)|^2 + |Z^L(n)|^2}{2} - r,
\end{aligned}$$

that can be solved in the same way as equation (17).

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