



BLIND IDENTIFICATION OF MIMO SYSTEMS BY A SYSTEM TO HOS BASED INVERSE FILTER RELATIONSHIP

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ABSTRACT

Higher-order statistics based inverse filter criteria (HOS-IFC) proposed by Tugnait and Chi et al. have been widely used for blind identification and deconvolution of multiple-input multiple-output (MIMO) linear time-invariant systems with a set of non-Gaussian measurements. Based on a relationship, that holds true for finite signal-to-noise ratio, between the optimum inverse filter associated with the HOS-IFC and the unknown MIMO system, an iterative FFT-based blind system identification (BSI) algorithm for MIMO systems is proposed in this paper, for which common subchannel zeros are allowed and the system order information is never needed, and meanwhile its performance is superior to the performance of Tugnait's HOS-IFC approach. Some simulation results are presented to support the efficacy of the proposed BSI algorithm.

1. INTRODUCTION

Blind identification of a discrete-time multiple-input multiple-output (MIMO) linear time-invariant (LTI) system, denoted $\mathbf{H}[n]$ ($P \times K$ impulse response matrix), is to estimate $\mathbf{H}[n]$ with only a set of *non-Gaussian* measurements $\mathbf{y}[n] = (y_1[n], y_2[n], \dots, y_P[n])^T$ given by

$$\mathbf{y}[n] = \mathbf{H}[n] * \mathbf{u}[n] + \mathbf{w}[n] = \sum_{i=-\infty}^{\infty} \mathbf{H}[i] \mathbf{u}[n-i] + \mathbf{w}[n] \quad (1)$$

where $\mathbf{u}[n] = (u_1[n], u_2[n], \dots, u_K[n])^T$ and $\mathbf{w}[n] = (w_1[n], w_2[n], \dots, w_P[n])^T$ are the driving input vector and measurement noise vector, respectively. Such models arise in digital multiuser/multiaccess communications, digital radio with diversity, multisensor systems, etc. [1, 2] and thus, MIMO blind system identification (BSI) is essential in these areas.

Tugnait [3] and Chi et al. [4] proposed higher-order statistics based inverse filter criteria (HOS-IFC) for

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blind deconvolution of MIMO systems (i.e., for estimation of $\mathbf{u}[n]$). Then the system estimate $\widehat{\mathbf{H}}[n]$ is obtained by cross-correlation of $\mathbf{y}[n]$ and the estimate $\widehat{\mathbf{u}}[n]$. This approach, referred to as the deconvolution followed by input-output cross-correlation (DEC-IOCC) method, results in biased system estimates for finite signal-to-noise ratio (SNR) due to the noise presence in $\mathbf{y}[n]$ and estimation error in $\widehat{\mathbf{u}}[n]$ [2].

Recently, Chi and Chen [5] established a connection between the optimal inverse filter associated with the HOS-IFC and the non-blind minimum mean-square error equalizer for MIMO systems. This connection further unveils a relationship, which holds true for finite SNR, between the unknown MIMO system and the optimal inverse filter associated with the HOS-IFC. Based on this relationship, Chi et al. [6] proposed a BSI algorithm for single-input multiple-output systems ($K = 1$), that outperforms Tugnait's DEC-IOCC method [3, 4] for finite SNR. Again, based on this relationship, this paper further proposes a BSI algorithm for MIMO systems that also outperforms the latter for finite SNR.

2. RELATIONSHIP BETWEEN THE SYSTEM AND THE OPTIMUM HOS BASED INVERSE FILTER

For ease of later use, let $\text{cum}\{x_1, x_2, \dots, x_p\}$ denote the joint cumulant of random variables x_1, x_2, \dots, x_p and $\mathcal{F}_n\{\cdot\}$ denote the discrete-time Fourier transform operator with respect to index n , and define the following notations

$\ \cdot\ $: Euclidean vector/matrix norm
$E\{\cdot\}$: Expectation operator
Superscript *	: Complex conjugation
$\text{cum}\{x : p, \dots\}$	$= \text{cum}\{x_1 = x, x_2 = x, \dots, x_p = x, \dots\}$
$C_{p,q}\{x\}$	$= \text{cum}\{x : p, x^* : q\}$
$\gamma_p\{x\}$	$= C_{p,p}\{x\} / C_{1,1}\{x\}$
$\mathbf{D}\{x_k; K\}$	$= \text{diag}\{x_1, \dots, x_K\}$ (diagonal matrix)

Some general assumptions for the noisy output $\mathbf{y}[n]$ of the MIMO system $\mathbf{H}[n]$ given by (1) are as follows:

- (A1) $u_k[n], k \in \{1, 2, \dots, K\}$, is a zero-mean, independent identically distributed (i.i.d.) non-Gaussian random process with $C_{p,q}\{u_k[n]\} \neq 0$ and statistically independent of $u_j[n]$ for all $j \neq k$.
- (A2) The K -input P -output system $\mathbf{H}[n]$ is stable and $P \geq K$.
- (A3) $\mathbf{w}[n]$ is zero-mean Gaussian, and statistically independent of $\mathbf{u}[n]$.

Processing $\mathbf{y}[n]$ by a P -input one-output inverse filter, denoted $\mathbf{v}[n] = (v_1[n], v_2[n], \dots, v_P[n])^T$, yields the deconvolved signal

$$e[n] = \mathbf{v}^T[n] * \mathbf{y}[n]. \quad (2)$$

Under the assumptions (A1) through (A3), Chi et al. [4] find the optimum $\mathbf{v}[n]$ by maximizing the following HOS-IFC (including Tugnait's HOS-IFC [3])

$$J_{p,q}(\mathbf{v}[n]) = \frac{|C_{p,q}\{e[n]\}|}{|C_{1,1}\{e[n]\}|^{(p+q)/2}}, \quad p+q \geq 3. \quad (3)$$

The optimum $e[n] = \hat{u}_\ell[n]$ (an estimate of $u_\ell[n]$ for finite SNR) is shown to be $\alpha_\ell u_\ell[n - \tau_\ell]$ for SNR equal to infinity, where α_ℓ and τ_ℓ are unknown scale factor and time delay, respectively, and $\ell \in \{1, 2, \dots, K\}$ is unknown. Tugnait's DEC-IOCC method [3] estimates the ℓ th column of $\mathbf{H}[n]$ by

$$\hat{\mathbf{h}}_\ell[n] = \frac{E\{\mathbf{y}[m+n]\hat{u}_\ell^*[m]\}}{E\{|\hat{u}_\ell[m]|^2\}}. \quad (4)$$

Through the multistage successive cancellation procedure reported in [3], all of the K driving inputs and the system $\mathbf{H}[n]$ can be estimated, though the resultant system estimate of $\mathbf{H}[n]$ is not unbiased for finite SNR [2].

Consider the HOS-IFC $J_{p,p}$ ($p = q \geq 2$) and define

$$\begin{aligned} \mathcal{H}(\omega) &= \mathcal{F}_n\{\mathbf{H}[n]\} \\ \mathbf{V}(\omega) &= \mathcal{F}_n\{\mathbf{v}[n]\} \text{ associated with } J_{p,p} \\ \mathcal{P}_y(\omega) &= \mathcal{F}_k\{E\{\mathbf{y}[n]\mathbf{y}^H[n-k]\}\} \\ \mathbf{A}_u &= \mathbf{P}_H^T \mathbf{D} \{x_k = C_{p,p}\{u_k[n]/\alpha_k\}; K\} \mathbf{P}_H \\ \tilde{\mathcal{H}}(\omega) &= \mathcal{H}(\omega) \mathbf{\Gamma}(\omega) \mathbf{A} \mathbf{P}_H = \mathcal{F}_n\{\tilde{\mathbf{H}}[n]\} \\ s_k[n] &: \text{kth entry of } \mathbf{s}[n] = \tilde{\mathbf{H}}^T[n] * \mathbf{v}[n] \end{aligned} \quad (5)$$

where $\mathbf{\Gamma}(\omega) = \mathbf{D}\{x_k = e^{-j\omega\tau_k}; K\}$ is a “time-shift” diagonal matrix, $\mathbf{A} = \mathbf{D}\{x_k = \alpha_k; K\}$ is a scaling diagonal matrix, and \mathbf{P}_H is a $K \times K$ permutation matrix. The nonlinear relationship between $\mathbf{V}(\omega)$ ($P \times 1$ filter) and $\mathcal{H}(\omega)$ is described by the following fact.

Fact 1: (Derived from Property 2 in [5]) For finite SNR and sufficient length of $\mathbf{v}[n]$,

$$\mathcal{P}_y^T(\omega) \mathbf{V}(\omega) \gamma_p\{e[n]\} = \tilde{\mathcal{H}}^*(\omega) \mathbf{\Lambda}_u \mathcal{G}(\omega) \quad (6)$$

where $\mathbf{G}(\omega)$ is a $K \times 1$ vector with the k th element $G_k(\omega) = \mathcal{F}_n\{|s_k[n]|^{2(p-1)} \cdot s_k[n]\}$. \square

Let $\mathbf{v}_k[n]$ (or $\mathbf{V}_k(\omega) = \mathcal{F}_n\{\mathbf{v}_k[n]\}$), $k = 1, 2, \dots, K$ denote the optimum inverse filters associated with the HOS-IFC $J_{p,p}$ with respect to the inputs $u_k[n], k = 1, 2, \dots, K$, respectively. Furthermore, let us define

$$\begin{aligned} \mathbf{V}[n] &= (\mathbf{v}_1[n], \mathbf{v}_2[n], \dots, \mathbf{v}_K[n]) \\ \mathbf{V}(\omega) &= \mathcal{F}_n\{\mathbf{V}[n]\} = (\mathbf{V}_1(\omega), \mathbf{V}_2(\omega), \dots, \mathbf{V}_K(\omega)) \\ \tilde{\mathbf{V}}(\omega) &= \mathbf{V}(\omega) \mathbf{P}_V \text{ (i.e., } \tilde{\mathbf{V}}[n] = \mathbf{V}[n] \mathbf{P}_V) \\ e_k[n] &= \mathbf{v}_k^T[n] * \mathbf{y}[n] \\ \mathbf{\Lambda}_e &= \mathbf{P}_V^T \mathbf{D} \{x_k = \gamma_p\{e_k[n]\}; K\} \mathbf{P}_V \\ s_{i,j}[n] &: (i, j)\text{th element of } \mathbf{S}[n] = \tilde{\mathbf{H}}^T[n] * \tilde{\mathbf{V}}[n] \end{aligned}$$

where \mathbf{P}_V is a $K \times K$ permutation matrix. Concatenating the K relations of (6) associated with $\mathbf{v}_k[n], k = 1, 2, \dots, K$ yields the nonlinear relationship between $\mathbf{V}(\omega)$ ($P \times K$ system) and $\mathcal{H}(\omega)$ as described by the following fact.

Fact 2: For finite SNR and sufficient length of $\mathbf{v}_k[n]$'s,

$$\mathcal{P}_y^T(\omega) \tilde{\mathbf{V}}(\omega) \mathbf{\Lambda}_e = \tilde{\mathcal{H}}^*(\omega) \mathbf{\Lambda}_u \mathcal{G}(\omega) \quad (7)$$

where $\mathcal{G}(\omega)$ is a $K \times K$ matrix with the (i, j) th element $[\mathcal{G}(\omega)]_{i,j} = \mathcal{F}_n\{|s_{i,j}[n]|^{2(p-1)} \cdot s_{i,j}[n]\}$. \square

Note that the relationship given by (7) implies that $\mathbf{H}[n]$ can be estimated (up to a time-shift matrix, a scaling matrix and a permutation matrix; see (5)) from the cross-spectral matrix $\mathcal{P}_y(\omega)$ and the optimal inverse filter $\tilde{\mathbf{V}}(\omega)$ (a permuted version of $\mathbf{V}(\omega)$). Next, let us present an iterative algorithm for estimating $\mathbf{H}[n]$ based on the relationship given by (7).

3. BLIND IDENTIFICATION OF MIMO SYSTEMS

Let $\omega_l = 2\pi l/\mathcal{L}, l = 0, 1, \dots, \mathcal{L} - 1$, and let

$$\begin{aligned} \mathcal{H}[l] &= \mathcal{H}(\omega_l) \text{ (similarly for } \mathcal{P}_y, \mathbf{V}, \mathcal{G}, \text{ etc.)} \\ \bar{\mathbf{a}}_k &= (\mathbf{a}_k[0]^T, \dots, \mathbf{a}_k[\mathcal{L}-1]^T)^T, \quad k = 1, 2, \dots, K \\ \bar{\mathbf{b}}_k &= (\mathbf{b}_k[0]^T, \dots, \mathbf{b}_k[\mathcal{L}-1]^T)^T, \quad k = 1, 2, \dots, K \end{aligned}$$

where $\mathbf{a}_k[l]$ and $\mathbf{b}_k[l]$ are the k th column of the matrices $(\mathcal{P}_y^T[l] \tilde{\mathbf{V}}[l] \mathbf{\Lambda}_e)$ and $(\tilde{\mathcal{H}}^*[l] \mathbf{\Lambda}_u \mathcal{G}[l])$, respectively.

Finding the MIMO system $\mathcal{H}(\omega)$ that satisfies the relationship given by (7) is equivalent to finding $\mathcal{H}[l], l = 0, 1, \dots, \mathcal{L} - 1$, such that

$$\mathbf{a} = (\bar{\mathbf{a}}_1^T, \bar{\mathbf{a}}_2^T, \dots, \bar{\mathbf{a}}_K^T)^T = \mathbf{b} = (\bar{\mathbf{b}}_1^T, \bar{\mathbf{b}}_2^T, \dots, \bar{\mathbf{b}}_K^T)^T. \quad (8)$$

Equation (8) implies that $\tilde{\mathcal{H}}(\omega)$ can be estimated up to a scale factor by maximizing the following objective function:

$$\mathcal{C}(\tilde{\mathcal{H}}) = \frac{|\mathbf{a}^H \mathbf{b}|}{\|\mathbf{a}\| \cdot \|\mathbf{b}\|} \quad (9)$$

because $0 \leq \mathcal{C}(\tilde{\mathcal{H}}) = \mathcal{C}(\beta \tilde{\mathcal{H}}) \leq 1$ for any non-zero scale factor β and $\mathcal{C}(\tilde{\mathcal{H}}) = 1$ as $\mathbf{a} = c\mathbf{b}$ for any non-zero constant c . Thus, $\mathbf{H}[n]$ can be estimated (up to a time-shift matrix, a scaling matrix and a permutation matrix) by the following iterative algorithm with a given set of finite measurements $\mathbf{y}[n]$.

BSI Algorithm:

Step 1. Blind Deconvolution and Cross-spectral Matrix Estimation

- (S1) Obtain the K optimum inverse filters $\hat{\mathbf{v}}_k[n]$ and $\hat{e}_k[n]$, $k = 1, 2, \dots, K$ associated with the HOS-IFC $J_{p,p}$, and then form $\tilde{\mathbf{V}}[n] = (\hat{\mathbf{v}}_1[n], \hat{\mathbf{v}}_2[n], \dots, \hat{\mathbf{v}}_K[n])$, $\tilde{\mathbf{\Lambda}}_e = \mathbf{D}\{x_k = \gamma_p\{\hat{e}_k[n]\}; K\}$ and $\tilde{\mathbf{\Lambda}}_u = \mathbf{D}\{x_k = C_{p,p}\{\hat{e}_k[n]\}; K\}$.
- (S2) Obtain the cross-spectral matrix estimate $\hat{\mathcal{P}}_{\mathbf{y}}[l]$ by multichannel power spectral estimation methods, and then compute $\mathcal{A}[l] = \hat{\mathcal{P}}_{\mathbf{y}}^T[l] \tilde{\mathbf{V}}[l] \tilde{\mathbf{\Lambda}}_e$.

Step 2. System Identification

- (S3) Set $i = 0$. Set initial values $\tilde{\mathcal{H}}_0[l]$ for $\tilde{\mathcal{H}}[l]$ and a convergence tolerance $\epsilon > 0$.
- (S4) Update i by $i + 1$. By **Fact 2**, compute $\mathcal{G}_{i-1}[l]$ from $\tilde{\mathcal{H}}_{i-1}[l]$ and $\tilde{\mathbf{V}}[l]$ and then obtain

$$\tilde{\mathcal{H}}_i[l] = \left(\mathcal{A}[l] \mathcal{G}_{i-1}^{-1}[l] \tilde{\mathbf{\Lambda}}_u^{-1} \right)^*$$

which is then normalized by $\sum_l \|\tilde{\mathcal{H}}_i[l]\|^2 = 1$.

- (S5) If $\mathcal{C}(\tilde{\mathcal{H}}_i) > \mathcal{C}(\tilde{\mathcal{H}}_{i-1})$, go to (S6); else compute $\Delta\tilde{\mathcal{H}}_i[l] = \tilde{\mathcal{H}}_i[l] - \tilde{\mathcal{H}}_{i-1}[l]$ and update $\tilde{\mathcal{H}}_i[l]$ via

$$\tilde{\mathcal{H}}_i[l] = \tilde{\mathcal{H}}_{i-1}[l] + \mu \cdot \Delta\tilde{\mathcal{H}}_i[l]$$

where μ is chosen such that $\mathcal{C}(\tilde{\mathcal{H}}_i) > \mathcal{C}(\tilde{\mathcal{H}}_{i-1})$, and then normalize $\tilde{\mathcal{H}}_i[l]$ by $\sum_l \|\tilde{\mathcal{H}}_i[l]\|^2 = 1$.

- (S6) If

$$\frac{\mathcal{C}(\tilde{\mathcal{H}}_i) - \mathcal{C}(\tilde{\mathcal{H}}_{i-1})}{\mathcal{C}(\tilde{\mathcal{H}}_{i-1})} > \epsilon,$$

then go to (S4); otherwise, the frequency response estimate $\tilde{\mathcal{H}}[l] = \tilde{\mathcal{H}}_i[l]$, $l = 0, 1, \dots, \mathcal{L} - 1$, and its \mathcal{L} -point inverse FFT $\hat{\mathbf{H}}[n]$ are obtained.

Several worthy remarks regarding the proposed BSI algorithm are as follows.

- (R1) In Step 1, Chi and Chen's fast algorithm (Algorithm 2 in [5]) with certain initial conditions for $\mathbf{v}_k[n]$'s suggested by Inouye and Tanebe [7] can be used to obtain the K optimum inverse filters $\hat{\mathbf{v}}_k[n]$'s. The Blackman-Turkey multichannel spectral estimator can be used to obtain $\hat{\mathcal{P}}_{\mathbf{y}}[l]$.
- (R2) In (S3) of Step 2, the initial values $\tilde{\mathcal{H}}_0[l]$ can be chosen as the FFT of the associated channel estimates $(\hat{\mathbf{h}}_1[n], \hat{\mathbf{h}}_2[n], \dots, \hat{\mathbf{h}}_K[n])$ obtained by Tugnait's DEC-IOCC method (see (4)). Moreover, the convergence of the BSI algorithm is guaranteed because $\mathcal{C}(\tilde{\mathcal{H}}_i)$ is upper bounded by unity and its value is increased at each iteration.
- (R3) The proposed BSI algorithm is never limited by the length of $\hat{\mathbf{H}}[n]$ as long as the FFT size \mathcal{L} is chosen sufficiently large such that aliasing effects on the resultant $\hat{\mathbf{H}}[n]$ are negligible. In other words, the knowledge of system order is never needed, and meanwhile, the system $\mathbf{H}[n]$ is also allowed to have common subchannel zeros.
- (R4) The obtained $\hat{\mathbf{H}}[n]$ is robust against Gaussian noise since **Fact 2** (i.e., (7)) is valid for any SNR, although $\hat{\mathbf{v}}_k[n]$'s and $\hat{\mathcal{P}}_{\mathbf{y}}[l]$ depend on SNR.

4. SIMULATION RESULTS

Two examples are provided to justify the efficacy of the proposed BSI algorithm for MIMO systems. In each example, fifty independent runs were performed for different data length N and noise being white Gaussian with $\text{SNR} \triangleq E\{\|\mathbf{y}[n] - \mathbf{w}[n]\|^2\}/E\{\|\mathbf{w}[n]\|^2\}$ over the range between 0 and 20 dB. Synthetic $\mathbf{y}[n]$ were processed by Tugnait's DEC-IOCC method [3] and by the proposed BSI algorithm with the inverse filters $\mathbf{v}_k[n]$ associated with the HOS-IFC $J_{2,2}$ assumed to be causal FIR filters of order equal to 24, FFT size $\mathcal{L} = 64$, and the convergence tolerance $\epsilon = 10^{-10}$. The overall normalized mean-square errors (ONMSE) (defined in [3]) between the estimate $\hat{\mathbf{H}}[n]$ and the true $\mathbf{H}[n]$ were calculated as the performance index.

Example 1. 2×2 MA(6) system (taken from [3])

The driving inputs $u_1[n]$ and $u_2[n]$ were assumed to be mutually independent, zero-mean i.i.d., binary sequences taking values ± 1 with probability 0.5 each. Figure 1 shows ONMSE versus SNR associated with the proposed BSI algorithm (solid lines) and the DEC-IOCC method (dashed lines), from which one can observe that the proposed BSI algorithm performs much better than the DEC-IOCC method (lower ONMSE).

Example 2. 3×2 MA(6) system (taken from [3])

The two driving input signals were assumed to be mutually independent, zero-mean i.i.d. sequences for

Case 1: $u_k[n], k = 1, 2$ are both binary and Case 2: $u_1[n]$ is exponentially distributed and $u_2[n]$ is binary. Figures 2(a) and 2(b) show ONMSEs associated with Cases 1 and 2, respectively. Again, one can observe, from these figures, that the proposed BSI algorithm performs much better than the DEC-IOCC method.

The above examples demonstrate the efficacy of the proposed BSI algorithm. As a final remark, typically, only $6 \sim 9$ iterations in Step 2 were spent in obtaining $\hat{\mathbf{H}}[n]$ in the above two examples.

5. CONCLUSIONS

Based on the relationship (see **Fact 2**) between the optimum inverse filters associated with the HOS-IFC $J_{p,p}$ (see (3) with $p = q \geq 2$) and the unknown MIMO system, a frequency-domain BSI algorithm has been presented which iteratively estimates the unknown MIMO system from the optimum inverse filters and an estimate of the multichannel cross-spectral matrix of measurements. The unknown MIMO system is allowed to have common subchannel zeros, and the system order information is never needed. Some simulation results were provided to support that the proposed BSI algorithm outperforms Tugnait's DEC-IOCC method.

6. REFERENCES

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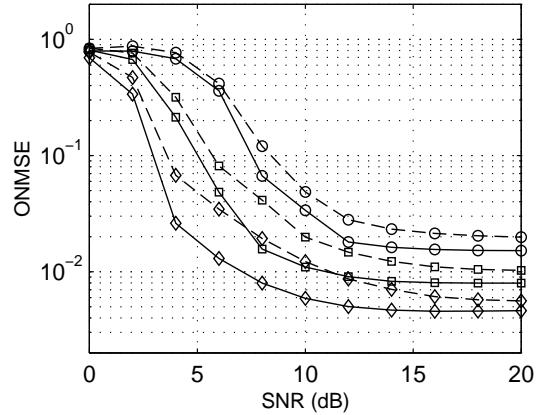
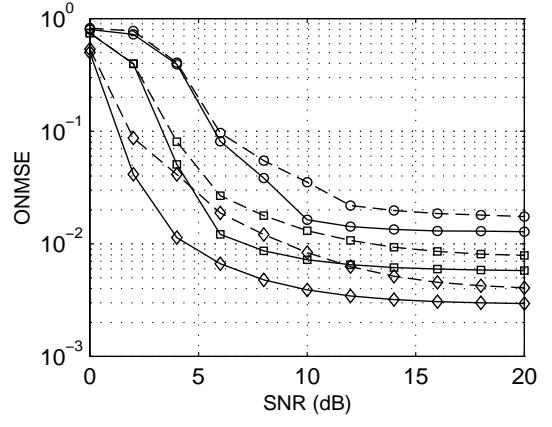
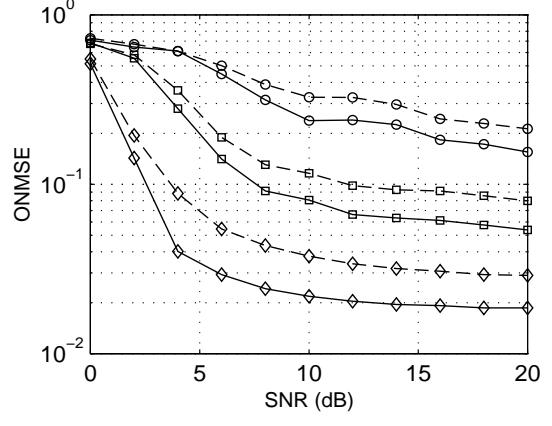


Figure 1. Simulation results of Example 1. Plots of ONMSEs versus SNR associated with the proposed BSI algorithm (solid lines) and the DEC-IOCC method (dashed lines), respectively, for data length $N = 1500$ (' \circ '), 3000 (' \square '), and 6000 (' \diamond ').



(a)



(b)

Figure 2. Simulation results of Example 2 for (a) Case 1, and (b) Case 2, respectively. Plots of ONMSEs versus SNR associated with the proposed BSI algorithm (solid lines) and the DEC-IOCC method (dashed lines), respectively, for data length $N = 1500$ (' \circ '), 3000 (' \square '), and 6000 (' \diamond ').