A CLOSED-FORM SOLUTION TO BLIND MIMO FIR CHANNEL EQUALIZATION FOR WIRELESS COMMUNICATION SYSTEMS BASED ON AUTOCORRELATION MATCHING

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ABSTRACT

The Autocorrelation Matching method is a second order statistics-based blind MIMO FIR channel equalization technique designed for wireless communication systems using multiple receiving antennas. This paper presents a closed-form solution to the optimal zero-forcing equalizer for the Autocorrelation Matching method. It can recover a transmitted signal with maximum SNR from an unknown MIMO FIR channel based on the autocorrelation coefficients of the transmitted signal and the autocorrelation matrices of the received signals. Numerical results obtained from intensive computer simulations are also given in this paper, which verify the effectiveness and accuracy of the closed-form solution.

1. INTRODUCTION

Blind MIMO (multiple-input multiple-output) FIR (finite input response) channel equalization techniques refer to a category of signal processing methods that are designed to recover a number of input signals distorted by an unknown MIMO FIR channel without using training signals to identify the channel. The recovery of input signals is in general based on some statistical information about the input signals, the output signals of the channel, and some structure information about the channel model.

The Autocorrelation Matching (AM) method is a second order statistics-based blind MIMO FIR channel equalization technique designed for wireless communication systems using multiple receiving antennas [1]. It requires (1) the autocorrelation functions of the signals transmitted into an MIMO FIR channel satisfy a linear shift-independence condition; and (2) the MIMO FIR channel itself satisfies an FIR invertible condition. With these conditions satisfied, a transmitted signal is equal to an output signal of a MISO (multiple-input single-output) FIR equalizer applied to the received signals from the MIMO FIR channel up to a unitary factor and a finite delay, if and only if the autocorrelation function of the output signal equals that of the transmitted signal.

The AM method can be used to design an interference-resist wireless communication system [2], such as a wireless local area network operating on an unlicensed spectrum where strong interference may exist. In such system, the signal sent by a transmitter is always filtered using a special filter and thus has a unique autocorrelation function different from all interfering signals; it is separated from the interfering signals and equalized using a MISO FIR equalizer applied to the received signals of multiple receiving antennas at a receiver. The AM method can also be used to design a distributed wireless MIMO communication system with high spectrum efficiency [1]. In such system, all transmitters send signals, each filtered using a distinct filter and thus having a distinct autocorrelation function, to a receiver over the same carrier at the same time; these signals are separated and recovered using a group of MISO FIR equalizers at the receiver, each recovering a transmitted signal.

The principle of the AM method has been proved in [1, 2, 3]. The algorithms used to evaluate the MISO FIR equalizers in [1, 2, 3] are iterative ones, which suffer from local minima. To solve this problem, this paper presents a closed-form algorithm to compute the optimal zero-forcing MISO FIR equalizers for the AM method. The MISO FIR equalizers are computed based on the autocorrelation functions of the transmitted signals and the autocorrelation functions of the received signals. The MIMO FIR channel itself, including the channel order, does not need to be known.

Compared with the closed-form solution given in [4], this solution is based on weaker conditions on the channel and thus has wider applicability.

2. PROBLEM FORMULATION

This paper is focused on the recovery of one transmitted signal from an unknown MIMO FIR channel using a MISO FIR equalizer. A communication system model based on the AM method for this scenario is shown in Fig. 1.

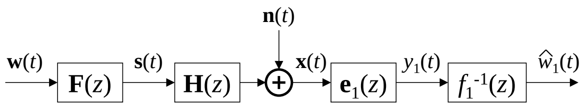

Fig.1 A communication system model based on the AM method

Where,

- $\mathbf{w}(t)$: a vector of N information signals. $\mathbf{w}(t) = [w_1(t), w_2(t), \cdots, w_N(t)]^T$
- $\mathbf{F}(z)$: a group of N pre-filters. $\mathbf{F}(z) = \mathbf{diag}(f_1(z), f_2(z), \cdots, f_N(z))$
- $\mathbf{s}(t)$: a vector of N transmitted signals. $\mathbf{s}(t) = [s_1(t), s_2(t), \cdots, s_N(t)]^T$
- $\mathbf{H}(z)$: an $M \times N$ FIR channel. $\mathbf{H}(z) = [\sum_{l=0}^{L_h} h_{mn(l)} z^{-l}]_{m,n=1,1}^{M,N}$
- $\mathbf{n}(t)$: a vector of M additive noises. $\mathbf{n}(t) = [n_1(t), n_2(t), \cdots, n_M(t)]^T$
- $\mathbf{x}(t)$: a vector of M received signals. $\mathbf{x}(t) = [x_1(t), x_2(t), \cdots, x_M(t)]^T$
- $\mathbf{e}_1(z)$: a vector of M FIR equalizers for recovery of $s_1(t)$. $\mathbf{e}_1(z) = [\sum_{l=0}^{L_e} e_{11(l)} z^{-l}, \cdots, \sum_{l=0}^{L_e} e_{1M(l)} z^{-l}]^T$

$y_1(t)$:	an estimate of $s_1(t)$.
$f_1^{-1}(z)$:	an FIR filter that reverses $f_1(z)$ approximately.
$\hat{w}_1(t)$:	an estimate of $w_1(t)$.

The above system can be described by,

$$\begin{cases} \mathbf{s}(t) = [\mathbf{F}(z)](\mathbf{w}(t)) \\ \mathbf{x}(t) = [\mathbf{H}(z)](\mathbf{s}(t)) + \mathbf{n}(t) \\ y_1(t) = [\mathbf{e}_1^T(z)\mathbf{H}(z)](\mathbf{s}(t)) + [\mathbf{e}_1^T(z)](\mathbf{n}(t)) \\ \hat{w}_1(t) = [f_1^{-1}(z)]y_1(t) \end{cases} \quad (1)$$

where polynomials in [ ] serve as convolution operators and $t \in Z$. The degree of the overall system is $L = L_h + L_e$.

The MIMO FIR channel response in Eq(1) can be formulated using matrices.

$$\bar{\mathbf{x}}(t) = \bar{\mathbf{H}}\bar{\mathbf{s}}(t) + \bar{\mathbf{n}}(t)$$

where, the $M(L_e+1)$-vector signal $\bar{\mathbf{x}}(t)$, the $N(L+1)$-vector signal $\bar{\mathbf{s}}(t)$, and the $M(L_e+1)$-vector signal $\bar{\mathbf{n}}(t)$ are re-arrangements of the received vector signal $\mathbf{x}(t)$, the transmitted vector signal $\mathbf{s}(t)$, and the noise vector signal $\mathbf{n}(t)$ respectively. In detail,

$$\begin{aligned} \bar{\mathbf{x}}(t) &= [\tilde{\mathbf{x}}_1^T(t), \tilde{\mathbf{x}}_2^T(t), \cdots, \tilde{\mathbf{x}}_M^T(t)]^T \\ \bar{\mathbf{s}}(t) &= [\tilde{\mathbf{s}}_1^T(t), \tilde{\mathbf{s}}_2^T(t), \cdots, \tilde{\mathbf{s}}_N^T(t)]^T \\ \bar{\mathbf{n}}(t) &= [\tilde{\mathbf{n}}_1^T(t), \tilde{\mathbf{n}}_2^T(t), \cdots, \tilde{\mathbf{n}}_M^T(t)]^T \end{aligned}$$

in which $\tilde{\mathbf{x}}_m(t)$, $\tilde{\mathbf{s}}_n(t)$, and $\tilde{\mathbf{n}}_m(t)$ are given by,

$$\begin{aligned} \tilde{\mathbf{x}}_m(t) &= [x_m(t), x_m(t-1), \cdots, x_m(t-L_e)]^T \\ \tilde{\mathbf{s}}_n(t) &= [s_n(t), s_n(t-1), \cdots, s_n(t-L)]^T \\ \tilde{\mathbf{n}}_m(t) &= [n_m(t), n_m(t-1), \cdots, n_m(t-L_e)]^T \end{aligned}$$

for $m = 1, 2, \cdots, M$; the $M(L_e+1) \times N(L+1)$ matrix $\bar{\mathbf{H}}$ is a matrix representation for the MIMO FIR channel $\mathbf{H}(z)$,

$$\bar{\mathbf{H}} = [\tilde{\mathbf{H}}_{mn}]_{m,n=1,1}^{M,N}$$

in which $\tilde{\mathbf{H}}_{mn}$'s are $(L_e+1) \times (L+1)$ Toeplitz matrices,

$$\begin{bmatrix} h_{mn(0)} & \cdots & h_{mn(L_h)} & \cdots & 0 & 0 \\ 0 & h_{mn(0)} & \cdots & h_{mn(L_h)} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & h_{mn(0)} & \cdots & h_{mn(L_h)} \end{bmatrix}$$

The channel matrix $\bar{\mathbf{H}}$ can also be deemed as a row of N Sylvester matrices,

$$\bar{\mathbf{H}} = [\bar{\mathbf{H}}_1, \bar{\mathbf{H}}_2, \cdots, \bar{\mathbf{H}}_N]$$

in which each Sylvester matrix $\bar{\mathbf{H}}_n$ contains a column of M Toeplitz matrices $\tilde{\mathbf{H}}_{mn}$'s.

The composite system response in Eq(1) can also be formulated using matrices.

$$y_1(t) = \bar{\mathbf{e}}_1^T\bar{\mathbf{H}}\bar{\mathbf{s}}(t) + \bar{\mathbf{e}}_1^T\bar{\mathbf{n}}(t)$$

where, the $M(L_e+1)$-vector $\bar{\mathbf{e}}_1$ denote the MISO FIR vector equalizer $\mathbf{e}_1(z)$ that recovers $s_1(t)$. In detail,

$$\bar{\mathbf{e}}_1 = [\tilde{\mathbf{e}}_{11}^T, \tilde{\mathbf{e}}_{12}^T, \cdots, \tilde{\mathbf{e}}_{1M}^T]^T$$

in which each entry is given by,

$$\tilde{\mathbf{e}}_{1m} = [e_{1m(0)}, e_{1m(1)}, \cdots, e_{1m(L_e)}]^T$$

for $m = 1, 2, \cdots, M$.

The objective of this paper is to design the optimal zero-forcing equalizer $\bar{\mathbf{e}}_1$ that minimizes $||\bar{\mathbf{e}}_1||$ subject to

$$\bar{\mathbf{e}}_1^T\bar{\mathbf{H}} = [\mathbf{f}_{L+1}^T(l_1), \mathbf{0}^T, \cdots, \mathbf{0}^T]$$

where $\mathbf{f}_{(L+1)}(l_1)$ is a unitary $(L+1)$-vector with the $(l_1+1)^{th}$ entry being one and all other entries being zero. This is done using the autocorrelation function of $\mathbf{x}(t)$ and the autocorrelation function of $s_1(t)$. The MIMO FIR channel $\mathbf{H}(z)$, the channel degree L_h, and the autocorrelation functions of $s_2(t), s_3(t), \cdots, s_N(t)$ do not need to be known.

The following assumptions are needed throughout this paper.

AS1: $\mathbf{w}(t)$ is a wide-sense stationary vector sequence with zero mean and unitary variance. It is temporally white and spatially uncorrelated, *i.e.*, $E(\mathbf{w}(t)\mathbf{w}^H(t-k) = \delta(k)\mathbf{I}$ for $k \in Z$.

AS2: $\mathbf{n}(t)$ is wide-sense stationary vector sequence with zero mean and unknown variance $[\sigma_1^2, \sigma_2^2, \cdots, \sigma_M^2]^T$. It is temporally white and spatially uncorrelated, *i.e.*, $E(\mathbf{n}(t)\mathbf{n}^H(t-k) = \delta(k)$ $\mathbf{diag}(\sigma_1^2, \sigma_2^2, \cdots, \sigma_M^2)$ for $k \in Z$.

AS3: $\mathbf{n}(t)$ and $\mathbf{w}(t)$ are temporally uncorrelated, *i.e.*, $E(\mathbf{w}(t)$ $\mathbf{n}^H(t-k)) = \mathbf{0}$ for $k \in Z$.

3. CONDITIONS

It may be impossible to recover a transmitted signal from a MIMO FIR channel using a MISO FIR equalizer. In addition, second order statistics are in general insufficient for blind MIMO FIR channel equalization [5]. In order to assure the validity of using the AM method in the wireless communication system shown in Fig. 1, the following conditions must be satisfied.

3.1. Channel Condition

In order to assure the existence of a MISO FIR equalizer $\mathbf{e}_1(z)$ that can recover $s_1(t)$ with delay l from a MIMO FIR channel $\mathbf{H}(z)$, the channel must satisfy Condition 1 below.

Condition 1

The $(l+1)^{th}$ column vector in $\bar{\mathbf{H}}_1$ is independent of all column vectors of $\bar{\mathbf{H}}_{2-N}$ and all other nonzero column vectors of $\bar{\mathbf{H}}_1$, where $\bar{\mathbf{H}}_{2-N} = [\bar{\mathbf{H}}_2, \bar{\mathbf{H}}_3, \cdots, \bar{\mathbf{H}}_N]$.

Theorem 1

A signal $s_1(t)$ can be equalized from a MIMO FIR channel $\mathbf{H}(z)$ with equalization delay l, $0 \leq l \leq L$, using a MISO FIR filter $\mathbf{e}_1(z)$ if and only if Condition 1 is satisfied. (proof omitted)

In order to equalize a signal $s_1(t)$ from a MIMO FIR channel $\mathbf{H}(z)$ using a blind equalization method, we need a stronger condition on the channel than Condition 1. It is stated below.

Condition 2

The $(l+1)^{th}$ column vector in $\bar{\mathbf{H}}_1$ is independent of all column vectors of $\bar{\mathbf{H}}_{2-n}$; and all nonzero column vectors of $\bar{\mathbf{H}}_1$ are linear independent.

Note that the above conditions are weaker than the necessary conditions often cited in the literature of blind MIMO FIR channel equalization, such as that $\mathbf{H}(z)$ is irreducible [6] and that the determinants of all $N \times N$ minors of $\mathbf{H}(z)$ is coprime [7]. The above conditions imply that one or more signals can be recovered from a MIMO FIR channel even though the entire MIMO FIR channel cannot be reversed using a MIMO FIR equalizer.

3.2. Signal Condition

In order to assure that a transmitted signal can be recovered by an output signal of a MISO FIR equalizer by matching the autocorrelation functions of them, the transmitted signals need to satisfy Condition 3 below.

Condition 3

The set of shifted autocorrelation functions of these signals, $\{r_n(k-l)|n=1,2,\cdots,N; l=0,\pm1,\pm2,\cdots,\pm L\}$, is linearly independent for $|k|>L_e$, where $r_n(k)=E(s_n(t)s_n^*(t-k))$.

Condition 3 is called a linear shift-independence condition. It can be equivalently stated as follows.

The $N(2L+1)\times N(2L+1)$ matrix $\mathbf{S}=[\mathbf{S}_1,\mathbf{S}_2,\cdots,\mathbf{S}_N]$ has full rank for some $T_1,T_2,\cdots,T_{N(2L+1)}\in Z$, with $|T_1|$, $|T_2|$, $\cdots$, $|T_{N(2L+1)}|>L_e$, where $\mathbf{S}_n$'s are given by,

$$\mathbf{S}_n=\begin{bmatrix} r_n(T_1+L) & \cdots & r_n(T_1) & \cdots & r_n(T_1-L) \\ r_n(T_2+L) & \cdots & r_n(T_2) & \cdots & r_n(T_2-L) \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

Theorem 2

In a noisy MIMO FIR system $y_1(t)=[\mathbf{e}_1^T(z)\mathbf{H}(z)](\mathbf{s}(t))+[\mathbf{e}_1^T(z)](\mathbf{n}(t))$ with $\mathbf{H}(z)$ satisfying Condition 1 and $\mathbf{s}(t)$ satisfying Condition 3, the transmitted signal $s_1(t)$ is recovered by the output signal $y_1(t)$ up to a unitary factor and a finite delay, *i.e.*,

$$y_1(t)=d_1s_1(t-l_1)+\bar{\mathbf{e}}_1^T\bar{\mathbf{n}}(t) \quad (2)$$

if and only if there exist $T_1,T_2,\cdots,T_{N(2L+1)}\in Z$ with $|T_1|\neq|T_2|\neq\cdots\neq|T_{N(2L+1)}|>L_e$ such that $E(y_1(t)y_1^*(t-k))=r_1(k)$ for $k=T_1,T_2,\cdots,T_{N(2L+1)}$. (see [3] for proof)

According to Theorem 2, a match of autocorrelation functions between $y_1(t)$ and $s_1(t)$ theoretically guarantees a perfect zero-forcing equalization regardless of the noise power. In practice, the accuracy of the zero-forcing equalization is still affected by the noise because the autocorrelation function of $y_1(t)$ is estimated from data samples of $\bar{\mathbf{x}}(t)$ and thus becomes less accurate as the noise power increases. Nonetheless, given a SNR, the accuracy of zero-forcing equalization can be improved by using a large number of data samples in the estimation.

Note that the information signal $\mathbf{w}(t)$ in general does not satisfy Condition 3. This is why the pre-filter $\mathbf{F}(z)$ is introduced in the AM method to convert $\mathbf{w}(t)$ into $\mathbf{s}(t)$ that satisfies Condition 3, as shown in Fig. 1.

Although Condition 3 looks very complicated, it is mild and thus there are many choices for the pre-filter [1]. In this paper, we use a two-tap pre-filter. It is given by,

$$f_n(z)=1+0.3z^{-n(2L+1)} \quad (3)$$

for $n=1,2,\cdots,N$.

4. A CLOSED-FORM SOLUTION

With the channel satisfying Condition 2 and the transmitted signal satisfying Condition 3, the following algorithm can be used to compute a MISO FIR equalizer $\bar{\mathbf{e}}_1$ that recovers the transmitted signal $s_1(t)$ from the MIMO FIR channel $\mathbf{H}(z)$ with delay l, based on the $N(2L+1)$ autocorrelation matrices of the received signal $\bar{\mathbf{x}}(t)$, *i.e.*, $\mathbf{R}_{\bar{\mathbf{x}}}(T_1)$, $\mathbf{R}_{\bar{\mathbf{x}}}(T_2)$, $\cdots$, $\mathbf{R}_{\bar{\mathbf{x}}}(T_{N(2L+1)})$, and the $N(2L+1)$ autocorrelation coefficients of the transmitted signal $s_1(t)$, *i.e.*, $r_1(T_1)$, $r_1(T_2)$, $\cdots$, $r_1(T_{N(2L+1)})$.

Algorithm 1

1. Compute the Singular Value Decomposition of $\mathbf{S}_1$ below,

$$\mathbf{S}_1=[\mathbf{U}_{S1S},\mathbf{U}_{S1N}]\mathbf{diag}(\mathbf{D}_{S1},\mathbf{0})[\mathbf{V}_{S1S},\mathbf{V}_{S1N}]^T$$

where $\mathbf{U}_{S1N}=[u_{ij}]$ is an $N(2L+1)\times(N-1)(2L+1)$ orthogonal matrix representing the null space of $\mathbf{S}_1^T$.

2. Using entries in $\mathbf{U}_{S1N}$ and autocorrelation matrices $\mathbf{R}_{\bar{\mathbf{x}}}(T_1)$, $\mathbf{R}_{\bar{\mathbf{x}}}(T_2)$, $\cdots$, $\mathbf{R}_{\bar{\mathbf{x}}}(T_{N(2L+1)})$ compose the following $M(L_e+1)\times M(L_e+1)$ matrices,

$$\mathbf{R}_w(j)=\sum_{i=1}^{N(2L+1)}u_{ij}\mathbf{R}_{\bar{\mathbf{x}}}(T_i)$$

for $j=1,2,\cdots,(N-1)(2L+1)$. Let $\mathbf{R}_w$ = [$\mathbf{R}_w(1)$, $\mathbf{R}_w(2)$, $\cdots$, $\mathbf{R}_w((N-1)(2L+1))$] and $\mathbf{R}_c=\mathbf{R}_w^H\mathbf{R}_w$. Note that

$$\mathbf{R}_c=\bar{\mathbf{H}}_{2-N}\mathbf{R}_{SCK}\bar{\mathbf{H}}_{2-N}^H$$

where $\mathbf{R}_{SCK}$ is a $(N-1)(L+1)\times(N-1)(L+1)$ positive definite matrix.

3. Compute the Eigenvalue Decomposition of $\mathbf{R}_c$ below,

$$\mathbf{R}_c=[\mathbf{U}_{RcS},\mathbf{U}_{RcN}]\mathbf{diag}(\mathbf{D}_{Rc},\mathbf{0})[\mathbf{U}_{RcS},\mathbf{U}_{RcN}]^H$$

where $\mathbf{U}_{RcN}$ is an orthogonal matrix representing the null space of $\bar{\mathbf{H}}_{2-N}$. Every column of $\mathbf{U}_{RcN}$ is an eigenvector corresponding to a zero eigenvalue.

4. Find an $N(2L+1)$-vector $\mathbf{v}=[v_1,v_2,\cdots,v_{N(2L+1)}]^T$ that minimizes $||\mathbf{v}||$ subject to $\mathbf{S}_1^T\mathbf{v}=\mathbf{f}_{2L+1}(L+1)$, where $\mathbf{f}_{2L+1}(L+1)$ is a unitary $(2L+1)$-vector with the $(L+1)^{th}$ entry being one and other entries being zero. Using $\mathbf{v}$ compute a matrix $\mathbf{R}_0$ as follows,

$$\mathbf{R}_0=\mathbf{U}_{RcN}^H(\sum_{i=1}^{N(2L+1)}v_i\mathbf{R}_{\bar{\mathbf{x}}}(T_i))\mathbf{U}_{RcN}$$

Note that

$$\mathbf{R}_0=\mathbf{U}_{RcN}^H\bar{\mathbf{H}}_1\bar{\mathbf{H}}_1^H\mathbf{U}_{RcN}$$

5. Find an $N(2L+1)$-vector $\mathbf{w}=[w_1,w_2,\cdots,w_{N(2L+1)}]^T$ that minimizes $||\mathbf{w}||$ subject to $\mathbf{S}_1^T\mathbf{w}=\mathbf{f}_{2L+1}(L+l+2)+\mathbf{f}_{2L+1}(l)$, where $\mathbf{f}_{2L+1}(L+l+2)=\mathbf{0}$ when $l=L$ and $\mathbf{f}_{2L+1}(l)=\mathbf{0}$ when $l=0$. Using $\mathbf{w}$ compute a matrix $\mathbf{R}_l$ as follows,

$$\mathbf{R}_l=\mathbf{U}_{RcN}^H(\sum_{i=1}^{N(2L+1)}w_i\mathbf{R}_{\bar{\mathbf{x}}}(T_i))\mathbf{U}_{RcN}$$

Note that

$$\mathbf{R}_l=\mathbf{U}_{RcN}^H\bar{\mathbf{H}}_1(\mathbf{J}^{(l+1)}+(\mathbf{J}^T)^{(L-l+1)})\bar{\mathbf{H}}_1^H\mathbf{U}_{RcN}$$

where $\mathbf{J}$ is a $(L+1)\times(L+1)$ shifting matrix defined below,

$$\mathbf{J}=\begin{bmatrix} 0 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 1 & 0 \end{bmatrix}$$

6. Find a vector $\mathbf{g}$ such that $\mathbf{R}_l\mathbf{g}=\mathbf{0}$ subject to $\mathbf{g}^H\mathbf{R}_0\mathbf{g}=1$. The MISO FIR equalizer $\bar{\mathbf{e}}_1$ is given by,

$$\bar{\mathbf{e}}_1=\mathbf{U}_{RcN}\mathbf{g}$$

Note that $\mathbf{g}$ is a generalized eigenvector of $\mathbf{R}_l$ and $\mathbf{R}_0$ that corresponds to the zero generalized eigenvalue of $\mathbf{R}_l$ and $\mathbf{R}_0$.

Using Algorithm 1, we can compute zero-forcing equalizers with all possible equalization delays (from 0 to L). Among them, the optimal zero-forcing equalizer is the one with the minimum norm, because it introduces the minimum noise power into the equalized signal (see Eq(2)).

5. COMPUTER SIMULATIONS

Three computer simulations are conducted to equalize two transmitted signals using four receiving antennas from a MIMO FIR channel with degree 1, 2, and 3 respectively.

In each simulation, 100 bursts are tested. In each burst test, two uncorrelated white random sequences on [-1, 1] are generated as two information signals, each of which has 10000 symbols and 10 of them serve as train symbols for the purpose of identifying the delay and polarization after the signal is recovered. These two information signals are filtered using two-tap pre-filters $f_1(z)$ and $f_2(z)$ (see Eq(3)) respectively, and then transmitted through a randomly generated MIMO FIR channel $\mathbf{H}(z)$. Every entry of $\mathbf{H}(z)$ yields the standard Gaussian distribution. The channel is quasi-static, *i.e.*, constant in every burst but different from one burst to another. The received signals are fed to a MISO FIR equalizer to recover the first transmitted signal. The equalizer is designed using Algorithm 1 based on the knowledge of $f_1(z)$ and the estimated autocorrelation matrices of the received signals. The degree of equalizer is 1, 2, and 3 in three simulations respectively. After the transmitted signal is equalized, it is filtered by an approximate reverse FIR filter, and then converted into hard symbols 1 or -1. At last, the delay and the polarization are identified by correlating the training symbols with the estimated information signal.

Fig. 2 shows the BER vs. SNR curves for these simulations. The upper curve is for the channel with degree 3, the middle one for the channel with degree 2, and the lower one for the channel with degree 1. In this figure, each sample point is an average over 10^6 runs. Note that these BER curves show raw bit error rates. No error correction coding techniques are applied.

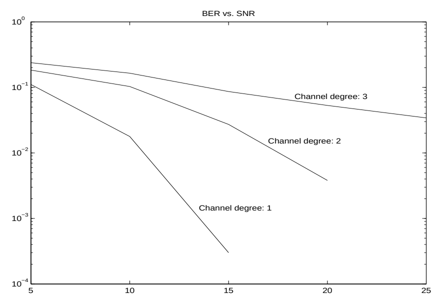

Fig.2 Performance of the closed-form algorithm

In one of these simulation runs, a MIMO FIR channel $\mathbf{H}(z) = [h_{mn}(z)]_{m,n=1,1}^{4,2}$ is randomly generated as follows,

$$\begin{aligned}
h_{11}(z) &= -0.2858 - 0.9109z^{-1} + 0.7621z^{-2} \\
h_{12}(z) &= -0.6707 - 0.2410z^{-1} - 0.4798z^{-2} \\
h_{21}(z) &= -0.5294 - 0.3219z^{-1} + 0.6242z^{-2} \\
h_{22}(z) &= -0.3053 + 1.1354z^{-1} - 0.3714z^{-2} \\
h_{31}(z) &= -0.1551 - 0.7845z^{-1} - 0.1249z^{-2} \\
h_{32}(z) &= 1.3780 - 0.0493z^{-1} - 0.6320z^{-2} \\
h_{41}(z) &= -1.4213 + 0.6630z^{-1} + 0.0607z^{-2} \\
h_{42}(z) &= -0.2695 + 0.3850z^{-1} - 0.9653z^{-2}
\end{aligned}$$

The corresponding MISO FIR equalizer $\mathbf{e}_1(z) = [e_{11}(z), e_{12}(z), e_{13}(z), e_{14}(z)]^T$ is evaluated using Algorithm 1. It is given below,

$$\begin{aligned}
e_{11}(z) &= -0.0998 - 0.1731z^{-1} + 0.2054z^{-2} \\
e_{12}(z) &= 0.0808 - 0.3043z^{-1} - 0.3108z^{-2} \\
e_{13}(z) &= -0.1434 - 0.0406z^{-1} - 0.3067z^{-2} \\
e_{14}(z) &= -0.7002 - 0.0289z^{-1} + 0.2553z^{-2}
\end{aligned}$$

The composite system response between $y_1(t)$ and $s_1(t)$ is,

$$\begin{aligned}
y_1(t) = \; & \mathbf{1.003}s_1(t) - 0.029s_1(t-1) + 0.009s_1(t-2) \\
& +0.004s_1(t-3) + 0.016s_1(t-4) \\
& +0.034s_2(t) + 0.015s_2(t-1) - 0.064s_2(t-2) \\
& -0.039s_2(t-3) - 0.036s_2(t-4)
\end{aligned}$$

6. CONCLUSIONS

The Autocorrelation Matching method is a second order statistics-based blind MIMO FIR channel equalization technique designed for wireless communication systems using multiple receiving antennas. This paper presents a closed-form solution to the optimal zero-forcing equalizer for the Autocorrelation Matching method. It can recover a transmitted signal with maximum SNR from an unknown MIMO FIR channel based on the autocorrelation coefficients of the transmitted signal and the autocorrelation matrices of the received signals. Numerical results obtained from intensive computer simulations are also given in this paper, which verify the effectiveness and accuracy of the closed-form solution.

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