BLIND DECONVOLUTION OF FIR CHANNELS WITH BINARY SOURCES: A GROUPING DECISION APPROACH

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ABSTRACT

This paper proposes a novel grouping decision approach for blind deconvolution of FIR channels with binary sources. First, necessary and sufficient conditions for recoverability are derived. For single-input systems, a new deterministic algorithm based on grouping and decision is proposed to recover the source up to a delay. Then the algorithm is extended to deal with high noise case and long decaying channel case. Furthermore Blind deconvolution for multi-input systems also can be carried out as with the case of single input systems. All sources can be recovered sequentially. Finally, the validity and performance of the algorithms are illustrated by several simulation examples.

1. INTRODUCTION

Binary signals play important roles in pattern recognition, digital signal processing, and especially wireless communications. When multiple binary signals are transmitted from different sources, the mixtures of them are often received by sensors. In this paper we consider a dynamically mixing model described as,

$$\mathbf{x}(k) = \sum_{i=1}^{n}\sum_{p=0}^{L}\mathbf{a}_{i,p}s_i(k-p) + \mathbf{v}(k), \tag{1}$$

where $\mathbf{s}(k) = [s_1(k), \cdots, s_n(k)]^T$ ($n \geq 1$) is an unknown source vector with mutually independent binary components; this can take only two discrete values $\{d_1, d_2\}$, typically $\{0, 1\}$ or $\{-1, +1\}$. $\mathbf{x}(k) = [x_1(k), \cdots, x_m(k)]^T$ ($m \geq 1$) is an available sensor signal vector (convolutive mixture), $\mathbf{a}_{i,p} = [a_{1i,p}, \cdots, a_{mi,p}]^T$, $i = 1, \cdots, n$, $p = 0, \cdots, L$ are unknown coefficient column vectors, $\mathbf{v} = [v_1, \cdots, v_m]^T$ is the additive white Gaussian noise vector with mutually independent components.

The task of blind deconvolution is to recover sources $s_1, \cdots, s_n$ up to an arbitrary delay and a scale from the observable convolutive mixture $\mathbf{x}$.

Recently, there have existed several references on instantaneous blind separation of digital sources, e.g., [1, 2, 3, 4]. These existing algorithms can be divided into two classes: deterministic algorithms and iterative algorithms. The computational burden of deterministic algorithms increases exponentially with respect to the number of sources, and the noise tolerance is very low. Iterative algorithms are limited by poor convergence and computational complexity.

There also have been many relevant references on blind deconvolution (i.e., equalization) with finite alphabet sources, e.g., [5, 6, 7]. In almost all other existing algorithms, a inverse filter system (equalizer) is designed under one or more of the following conditions:

1. There are no zeros on the unit circle for the case of mixing dynamic systems; that is, the mixing systems are assumed to be non-singular systems. If the algorithm is executed online, the mixing systems should be minimum-phase systems. 2. For MIMO systems, the sensor number is larger than the source number. 3. The sources are temporarily independent (e.g., i.i.d. sequence), or are at least temporally uncorrelated.

The present paper discusses blind deconvolution for single input systems and multi-input systems with binary sources. We develop a new grouping decision algorithm that addresses some of the aforementioned limitations of previous approaches. Using this approach, rather than an approach based on inverse filtering, blind deconvolution can be carried out without imposing the three conditions stated above.

2. SOLVABILITY ANALYSIS

In this section, we discuss the solvability of blind deconvolution for the following noise-free model corresponding to (1),

$$\mathbf{x}(k) = \sum_{i=1}^{n}\sum_{p=0}^{L}\mathbf{a}_{i,p}s_i(k-p). \tag{2}$$

Definition 1 *The model (2) is said to be well-posed, if and only if there exists a set of delays* $p_1, \cdots, p_n \in \{0, \cdots, L\}$, *such that* $\forall k$, $\sum_{i=1}^{n}\sum_{p=0}^{L}\mathbf{a}_{i,p}\ s_i(k-p) = \sum_{i=1}^{n}\sum_{p=0}^{L}\mathbf{a}_{i,p}s'_i(k-p)$ *implies that* $[s_1(k-p_1), \cdots, s_n(k-p_n)]^T = [s'_1(k-$

$p_1), \cdots, s'_n(k-p_n)]^T$, *where* $\mathbf{s}, \mathbf{s}'$ *are two binary source vectors.*

Theorem 1 *The model (2) is well-posed, if and only if there exists* $p_1, \cdots, p_n \in \{0, \cdots, L\}$, *such that*

$$\begin{aligned} c_{1p_1}\mathbf{a}_{1,p_1} + \cdots + c_{np_n}\mathbf{a}_{n,p_n} + \sum_{j_1 \neq p_1, j_1=0}^{L} c_{1j_1}\mathbf{a}_{1,j_1} \\ + \cdots + \sum_{j_n \neq p_n, j_n=0}^{L} c_{nj_n}\mathbf{a}_{n,j_n} \neq 0, \end{aligned} \tag{3}$$

where constants $c_{ij} \in \{-1, 0, 1\}, i = 1, \cdots, n, j = 0, \cdots, L$, *specifically,* $\{c_{1p_1}, \cdots, c_{np_n}\}$ *has at least a nonzero entry.*

Remark 1: By Theorem 1, it is possible to separate all sources, even if the number of sensors is less than the number of sources (see Example 4).

3. BLIND DECONVOLUTION ALGORITHM FOR SINGLE-INPUT DYNAMICAL SYSTEMS

In this section, we consider model (1) with single input ($n = 1$). Suppose that the solvability condition (3) in Theorem 1 is satisfied throughout this section. First, we consider the noise free case when the length L of the mixing channel is not large, say $L \leq 10$ (the low noise case can be dealt with similarly). A deterministic grouping decision approach is proposed for blind deconvolution of single input systems. The algorithm is then extended for dealing effectively with the high-noise case and long decaying channel case.

3.1. Noise free case

Since the source is binary, there are at most $2^{(L+1)}$ different output vectors of the noise free model (2) with only input, denoted as a set $\mathbf{X} = \{\mathbf{x}_1, \cdots, \mathbf{x}_N\}$, where $N \leq 2^{(L+1)}$.

The following assumption is necessary for the grouping decision algorithm to be presented shortly.

Assumption 1: For the model (**??**) ($n = 1$), there exists a column vector $\mathbf{a}(q)$, which satisfies (3) and the following inequalities:

$$\mathbf{a}_q \quad \neq \quad \frac{1}{2} \sum_{p \neq q, p=0}^{L} c_p \mathbf{a}_p, \tag{4}$$

where $c_0, \cdots, c_{q-1}, c_{q+1} \cdots, c_L \in \{1, 0, -1\}$.

In fact, (4) is satisfied with probability of one.

Now we present the grouping decision algorithm.

Step 1. (Estimating a column vector of the convolutively mixing matrix) Choose a row of the matrix $\mathbf{X}$ with at least two non-zero components assumed to be the first row, and then determine the largest and the second largest components assumed to be x_{11} and x_{12}. Set

$$\bar{\mathbf{a}}_1 = \frac{1}{d_2 - d_1}[\mathbf{x}_1 - \mathbf{x}_2].$$

It is not difficult to prove that $\bar{\mathbf{a}}_1$ is one of the columns of $\mathbf{A}$, up to a sign.

Step 2. (Matching and Grouping) Match the columns of $\mathbf{X}$ in a pairwise manner. That is, if $||\mathbf{x}_i - \mathbf{x}_j - (d_2 - d_1)\bar{\mathbf{a}}_1|| < \epsilon_0$, then $(\mathbf{x}_i, \mathbf{x}_j)$ is defined as a pair. There exist $\frac{N}{2}$ pairs denoted as $(\mathbf{x}_{i_1}, \mathbf{x}_{i_2}), \cdots, (\mathbf{x}_{i_{(N-1)}}, \mathbf{x}_{i_N})$.

According to the pairs above, divide $\{\mathbf{x}_i\}$ into two groups denoted as,

$$\mathbf{X}^{11} = \{\mathbf{x}_{i_1}, \mathbf{x}_{i_3}, \cdots, \mathbf{x}_{i_{(N-1)}}\},$$

$$\mathbf{X}^{12} = \{\mathbf{x}_{i_2}, \mathbf{x}_{i_4}, \cdots, \mathbf{x}_{i_N}\}.$$

Step 3. (Deconvolution) For an output $\mathbf{x}(k)$ of (2), if it belongs to $\mathbf{X}^{11}$, then set $\bar{s}(k) = d_2$, otherwise, set $s(k) = d_1$. Thus we obtain the source up to a delay and an exchange of d_1 and d_2;

For example, for the noise-free SISO model,

$$x(k) = [0.1, 0.15, 0.1][s(k), s(k-1), s(k-2)]^T, \tag{5}$$

the solvability condition (3) and Assumption 1 are satisfied, the blind deconvolution can be carried using the algorithm above.

Remarks 1: 1. Note that two zeros of the system (5) are on the unit circle. standard Blind deconvolution can not be carried out in this case using a general inverse filtering approach. 2. Obviously, if the noise level is sufficiently low, there are N different clusters formed by the outputs of (1). The centers of these clusters are $\{\mathbf{x}_1, \cdots, \mathbf{x}_N\}$. Thus the above algorithm can be used in low noise case.

There exist two limitations of the deterministic algorithm: the first is that the noise level should be low such that all cluster centers representing different outputs of noise-free model (2) are discriminated easily; the second is that the computational burden increases exponentially with respect to the tap number of channel. The tasks in the next subsection are to extend the algorithm to the high-noise and long-channel cases.

3.2. High noise and long decaying channel cases

When the noise is high and gaussian, the outputs will form clusters. It is not difficult to find that if the cluster centers can be estimated correctly in advance, then the proposed deterministic algorithm still works effectively.

The deconvolution strategy for the high-noise case is divided into two steps. The first step is to estimate the cluster centers; the second is to carry out the deconvolution, as in Subsection 3.1. Under assumption of Gaussian noise, the pdf of the output of (1) (n=1) has a local maximum in the cluster centers, as illustrated in the first subplot of Fig. 2 for a one-dimensional mixture.

Thus the cluster centers can be obtained as the following processing briefly: 1. Estimate the pdf $p(x_1, \cdots, x_m)$

of the output $\mathbf{x}$; 2. Use ML iterative approach to find the cluster centers which are corresponding to the peaks of the pdf. After the cluster centers are estimated, we can use the deterministic algorithm in subsection 3.1 for blind deconvolution of the binary source.

For long-channel case, we only consider decaying channel which satisfies the following assumption.

Assumption 2: For the FIR channel $[a_0, \cdots, a_L]$, suppose there exists a k_0, $0 \leq k_0 << L$, such that: 1. $\sum_{p=k_0+1}^{L} |a_p| < \min\{|a_0|, \cdots, |a_{k_0}|\}$; 2. $\sum_{p=k_0+1}^{L} |a_p| < \min\{|a_i - a_j|, i, j = 0, \cdots, k_0, i \neq j\}$.

For instance, the exponentially decaying channel $\{a_j = \alpha\beta^j\}$, satisfies the assumption above if $|\beta| < \frac{1}{2}$, where α, β are constants.

Under the Assumption 2, the pdf and clusters of outputs are similar to those in the high noise case (Example 3). The blind deconvolution can be carried out using the same method as in high noise case. That is, find the cluster centers by estimating the pdf and using ML iterative approach, and then recover the source.

4. SEQUENTIAL BLIND EXTRACTION FOR MULTI-INPUT DYNAMIC SYSTEMS

For multi-input systems, when the solvability conditions (3) is satisfied, all sources can be recovered by sequential blind extraction approach. The algorithm above can be used in each single extraction. The sequential extraction steps are omitted here which can be seen in Example 4. It should be pointed out that all extractions are based on the original mixture, and there is no deflation process. Thus, it is unnecessary for all sources to be temporarily independent (or temporally uncorrelated).

5. SIMULATION RESULTS

Simulation results presented in this section are divided into four categories. Example 1 is concerned with blind deconvolution of a text source that has only one convolutive mixture. In Examples 2 and 3, the high-noise and long-channel cases are considered for SISO systems with a $4-QAM$ source, respectively. Example 4 concerns the sequential blind extraction for a two-input, single-output system with image sources.

Example 1: Consider the following model,

$$x(k) = [3.5, 3, 4.2, 3.5, 4.2, 7.3][s(k), s(k-4), s(k-8), s(k-12), s(k-16), s(k-22)]^T, \quad (6)$$

where s is a binary (black and white) text image with 250×250 pixels. Of course, only the convolutive mixture x is available. Fig. 1 shows the blind deconvolution results.

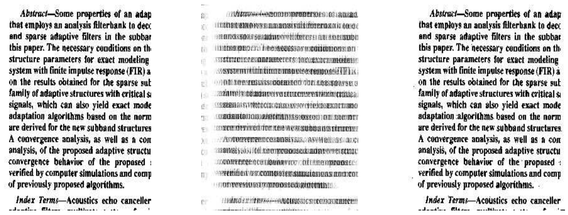

Fig. 1. Blind deconvolution for SISO systems considered in Example 1. Left, black and white text image source; Middle, available convolutive mixture corrupted by low additive noise; Right, the recovered source.

Example 2: Consider the model (1) with an output, a $4-QAM$ source and additive Gaussian, complex-valued noise. The channel parameter vector $a = [3.5, 2, 4, 3.5]$. Using the deconvolution algorithm for the high-noise case considered in Subsection 3.2, 6 simulation experiments were carried out in different noise situations.

Although the source is not binary, its real part and imaginary part are binary. And the real part and imaginary part of mixture are from the real and imaginary parts of the source, respectively. Thus we can deal with real and imaginary parts of mixture respectively, and then integrate the results to carry out blind deconvolution. Fig. 2 shows the simulation results. The left and middle subplots show the estimated pdf and iterative result of cluster centers (SNR= $18.328dB$), respectively. The right subplot shows the curve for the bit error rate with respect to SNR, calculated from the six simulations.

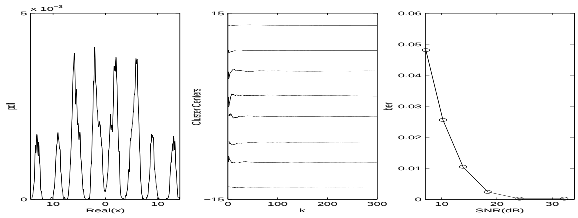

Fig. 2. Blind deconvolution results in different noise situations considered in Example 2.

Example 3: Consider the noise free model (1) with single input and single output. The channel parameter vector $\mathbf{a} = [16, 13, 7, 0, 0, 0.3125\ 0.9, 0, 0.0391, 0.8779, 0, 0.0049, 0.3902, 0, 0.0006, 0.1734, 0, \cdots, 0.0001]$ with length of 45, the source is a $4-QAM$ signal valued randomly in $\{-1-i, -1+i, 1-i, 1+i\}$, noise v is complex valued with its real and imaginary components equal to $0.01n(k)$, $n(k)$ is Gaussian white noise with mean of zero and variance of 1. The blind deconvolution can be carried out as in Example 2, Fig. 3 shows the result.

Example 4: Consider the model (2) with two inputs of 250×250 binary text images and a single output. Channel parameters $\mathbf{a_1}$ and $\mathbf{a_2}$ are set randomly as $[3.1536, 5.4027, 0.1571, 5.1255]$ and $[5.7722, 4.0325, 2.3827, 4.0842]$, respectively. Using the deterministic algorithm presented in Subsection 3.1, a channel parameter $\bar{a}_1$ is estimated as 0.1571, and a source $\bar{s}_1$ is recovered first with a bit error rate of 0.0099.

For estimating the second channel parameter, set

$$\bar{x}(i,j) = x(i,j) - \bar{a}_1\bar{s}_1(i,j), \tag{7}$$

where $x(i,j)$ is the mixture.

Based on the new mixture $\bar{x}$, another channel parameter $\bar{a}_2$ is obtained as 2.3827. Based on $\bar{a}_2$ and the original mixture x, another source $\bar{s}_2$ is obtained from the second extraction (Fig. 4).

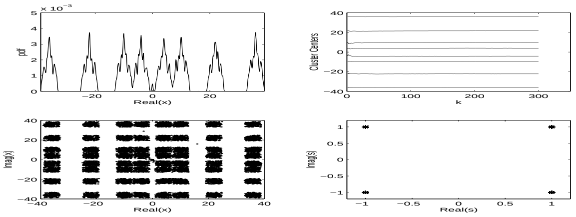

Fig. 3. Blind deconvolution for one convolutive mixture of one $4-QAM$ source considered in Example 3. Top left, estimated pdf of real component of the mixture; Top right, estimated cluster centers of the real component of the mixture; Bottom left, the mixture; Bottom right, the recovered $4-QAM$ source.

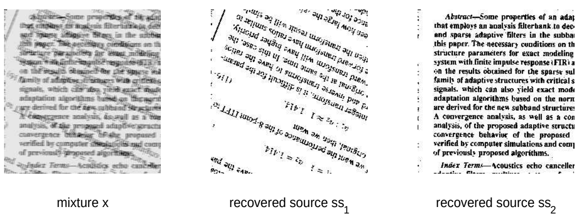

Fig. 4. Sequential blind extraction for one convolutive mixture of two text image sources considered in Example 4. Left, the mixture; Middle, source extracted after the first extraction; Right, another source obtained after the second extraction.

6. CONCLUDING REMARKS

A novel approach for blind deconvolution of mixed, convolutive, binary sources was proposed. Necessary and sufficient conditions for recoverability were established. For the low-noise and noise-free cases, a deterministic grouping decision algorithm was presented for blind deconvolution of single-input dynamical systems having a binary source; For multi-outputs systems, all sources can be recovered sequentially using this approach even if the sources are temporarily correlated. Compared with existing blind deconvolution algorithms generally based on inverse filtering, the grouping decision algorithm has three advantages. First, using the proposed algorithm, there is no condition imposed on the distribution of zeros of convolutive systems. Even though the system has zeros on the unit circle or outside the unit circle, the source can be recovered online. Second, the number of sensors can be less than the number of sources. Third, the algorithm can realize blind deconvolution of temporarily dependent sources (e.g., image sources), even non-stationary sources. By by estimating the pdf of the outputs and cluster centers, the approach is extended to deal with high-noise case and long decaying channel case, respectively. The validity and performance of the proposed algorithms were illustrated by four simulation examples finally.

7. REFERENCES

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