CONVERGENCE BEHAVIOR OF ITERATIVE SOURCE-CHANNEL DECODING

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ABSTRACT

Whenever digital communication is subject to transmission errors, the utilization of reliability information at the receiver is beneficial. Soft-channel decoding techniques improve the *error correcting* capabilities and in addition, *softbit source decoding* performs *error concealment*. While the reliability gain of channel decoding is based on artificial diversities *explicitly* introduced at the transmitter, *softbit source decoding* exploits *implicit* redundancies remaining in the bitstream after state-of-the-art source encoding.

In an iterative source-channel decoding scheme the reliability gains of both are exchanged iteratively in order to enhance the common error resistance. However, first investigations have shown that the number of profitable iterations is limited to small values.

In this paper, the convergence behavior is analyzed using a modern tool called EXIT charts [1]. EXIT charts permit to predict the convergence behavior by studying the individual components. As a novelty, EXIT characteristics are specified for *softbit source decoding*. The convergence analysis is confirmed by simulation.

1. ITERATIVE SOURCE-CHANNEL DECODING

At time instant τ, a source encoder determines a set $\underline{u}_\tau$ of M source codec parameters $u_{\kappa,\tau}$ with $\kappa = 1, \dots M$ denoting the position within the set. The single components $u_{\kappa,\tau}$ in the set $\underline{u}_\tau$ are assumed to be statistically independent among each other. Each value $u_{\kappa,\tau}$, which is continuous in magnitude but discrete in time, is individually quantized by 2^{K_κ} reproduction levels $\bar{u}_\kappa^{(i)}$. The reproduction levels are invariant with respect to τ and the whole set is given by $\mathbb{U}_\kappa$. To each quantizer reproduction level $\bar{u}_\kappa^{(i)}$ specified at time instant τ a unique bit pattern $\mathbf{x}_{\kappa,\tau}$ is assigned. Therein, K_κ denotes the length of $\mathbf{x}_{\kappa,\tau}$. The complete set of bit patterns specified at time instant τ is given by $\underline{\mathbf{x}}_\tau$ (see Figure 1).

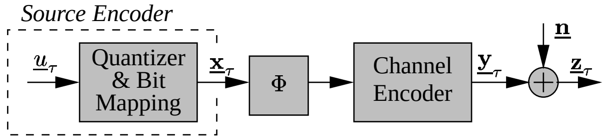

Figure 1: Transmitter for iterative source-channel decoding

A bit interleaver Φ scrambles the incoming data bit stream in a deterministic manner. Thereby, the interleaver might be sized such that several consecutive sets $\underline{\mathbf{x}}_\tau$ are rearranged in common. If so, the interleaver introduces a delay, which might be unacceptable in practical applications. However, the interleaver has to be designed such that the diversities resulting either from source or from channel encoding allow independent reliability gains at the receiver. As diversity of source encoding is based on residual redundancy in source codec parameters $u_{\kappa,\tau}$, independence will be ensured when channel encoding is performed across uncorrelated bit patterns $\mathbf{x}_{\kappa,\tau}$. To simplify matters, channel encoding is assumed to be realized in *systematic* form, i.e., the single data bits $x_{\kappa,\tau}(\lambda)$ with $\lambda = 1, \dots K_\kappa$ are part of the code words $\underline{\mathbf{y}}_\tau$.

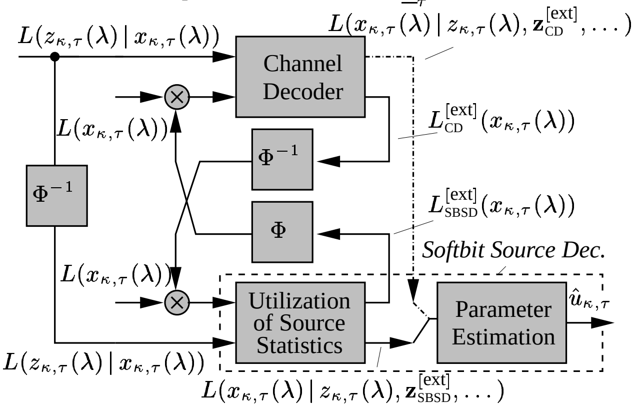

Figure 2: Receiver for iterative source-channel decoding; To simplify matters notation regards the deinterleaved domain

At the receiver reliability information about the transmission of single data bits $x_{\kappa,\tau}(\lambda)$ is utilized. Such reliability information can either be evaluated in terms of probabilities or in the so-called *log-likelihood* algebra. In *log-likelihood* algebra, reliabilities in terms of probabilities are transformed into *log-likelihood* ratios, or short L-values, e.g., [2]. If the transformation is restricted to a binary random variable in bipolar form, i.e., $x_{\kappa,\tau}(\lambda) \in \{+1, -1\}$, the *a posteriori log-likelihood* ratio is defined as [2]

$$L(x_{\kappa,\tau}(\lambda)\,|\,\underline{\mathbf{z}}_\tau) = \log \frac{P(x_{\kappa,\tau}(\lambda) = +1\,|\,\underline{\mathbf{z}}_\tau)}{P(x_{\kappa,\tau}(\lambda) = -1\,|\,\underline{\mathbf{z}}_\tau)} \quad . \tag{1}$$

The term $L(x_{\kappa,\tau}(\lambda)\,|\,\underline{\mathbf{z}}_\tau)$ represents a conditional L-value which is (at first) restricted to the evaluation of the observations $\underline{\mathbf{z}}_\tau$. "log" denotes the natural logarithm and "$+1$" signifies the "null" element of the binary *modulo-2* addition $\oplus$ in GF(2). The sign of the real-valued *log-likelihood* ratio $L(x_{\kappa,\tau}(\lambda)\,|\,\underline{\mathbf{z}}_\tau)$ yields the hard decision and the magnitude $|L(x_{\kappa,\tau}(\lambda)\,|\,\underline{\mathbf{z}}_\tau)|$ represents the reliability of this decision.

The *a posteriori* L-value for the **iterative source-channel decoding (ISCD)** scheme can be separated into four additive terms, if a memoryless transmission channel is assumed

$$\begin{aligned} L(x_{\kappa,\tau}(\lambda)\,|\,\underline{\mathbf{z}}_\tau) &= L(z_{\kappa,\tau}(\lambda)\,|\,x_{\kappa,\tau}(\lambda)) + L(x_{\kappa,\tau}(\lambda)) \\ &\quad + L_{\text{CD}}^{[\text{ext}]}(x_{\kappa,\tau}(\lambda)) + L_{\text{SBSD}}^{[\text{ext}]}(x_{\kappa,\tau}(\lambda)) \quad . \end{aligned} \tag{2}$$

The first term in (2) represents the channel-related L-value. If the transmission channel is considered to be AWGN, the conditional *log-likelihood* ratio is given by [2]

$$L(z_{\kappa,\tau}(\lambda)\,|\,x_{\kappa,\tau}(\lambda)) = 4 \cdot E_s/N_0 \cdot z_{\kappa,\tau}(\lambda) \quad . \tag{3}$$

E_s denotes the energy spent to transmit a single BPSK modulated channel encoded bit and $\sigma_n^2 = N_0/2$ the double sided power spectral density of the AWGN. The second term in (2) represents the *a priori* knowledge for bit $x_{\kappa,\tau}(\lambda)$. Both terms in the first line mark so-called *intrinsic* information about $x_{\kappa,\tau}(\lambda)$. In contrast, the two terms in the second line of (2) gain information from received values other than $z_{\kappa,\tau}(\lambda)$ and consequently, comprise *extrinsic* L-values of both diversity transmissions.

The channel decoder (CD) in Figure 2 restores *extrinsic* information from the diversity, which has *explicitly* been introduced by channel encoding. The set of received values $z_{\kappa,\tau}(\lambda)$, which contribute to the *extrinsic* information of the desired bit $x_{\kappa,\tau}(\lambda)$, is abbreviated by $\mathbf{z}_{\text{CD}}^{[\text{ext}]}$. Since determination rules for the $L_{\text{CD}}^{[\text{ext}]}(x_{\kappa,\tau}(\lambda))$ are well known already, we refer the reader to literature, e.g., [2].

The constituent decoder performs ***softbit source decoding* (SBSD)** [3]. SBSD is a parameter estimation technique which utilizes *implicit* redundancies in the source codec parameters $u_{\kappa,\tau}$. Due to delay and complexity constraints source codec parameters determined by modern source encoders exhibit residual redundancy, either in terms of non-uniform distributions or correlation. The algorithm of *softbit source decoding* by parameter estimation is in principle separable into two parts.

First, residual redundancy in the source codec parameters $u_{\kappa,\tau}$ is utilized for the determination of parameter *a posteriori* probabilities $P(\mathbf{x}_{\kappa,\tau}\,|\,\mathbf{z}_{\text{SBSD}})$. Therein, $\mathbf{z}_{\text{SBSD}}$ denotes all these received values $z_{\kappa,\tau}(\lambda)$ which participate at the utilization of residual redundancies. For instance, if a non-uniform distribution shall be utilized, $\mathbf{z}_{\text{SBSD}} = \mathbf{z}_{\kappa,\tau}$. If an additional auto-correlation property is considered, $\mathbf{z}_{\text{SBSD}} = \mathbf{z}_{\kappa,1}^{\tau}$. The short hand notation $\mathbf{z}_{\kappa,1}^{\tau}$ represents the entire history of received patterns $\mathbf{z}_{\kappa,\tau} \ldots \mathbf{z}_{\kappa,1}$.

Secondly, this parameter *a posteriori* knowledge is combined with quantizer reproduction levels to provide the parameter estimates $\hat{\underline{u}}_{\tau}$. If the *minimum mean squared error* (MMSE) serves as fidelity criterion, the individual estimates are given by [3]

$$\hat{u}_{\kappa,\tau} = \sum_{\bar{u}_{\kappa}^{(i)} \in \mathbb{U}_{\kappa}} \bar{u}_{\kappa}^{(i)} \cdot P(\mathbf{x}_{\kappa,\tau} \hat{=} i\,|\,\mathbf{z}_{\text{SBSD}}) \quad . \tag{4}$$

The first step, determination of $P(\mathbf{x}_{\kappa,\tau}\,|\,\mathbf{z}_{\text{SBSD}})$, is a soft-input/soft-output process, which permits the applicability to ISCD, since bitwise *extrinsic* information is extractable from the parameter *a posteriori* knowledge [4, 5]. In order to quantify the gain due to *softbit source decoding* for single bits, the bitwise *a posteriori* probabilities $P(x_{\kappa,\tau}(\lambda)\,|\,\mathbf{z}_{\text{SBSD}})$ with $\lambda = 1, \ldots K_{\kappa}$ can easily be obtained by the marginal distribution of the conditional probability of $P(\mathbf{x}_{\kappa,\tau}\,|\,\mathbf{z}_{\text{SBSD}})$ for $x_{\kappa,\tau}(\lambda)$. When marginal distributions are determined separately for $x_{\kappa,\tau}(\lambda) = \pm 1$, the bitwise *a posteriori* L-value of SBSD is given by

$$L(x_{\kappa,\tau}(\lambda)\,|\,\mathbf{z}_{\text{SBSD}}) = \log \frac{P(x_{\kappa,\tau}(\lambda) = +1\,|\,\mathbf{z}_{\text{SBSD}})}{P(x_{\kappa,\tau}(\lambda) = -1\,|\,\mathbf{z}_{\text{SBSD}})} \quad . \tag{5}$$

An iterative process using SBSD as one component is realizable, if the soft-output given in (5) is separable into a channel-related part $L(z_{\kappa,\tau}(\lambda)\,|\,x_{\kappa,\tau}(\lambda))$, an *a priori* knowlegde $L(x_{\kappa,\tau}(\lambda))$ as well as an *extrinsic* information [2]. That is, the observations $\mathbf{z}_{\text{SBSD}} = \{z_{\kappa,\tau}(\lambda), \mathbf{z}_{\text{SBSD}}^{[\text{ext}]}\}$ have to be evaluated separately in order to determine *intrinsic* and *extrinsic* knowledge.

As indicated in Figure 2, feedback of *extrinsic* information to the constituent decoder marks one of the key elements in an iterative process. The *extrinsic* information of the one decoder serves as additional *a priori* knowledge for the other one and vice versa. In the first iteration no *extrinsic* information is available for the channel decoder, so that the additional knowledge provided by the *softbit source decoder* in higher numbers of iterations usually permits remarkable refinements of the *a posteriori* L-values.

2. DETERMINATION OF *EXTRINSIC* INFORMATION

If a non-uniform distribution and a 1st order correlation property in the source codec parameters $u_{\kappa,\tau}$ shall be exploited by SBSD, the amount of bits $\mathbf{x}_{\text{SBSD}}^{[\text{ext}]}$ exhibiting *extrinsic* mutual dependencies to bit $x_{\kappa,\tau}(\lambda)$ is separable into two parts. First, the utilization of a non-uniform parameter distribution is based on the *extrinsic* bits $\mathbf{x}_{\kappa,\tau}^{[\text{ext}]}$ of the desired bit pattern $\mathbf{x}_{\kappa,\tau} = \{x_{\kappa,\tau}(\lambda), \mathbf{x}_{\kappa,\tau}^{[\text{ext}]}\}$. Secondly, with respect to the 1st order Markov property, the entire history $\mathbf{x}_{\kappa,1}^{\tau-1}$ and probably some future bit patterns $\mathbf{x}_{\kappa,\tau+1}^{\Lambda}$ have to be considered as well. If future bit patterns are taken into account, an interpolation technique allows significant quality improvements, but at the expense of a delay of $\Lambda - \tau$ parameters $u_{\kappa,\tau}$.

In consequence, the *extrinsic* information of SBSD has to be evaluated for $L_{\text{SBSD}}^{[\text{ext}]}(x_{\kappa,\tau}(\lambda)) = L(\mathbf{z}_{\kappa,\tau+1}^{\Lambda}, \mathbf{z}_{\kappa,\tau}^{[\text{ext}]}, \mathbf{z}_{\kappa,1}^{\tau-1}\,|\,x_{\kappa,\tau}(\lambda))$. The overall determination rule is given by (6) [4]. Therein, the channel-related L-values of all bits contributing to the diversity transmission $L(z_{k,t}(l)\,|\,x_{k,t}(l))$ and the additional *a priori* knowledge of soft-output channel decoding $L_{\text{CD}}^{[\text{ext}]}(x_{k,t}(l))$ serve as soft-input values. The L-values for $\mathbf{z}_{\kappa,\tau}^{[\text{ext}]}$ become self-evident in

$$\Theta(\mathbf{z}_{\kappa,\tau}^{[\text{ext}]}\,|\,\mathbf{x}_{\kappa,\tau}^{[\text{ext}]}) = \exp \sum_{x_{k,t}(l) \in \mathbf{x}_{\kappa,\tau}^{[\text{ext}]}} \frac{x_{k,t}(l)}{2} \cdot \Big(L(z_{k,t}(l)\,|\,x_{k,t}(l)) + L_{\text{CD}}^{[\text{ext}]}(x_{k,t}(l)) \Big). \tag{7}$$

In a similar fashion, past and future observations $\mathbf{z}_{\kappa,1}^{\tau-1}$, $\mathbf{z}_{\kappa,\tau+1}^{\Lambda}$ are included in an efficient *forward-backward* algorithm. Both recursive formulas are determinable by

$$\alpha_{\tau-1}(\mathbf{x}_{\kappa,\tau-1}) = \sum_{\mathbf{x}_{\kappa,\tau-2}} P(\mathbf{x}_{\kappa,\tau-1}\,|\,\mathbf{x}_{\kappa,\tau-2}) \cdot \Theta(\mathbf{z}_{\kappa,\tau-1}\,|\,\mathbf{x}_{\kappa,\tau-1}) \cdot \alpha_{\tau-2}(\mathbf{x}_{\kappa,\tau-2}) \tag{8}$$

$$\beta_{\tau}(\mathbf{x}_{\kappa,\tau}) = \sum_{\mathbf{x}_{\kappa,\tau+1}} P(\mathbf{x}_{\kappa,\tau+1}\,|\,\mathbf{x}_{\kappa,\tau}) \cdot \Theta(\mathbf{z}_{\kappa,\tau+1}\,|\,\mathbf{x}_{\kappa,\tau+1}) \cdot \beta_{\tau+1}(\mathbf{x}_{\kappa,\tau+1}) \tag{9}$$

with the initializations $\alpha_0(\mathbf{x}_{\kappa,0}) = P(\mathbf{x}_{\kappa,0})$ and $\beta_{\Lambda}(\mathbf{x}_{\kappa,\Lambda}) = 1$. With respect to the defined size of the interleaver Φ, throughout the refinement of bitwise *log-likelihood* values $\mathrm{T} + 1$ consecutive bit patterns $\mathbf{x}_{\kappa,\tau}$ with $\tau = \Lambda - \mathrm{T}, \ldots \Lambda$ are regarded in common. In consequence, the forward recursion needs not to be recalculated from the very beginning $\alpha_0(\mathbf{x}_{\kappa,0})$ in each iteration. All terms $\mathbf{x}_{\kappa,1}^{\Lambda-\mathrm{T}-1}$, which are scheduled before the first interleaved bit pattern $\mathbf{x}_{\kappa,\Lambda-\mathrm{T}}$, will not be updated during the iterative feedback of *extrinsic* information and can be measured once in advance.

The 1st order Markov property $P(\mathbf{x}_{\kappa,\tau}\,|\,\mathbf{x}_{\kappa,\tau-1})$ is explicitly evaluated in the recursions (8), (9). In contrast, in order to extract the bitwise *a priori* knowledge $L(x_{\kappa,\tau}(\lambda))$ of (2), reduced source statistics remain for (6). They result from the approximation

$$P(\mathbf{x}_{\kappa,\tau}^{[\text{ext}]}\,|\,\mathbf{x}_{\kappa,\tau-1}, x_{\kappa,\tau}(\lambda)) \approx \frac{P(\mathbf{x}_{\kappa,\tau}\,|\,\mathbf{x}_{\kappa,\tau-1})}{P(x_{\kappa,\tau}(\lambda))}. \tag{10}$$

Determination rule for the *extrinsic* L-value of *softbit source decoding*:

$$L_{\text{SBSD}}^{[\text{ext}]}(x_{\kappa,\tau}(\lambda)) = \log \frac{\sum\limits_{\mathbf{x}_{\kappa,\tau}^{[\text{ext}]}} \beta_{\tau}(\mathbf{x}_{\kappa,\tau}) \cdot \Theta(\mathbf{z}_{\kappa,\tau}^{[\text{ext}]}\,|\,\mathbf{x}_{\kappa,\tau}^{[\text{ext}]}) \sum\limits_{\mathbf{x}_{\kappa,\tau-1}} P(\mathbf{x}_{\kappa,\tau}^{[\text{ext}]}\,|\,\mathbf{x}_{\kappa,\tau-1}, x_{\kappa,\tau}(\lambda) = +1) \cdot \alpha_{\tau-1}(\mathbf{x}_{\kappa,\tau-1})}{\sum\limits_{\mathbf{x}_{\kappa,\tau}^{[\text{ext}]}} \beta_{\tau}(\mathbf{x}_{\kappa,\tau}) \cdot \Theta(\mathbf{z}_{\kappa,\tau}^{[\text{ext}]}\,|\,\mathbf{x}_{\kappa,\tau}^{[\text{ext}]}) \sum\limits_{\mathbf{x}_{\kappa,\tau-1}} P(\mathbf{x}_{\kappa,\tau}^{[\text{ext}]}\,|\,\mathbf{x}_{\kappa,\tau-1}, x_{\kappa,\tau}(\lambda) = -1) \cdot \alpha_{\tau-1}(\mathbf{x}_{\kappa,\tau-1})} \tag{6}$$

3. ANALYSIS OF *EXTRINSIC* INFORMATION

In order to predict the convergence behavior of iterative processes, in [1] a powerful method has been proposed which applies the *mutual information* measure to the input/output relations of the individual constituent soft-input/soft-output decoders. On the one hand, the information exhibited by the *a priori log-likelihood* ratio, and on the other hand, the information comprised in the *extrinsic* L-values after soft-output decoding is closely related to the information content of the originally transmitted data bits $x_{\kappa,\tau}(\lambda)$.

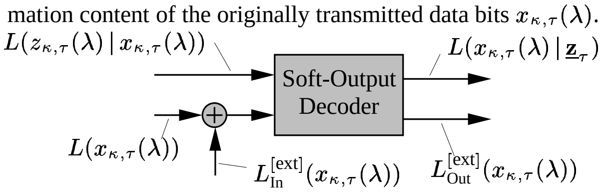

Figure 3: Soft-input/soft-output decoder using L-values

With respect to the *mutual information* measure the input/output relations can be quantified by $\mathcal{I}(\mathcal{X};\mathcal{L}^{[\text{apri}]})$ and $\mathcal{I}(\mathcal{X};\mathcal{L}^{[\text{ext}]})$. The term $\mathcal{X}$ specifies the alphabet of the discrete binary random variable in bipolar form $x_{\kappa,\tau}(\lambda) \in \mathcal{X}$ with $\mathcal{X} = \{-1, +1\}$ and $\mathcal{L}^{[\text{apri}]}$ resp. $\mathcal{L}^{[\text{ext}]}$ denotes the random process of the real valued L-value $\mathcal{L}^{[\cdot]} \in \mathbb{R}$ either at the *a priori* input or *extrinsic* output of the soft-output decoder. To simplify notation we define:

- $\mathcal{I}^{[\text{apri}]}$ quantifies the *mutual information* $\mathcal{I}(\mathcal{X};\mathcal{L}^{[\text{apri}]})$ in the distributions of the data bit $x_{\kappa,\tau}(\lambda)$ and the overall *a priori* L-value $L(x_{\kappa,\tau}(\lambda)) + L_{\text{In}}^{[\text{ext}]}(x_{\kappa,\tau}(\lambda))$.
- $\mathcal{I}^{[\text{ext}]}$ denotes the *mutual information* $\mathcal{I}(\mathcal{X};\mathcal{L}^{[\text{ext}]})$ between $x_{\kappa,\tau}(\lambda)$ and the *extrinsic* information $L_{\text{Out}}^{[\text{ext}]}(x_{\kappa,\tau}(\lambda))$.

The upper limit for both measures is constrained to the information content of $\mathcal{X}$. That is, $\mathcal{I}^{[\text{apri}]}$ and $\mathcal{I}^{[\text{ext}]}$ are bound to values less than or equal to the entropy $\mathcal{H}(\mathcal{X})$.

3.1. Extrinsic Information Transfer (EXIT) Characteristics

The *mutual information* measure at the output of the decoder $\mathcal{I}^{[\text{ext}]}$ depends on the settings applied at the input. Thereby, the channel-related input ratio $L(z_{\kappa,\tau}(\lambda)\,|\,x_{\kappa,\tau}(\lambda))$ is mainly adjusted by the E_s/N_0 value (see (3)). Observations obtained by simulation reveal [1] that the *a priori log-likelihood* ratio can be modeled by a Gaussian distributed random variable with variance $\sigma_L^2 = 4/\sigma_n^2$ and mean $\mu_L = \sigma_L^2/2 \cdot x_{\kappa,\tau}(\lambda)$. Since both are adjustable by a single variable σ_L^2, for arbitrary σ_L^2 the *a priori* relation $\mathcal{I}^{[\text{apri}]}$ can immediately be evaluated by numerical integration. Thus, the EXIT characteristics $\mathcal{T}$ of soft-output decoders are defined as [1]

$$\mathcal{I}^{[\text{ext}]} = \mathcal{T}\left(\mathcal{I}^{[\text{apri}]}, E_s/N_0\right) \quad . \tag{11}$$

If defined settings for $\mathcal{I}^{[\text{apri}]}$ resp. σ_L^2 and for E_s/N_0 are adjusted, $\mathcal{I}^{[\text{ext}]}$ is quantifiable by means of Monte-Carlo simulation.

The combination of EXIT characteristics of two soft-output decoders is referred to as EXIT chart [1]. The main contribution of EXIT charts is that an analysis of the convergence behavior of an entire concatenated scheme is realizable by solely studying the EXIT characteristics of the single components. Further on, the EXIT characteristics of SBSD will be analyzed.

3.2. EXIT characteristics of Softbit Source Decoding

Figure 4 depicts typical EXIT characteristics of SBSD when either a non-uniform distribution of the source codec parameters $u_{\kappa,\tau}$ or additionally an auto-correlation property is exploited. The $u_{\kappa,\tau}$ are modeled by a 1st order Gauss-Markov process with $\rho = 0.0$ or $\rho = 0.8$ and quantized by a Lloyd-Max quantizer using $K_\kappa = 3$ bits/parameter each. As index assignment serves *natural binary* and with this, entropy $\mathcal{H}(\mathcal{X}) = 1$ bit.

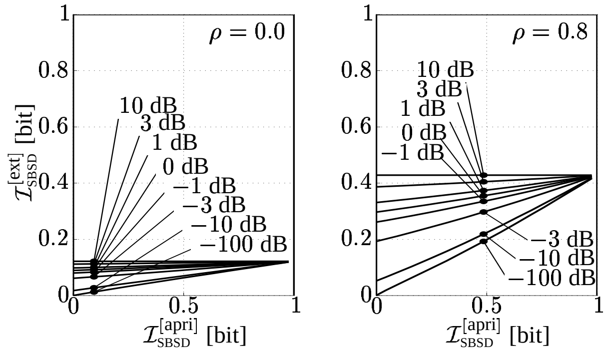

Figure 4: EXIT characteristics of SBSD for various E_s/N_0

The left subplot shows the results, when the *mutual information* measure $\mathcal{I}_{\text{SBSD}}^{[\text{ext}]}$ evaluates a non-uniform parameter distribution only. In contrast to sophisticated soft-output channel decoding none of the curves reaches entropy $\mathcal{I}_{\text{SBSD}}^{[\text{ext}]} = \mathcal{H}(\mathcal{X})$ even if the *mutual information* at the input $\mathcal{I}_{\text{SBSD}}^{[\text{apri}]}$ approaches 1 bit. The right subplot shows the results, when in addition a correlation property is utilized. Due to the correlation more information about $x_{\kappa,\tau}(\lambda)$ is available and thus, *mutual information* $\mathcal{I}_{\text{SBSD}}^{[\text{ext}]}$ increases.

However, for all EXIT characteristics it can be stated that the *mutual information* at the output increases approximately linear in the *mutual information* at the input. Thereby, the slope is usually rather flat. Similar characteristics are obtainable for different correlation properties ρ, quantizer codebook sizes 2^{K_κ}, bit mappings, and interpolation depths $\Lambda - \mathrm{T}$. Moreover, for a fixed but arbitrary parameter setting all curves merge in a single point.

Hence, for every parameter setting this maximum *mutual information* value $\mathcal{I}_{\text{SBSD,max}}^{[\text{ext}]}$ is quantifiable analytically. Whenever the input relation $\mathcal{I}_{\text{SBSD}}^{[\text{apri}]}$ increases to $\mathcal{H}(\mathcal{X}) = 1$ bit (or the channel quality increases to approximately $E_s/N_0 \approx 10$ dB), the terms $\Theta(\mathbf{z}_{\kappa,\tau}^{[\text{ext}]}\,|\,\mathbf{x}_{\kappa,\tau}^{[\text{ext}]})$, $\alpha_{\tau-1}(\mathbf{x}_{\kappa,\tau-1})$, and $\beta_\tau(\mathbf{x}_{\kappa,\tau})$ of (6) are generally valued such that all summations in the numerator and denominator degenerate to single elements. In consequence, the theoretically attainable $L_{\text{SBSD}}^{[\text{ext}]}(x_{\kappa,\tau}(\lambda))$ are given for all possible combinations of $\mathbf{x}_{\kappa,\tau+1}^{\Lambda}, \mathbf{x}_{\kappa,\tau}^{[\text{ext}]}, \mathbf{x}_{\kappa,1}^{\tau-1}$ by

$$L_{\text{SBSD}}^{[\text{ext}]}(x_{\kappa,\tau}(\lambda)) = \log \frac{P(\mathbf{x}_{\kappa,\tau}^{[\text{ext}]}\,|\,\mathbf{x}_{\kappa,\tau-1}, x_{\kappa,\tau}(\lambda){=}{+1}) \cdot P(\mathbf{x}_{\kappa,\Lambda-\mathrm{T}-1}) \prod\limits_{\substack{t=\Lambda-\mathrm{T},\\ t\neq\tau}}^{\Lambda} P(\mathbf{x}_{\kappa,t}\,|\,\mathbf{x}_{\kappa,t-1})}{P(\mathbf{x}_{\kappa,\tau}^{[\text{ext}]}\,|\,\mathbf{x}_{\kappa,\tau-1}, x_{\kappa,\tau}(\lambda){=}{-1}) \cdot P(\mathbf{x}_{\kappa,\Lambda-\mathrm{T}-1}) \prod\limits_{\substack{t=\Lambda-\mathrm{T},\\ t\neq\tau}}^{\Lambda} P(\mathbf{x}_{\kappa,t}\,|\,\mathbf{x}_{\kappa,t-1})} . \tag{12}$$

If the discrete probability distribution of all attainable values $L_{\text{SBSD}}^{[\text{ext}]}(x_{\kappa,\tau}(\lambda))$ is quantifed, the evaluation of its *mutual information* to $x_{\kappa,\tau}(\lambda)$ provides the upper bound for $\mathcal{I}_{\text{SBSD,max}}^{[\text{ext}]}$.

Table 1 summarizes the upper bounds for the example situation with $K_\kappa = 3$ bits/parameter and some of the most frequently used

	Correlation Property ρ			
	$\rho = 0.0$	$\rho = 0.7$	$\rho = 0.8$	$\rho = 0.95$
natural bin.	0.123	0.330	0.429	0.684
folded bin.	0.036	0.213	0.293	0.551
Gray enc.	0.054	0.226	0.299	0.515

Table 1: Theoretical bounds for $\mathcal{I}_{\text{SBSD,max}}^{[\text{ext}]}$

index assignments [6]. To simplify matters, *softbit source decoding* is restricted to parameter extrapolation, i.e., $\Lambda - \mathrm{T} = 1$. The theoretical upper bounds confirm the corresponding simulation results of Figure 4. Note, the bounds $\mathcal{I}^{\text{[ext]}}_{\text{SBSD,max}}$ do not comprise any information about the adverse effects of different mappings on instrumental quality measures like the parameter *signal-to-noise ratio* (SNR).

4. CAPABILITIES AND CONVERGENCE BEHAVIOR

The capabilities and convergence behavior of ISCD shall be demonstrated by simulation. Therefore, $M = 50$ components $u_{\kappa,\tau}$ are individually modeled by 1st order Gauss-Markov processes with correlation $\rho = 0.8$. The parameters $u_{\kappa,\tau}$ are scalarly quantized by an 8-level Lloyd-Max quantizer using $K_\kappa = 3$ bits/parameter each. *Natural binary* is used as index assignment. A classical $K_\kappa \times M$ block interleaver Φ serves for spreading of information bits. In Φ, at each time instant τ the incoming M bit patterns $\mathbf{x}_{\kappa,\tau}$ are written into the M rows and the interleaved set is read out along the columns. For channel encoding, a terminated memory $J = 2$ *recursive systematic convolutional* (RSC) code with generator polynomial $\mathbf{G}(D) = (1\,;\, \frac{1+D^2}{1+D+D^2})$ is used. For termination $J = 2$ tail bits are appended to each block of 3×50 data bits, which force the encoder back to zero state. The overall code rate of the iterative source-channel decoding scheme amounts $r = 150/304$. A BCJR decoder serves as component decoder in the ISCD scheme which is slightly modified with respect to the recursive structure of RSC codes [2].

Figure 5 shows the simulation results for various numbers of iterations. The curves obtained after soft-output channel decoding are marked by an upper "+" (switch to dash-dotted line in Figure 2). The curve labeled "0. iteration" neglects the reliability gains of soft-channel and *softbit source decoding*. A stepwise integration of both reliability gains permits to increase the robustness of the transmission system significantly. The *explicit* diversity of channel encoding ("0^+. iter.") as well as the *implicit* diversity due to correlation ("1. iteration"), improves the parameter SNR by several dB each.

Feedback of SBSD's *extrinsic* information to the BCJR channel decoder enables the soft-channel decoder to refine the $L^{\text{[ext]}}_{\text{CD}}(x_{\kappa,\tau}(\lambda))$ so far, that in the "2. iteration" the *softbit source decoder* reveals an additional quality gain. In moderately disturbed channel conditions of about $E_s/N_0 = -3$ dB the gain amounts $\Delta_{\text{SNR}} = 2.23$ dB. However, no noteworthy improvements are achievable with higher numbers of iterations.

The fast convergence can be confirmed, e.g., at $E_s/N_0 = -3$ dB (see vertical line), with the corresponding EXIT chart and a *decoding trajectory*. The EXIT characteristic of SBSD is taken from Figure 4, but with swapped axes. The *decoding trajectory* denotes the step curve in Figure 5 which visualizes the increase in *mutual information* gainable in the single iterations.

Decoding starts with the BCJR decoder while the *a priori* knowledge amounts $\mathcal{I}^{\text{[apri]}}_{\text{CD}} = 0$ bit. Due to the reliability gain of soft-output decoding, the decoder is able to provide $\mathcal{I}^{\text{[ext]}}_{\text{CD}} = 0.45$ bit. The instantaneous value of the trajectory is labeled "0^+. iter.".

The new information serves as *a priori* knowledge for SBSD, i.e., $\mathcal{I}^{\text{[apri]}}_{\text{SBSD}} = \mathcal{I}^{\text{[ext]}}_{\text{CD}}$, and thus its *extrinsic mutual information* is determinable $\mathcal{I}^{\text{[ext]}}_{\text{SBSD}} = 0.29$ bit ("1. iteration"). Iteratively decoding both soft-input/soft-output decoders allows to increase the information content step-by-step.

No further information is gainable, when an intersection in the enveloping EXIT characteristics is reached. In ISCD schemes intersections typically appear due to the upper bound $\mathcal{I}^{\text{[ext]}}_{\text{SBSD,max}}$. With respect to the flatness of SBSD's EXIT characteristic such an intersection is reached quite close after "2. iteration".

5. CONCLUSIONS

In this paper, the convergence behavior of iterative source-channel decoding is studied using EXIT charts. They permit to understand the capabilities of ISCD schemes. EXIT characteristics are determined for *softbit source decoding*, which are typically rather flat for various parameter settings. For a fixed parameter setting, the EXIT characteristics merge in a single prominent value. This value can be confirmed by analytical considerations. Due to the flatness of SBSD's EXIT characteristic usually a small number of iterations is sufficient to gain the highest possible *mutual information*. However, the capabilities might be improvable if channel codes are adapted to the source properties.

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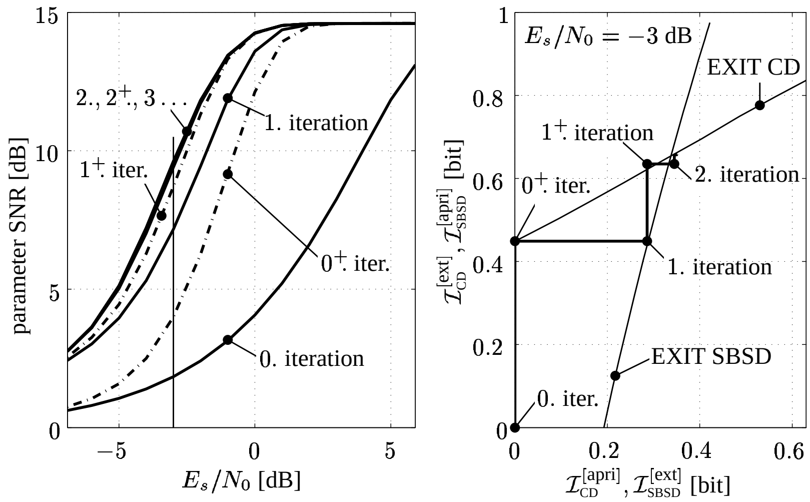

Figure 5: Improvements in *parameter* SNR and corresponding EXIT chart