A NOVEL DECODING PROCEDURE FOR REAL FIELD ERROR CONTROL CODES IN THE PRESENCE OF QUANTIZATION NOISE

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Abstract--**In this paper, we propose a novel decoding technique for DFT-based error control codes, which is robust against quantization and additive noise. The proposed algorithm simultaneously determines the number and the positions of the corrupted samples. We show that in contrast to the conventional decoding techniques, the proposed decoding method is stable in the presence of the quantization and additive noise.**

I. INTRODUCTION

In one of the most commonly used real field error control coding techniques, i.e., DFT-Based codes, the codeword is produced by padding enough zeros in the discrete Fourier transform (DFT) of the information vector [1]-[4]. It is shown that the decoding of this coding scheme reduces to solving a set of t Toeplitz equations, where t is the number of errors or missing samples [3]-[4].

Theoretically, the number of corrupted samples by impulsive noise, t, can be estimated using the well-known recursive methods such as Levinson-Durbin, Berlekamp-Massey, and Euclidian algorithms [5]. But in practice, because of the presence of additive quantization noise, which is inherent in the quantization process, these algorithms become unstable and fail to estimate the number of errors.

This deficiency has encouraged us to consider a novel decoding technique for real field error control codes under the impulsive channel model. It will be shown that the proposed method can estimate the number and positions of corrupted samples in the presence of quantization additive noise.

By impulsive noise we mean that a finite number of samples are either erased or drastically changed. While the remaining samples are almost similar to the original samples and are only different slightly due to quantization or Additive White Gaussian Noise.

II. THE PROPOSED DECODING METHOD

Using the DFT-based error control codes, in the encoder, the information vector, K-tuple $\mathbf{u}$, is encoded into an N-tuple $\mathbf{v}$, called a codevector (codeword), where we have $N > K$. In these codes, the encoding procedure is as follows,

1- We take DFT of the $\mathbf{u}$ to get K-tuple $\mathbf{U}$.
2- We insert N-K consecutive zeros to get N-tuple $\mathbf{V}$.
3- We take its inverse DFT to get N-tuple codeword $\mathbf{v}$.

In the receiver, let $\mathbf{r}$ be the received vector and suppose that an unknown error vector $\mathbf{e}$ is introduced as follows,

$$\mathbf{r} = \mathbf{v} + \mathbf{e} \tag{1}$$

where $\mathbf{v}$ is the transmitted codeword. We assume that the error vector $\mathbf{e}$ is due to lost samples in an impulsive channel. In this case, the ith component of the error vector, $\mathbf{e}(i)$, is zero where i is not equal to the positions of corrupted samples due to loss samples in the impulsive channel.

Let $\mathbf{E}$ be the DFT of the error vector $\mathbf{e}$ that coincides with the DFT of the received vector $\mathbf{R}$ in the positions that zeros are added in the encoding procedure. In the proposed decoding method, we produce an error locator polynomial as follows,

$$S(z) = \sum_{r=0}^{\left\lfloor \frac{N-K}{2} \right\rfloor} h_r z^r = 1 + \sum_{r=1}^{\left\lfloor \frac{N-K}{2} \right\rfloor} h_r z^r \tag{2}$$

where $h_i \quad i = 1,2,...,\left\lfloor \frac{N\text{-}K}{2} \right\rfloor$ satisfies the following equations,

$$S(e^{j\frac{2p}{N} i_m}) = \sum_{r=0}^{\left\lfloor \frac{N-K}{2} \right\rfloor} h_r e^{j\frac{2p}{N} i_m r} = 0, \tag{3}$$

where $\left\{ i_m : m = \{1,2,...,t\} \quad t \le \left\lfloor \frac{N-K}{2} \right\rfloor \right\}$ denotes the positions of the lost samples in the impulsive channel and t denotes the number of lost samples. In this case, $S(z)$ can be written as follows,

$$S(z)=\left\{\prod_{m=1}^{t}\left(1-ze^{-j\frac{2pi_m}{N}}\right)\right\}P(z) \quad (4)$$

where $P(z)$ is a polynomial of z such that its order is

$$\left\lfloor\frac{N-K}{2}\right\rfloor-t \quad (5)$$

and we have,

$$P(0)=1 \quad (6)$$

It should be noted that in the conventional decoding method, the upper limit of summations (2) and (3) is *t*. Therefore the proposed method is a generalization of the conventional technique and if we fix $P(z)=1$, the proposed method reduces to the conventional method, which is discussed in [1]-[4]. On the other hand, in contrast to the conventional method, in the proposed technique, to find the error locator polynomial, it is not necessary to know the number of lost samples, *t*, beforehand.

Similar to the conventional technique, multiplying (3) by $e(i_m)e^{-j\frac{2p}{N}i_m p}$ and summing over i_m, we get the following recursive equation,

$$\sum_{r=0}^{\left\lfloor\frac{N-K}{2}\right\rfloor} h_r E(p-r)=0 \qquad p=0\ ,1\ ,\dots,N \quad (7)$$

In the conventional decoding method, there is a recursive equation similar to (7) in which the upper limit of the summation is the number of lost samples that is *t*. Therefore, in contrast to the conventional method, in the proposed method, it is not necessary to know the number of lost samples. Similar to the conventional decoding technique, in the proposed procedure, the idea is to use equations (7) to determine $\left\lfloor\frac{N-K}{2}\right\rfloor$ unknown coefficients h_r. To solve equations (7), We only need to know *N-K* samples of **E** in the positions where zeros are added in the encoding procedure. Furthermore, because e is real, the equations are Toeplitz and Hermitian and therefore we have,

$$E(N/2-r)=E^*(N/2+r)\ \ r=0,1,\dots,N/2 \quad (8)$$

Thus the set of linear equations shown in (7) are Yule-Walker equations and can be rewritten as,

$$\mathbf{RH}=-\mathbf{E} \quad (9)$$

where we have,

$$\mathbf{E}=\begin{bmatrix}E_{N/2} & E_{N/2} & \dots & E_{N/2-\left\lfloor\frac{N-K}{2}\right\rfloor+1}\end{bmatrix}^T \quad (10)$$

$$\mathbf{R}=\lfloor R_{ij}\rfloor$$

$$R_{ij}=E_{N/2-|i-j|+1} \qquad 1\le i,j\le\left\lfloor\frac{N-K}{2}\right\rfloor \quad (11)$$

and

$$\mathbf{H}=\begin{bmatrix}h_1 & h_2 & \dots & h_{\left\lfloor\frac{N-K}{2}\right\rfloor}\end{bmatrix}^T \quad (12)$$

where $h_i\ \ i=1,2,\dots,\left\lfloor\frac{N\text{-}K}{2}\right\rfloor$ are unknown coefficients of the equations (2) and (7).

As it is shown in [1]-[4], if there is no quantization process, where the exact values of E_i in the positions where zeros are added in the encoding procedure are accessible, **E** will be an Auto Regressive (AR) random process with order *t*. Therefore, the matrix **R** will be singular. In this case, there are infinite solutions for **H** that in the proposed decoding method, each of them can be used as an acceptable solution. One way to find a proper solution is to put an additional constraint on **H.** It has been shown that if the constraint is to require the vector **H** to have the smallest possible magnitude, the unique solution is given by the Moore-Penrose pseodu-inverse that can be found as follows [6],

$$\mathbf{H}=-Pinv(\mathbf{R})*\mathbf{E} \quad (13)$$

where $Pinv(\mathbf{R})$ is a matrix of the same dimension as **R** so that we have,

$$\begin{aligned}&\mathbf{R}*Pinv(\mathbf{R})*\mathbf{R}=\mathbf{R}\\&Pinv(\mathbf{R})*\mathbf{R}*Pinv(\mathbf{R})=Pinv(\mathbf{R})\end{aligned} \quad (14)$$

In the presence of the quantization noise case, when the components of **e** are not zero even if they do not correspond to the positions of lost samples in the impulsive channel, the simulation results show that **R** will be a full rank symmetric Toeplitz matrix and therefore, there will be several fast inversion algorithms for it with any rank profile [5],[7].

From (3), it can be seen that i_m denotes the location of an error if $e^{j\frac{2p}{N}i_m}$ is a zero of the polynomial $S(z)$. Furthermore, if an *N*-tuple vector **HR** is constructed as follows,

$$\mathbf{HR}=\begin{bmatrix}1 & h_1 & h_2 & \dots & h_P & 0 & \dots & 0\end{bmatrix}^T \quad (15)$$

where h_i are the components of the solution of the vector equation (9), it can be easily seen that the zeros of the DFT of HR are in the location of errors.

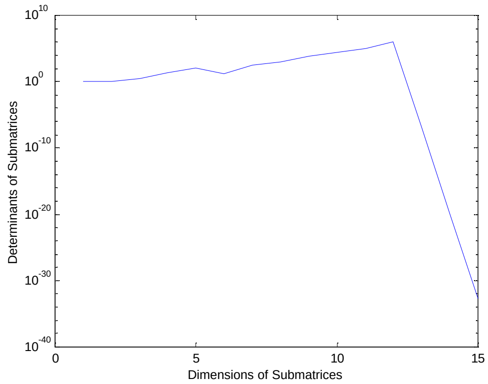

Figure 1: The determinant of submatrices versus the dimensions of submatrices in no quantization noise case.

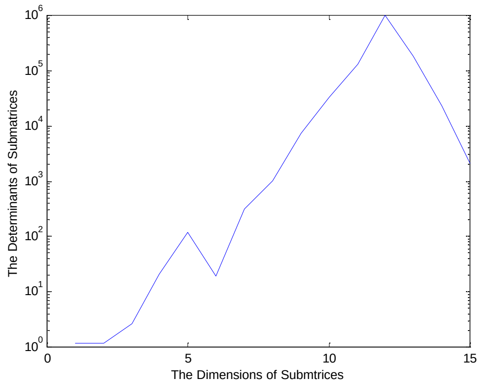

Figure 2: The determinant of submatrices versus the dimensions of submatrices in 8-bit quantization case

After computing **H,** using the identity of $E(i)=R(i)$ for i denoting the positions of *N-K* consecutive zeros in the DFT, the remaining values of E can be found by the recursive equation (7).

III. Numerical Results

As it was stated in the previous sections, if there is no quantization process, the vector **E** will be an AR random process with order t, which denotes the number of lost samples. In the conventional decoding method, the Levinson-Durbin, Berlekamp, and Euclian algorithms can be used to find the number of errors. The idea of these algorithms is to recursively compute the solutions of the Yule-walker equations for the top principal submatrixes in the equations (7). In the conventional technique, to properly estimate the number of errors t, these algorithms requires that the top $t\times t$ principal matrix to be nonsingular and the top $(t+1)\times(t+1)$ principal matrix to be singular. In other words, the curve of determinants of the top principal submatrices versus their dimensions should rapidly decrease at the point of t.

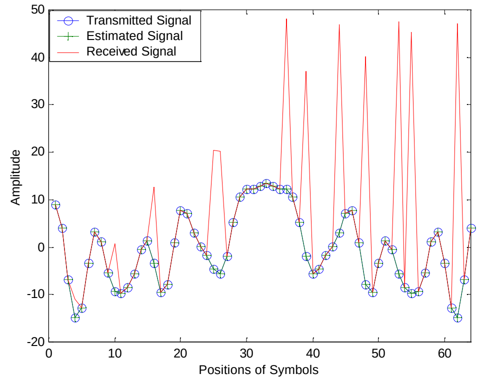

Figure 3: The transmitted, received, and estimated signals in no quantization case.

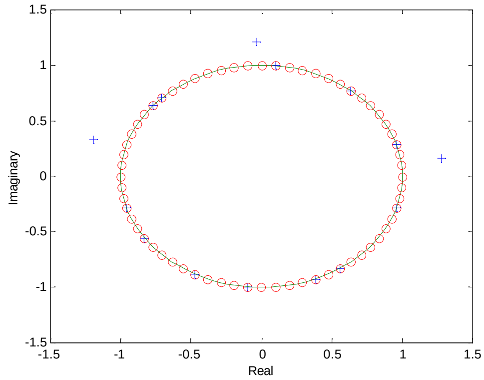

Figure 4: The positions of zeros of the polynomial $S(z)$ in no quantization noise case. The positions of zeros which are on the unit circle, show the location of errors

In order to show the sensitivity of the conventional decoding technique to the additive noise, which is generated by quantization process of the known samples of **E**, for a system with N=64, N-K=31, and t=12, in Figures 1-2, the curves of determinants versus dimensions of top principle submatrices are plotted in no quantization, and 8-bit quantization cases, respectively. It can be seen that in the case of no quantization noise, because of rapid fall off the values of determinants at dimensions greater than 12, the conventional algorithm can estimate the number of errors that is 12. But in the presence of the quantization noise, the conventional algorithm cannot exactly predict the number of errors. To evaluate the performance of the proposed method, the transmitted, the received, and the reconstructed signals using the proposed method in the case of no quantization are plotted in Figure 3. It can be seen that the proposed method can exactly estimate the transmitted signal. In Figure 4, the positions of zeros of the polynomial $S(z)$ are shown. It can be seen that the positions of zeros with the form of $e^{j\frac{2p}{N}m}$ exactly show the location of errors. In Figure 5, the amplitude of

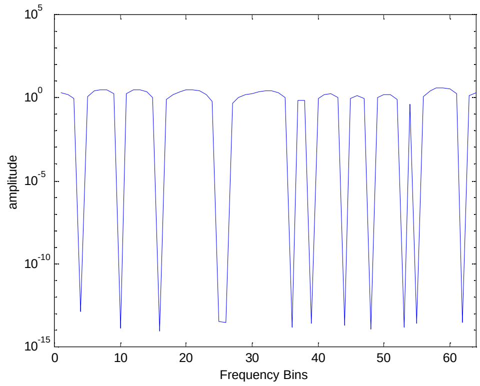

Figure 5: The amplitude of the DFT of **HR** in quantization noise case.

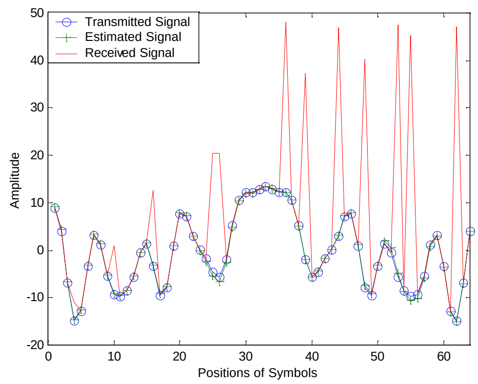

Figure 6: The transmitted, received, and estimated signals in 8-bit quantization case.

DFT of the **HR** is shown. We see that the minima of this curve are at the location of errors.

In Figure 6, the reconstructed signal is compared with the received and transmitted signals in 8-bit quantization case. It can be seen that the proposed method works well and can reconstruct the transmitted signal. In Figure 7, the positions of zeros of $S(z)$ are shown. It can be seen that the positions of those zeros whose locations denote the positions of errors are insensitive to the quantization noise. Figure 8 shows the amplitude of the DFT of **HR** in 8-bit quantization case. It can be seen that the presence of quantization noise does not change the locations of the minima of the curve.

CONCLUSIONS

In this paper, a novel decoding technique for real field error control codes has been proposed. The proposed algorithm determines the number and the positions of the error samples simultaneously. It has been shown that in contrast to the conventional decoding techniques, the proposed method can work in the presence of the additive or quantization noise.

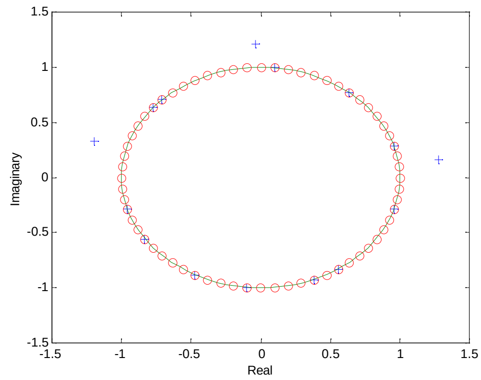

Figure 7: The positions of zeros of the polynomial $S(z)$ in 8-bit quantization case. The positions of zeros with the form of $e^{j\frac{2\pi}{N}m}$, which are on the unit circle, exactly show the location of errors

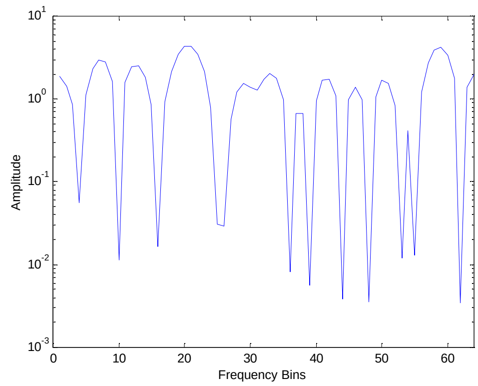

Figure 8: The amplitude of the DFT of **HR** in 8-bit quantization case.

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