OVERSAMPLED FILTERBANKS SEEN AS CHANNEL CODES: IMPULSE NOISE CORRECTION

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ABSTRACT

In this paper, an analogy between oversampled filterbanks and channel codes is considered. Parity-check polynomial matrices and syndromes can be defined. Techniques for detecting and correcting impulse noise are provided. Their performances are illustrated on an example.

1. INTRODUCTION

This paper is concerned with impulse noise detection and correction by mean of oversampled filterbanks (OFB). Recently, OFB have gained growing interest in many area of signal processing, mainly due to three advantages over critically sampled filterbanks (see, *e.g.*, [1] and [2]). They provide increased design freedom, improved noise immunity and have shown, due to the redundancy between their subbands, to be robust to some subbands erasures [3]. This kind of subband losses appears, *e.g.*, in the framework of packet transmission with losses.

In this paper, all subbands are assumed to be available but some of them may be corrupted by impulse errors. This situation can be modeled as a communication channel corrupted by the sum of gaussian plus Bernoulli gaussian noises. Such channels have already been studied by [4] and were shown to be able to model the process of quantization (background noise) and transmission errors (impulse noise).

The corresponding channel is memoryless, and its input ($y(n)$) - output ($\widetilde{y}(n)$) relation is

$$\widetilde{y}(n) = y(n) + a(n) + b(n). \quad (1)$$

The gaussian noise $b(n)$ has zero mean and variance σ_g^2, while the impulse noise $a(n)$ is modeled as Bernoulli gaussian $a(n) = \xi(n)b'(n)$. Here $\xi(n)$ stands for a Bernoulli process, an i.i.d. sequence of ones with prob$(\xi(n) = 1) = p$ and zeros, and $b'(n)$ represents a gaussian noise with zero mean and variance σ_i^2, such that $\sigma_i^2 \gg \sigma_g^2$ [4]. The pdf of the channel noise $c(n) = a(n) + b(n)$ can be expressed as

$$p(c) = (1-p)\, G\left(c,0,\sigma_g^2\right) + pG\left(c,0,\left(\sigma_g^2 + \sigma_i^2\right)\right), \quad (2)$$

with $G(c,m_c,\sigma_c^2)$ denoting a gaussian density with mean m_c and variance σ_c^2.

Previous results on reconstruction of subbands impaired by channel transmission errors have been presented in [5] for critically sampled filterbanks. This work makes use of residual redundancy in the subbands in order to recover the errors. Hence, the efficiency of the source coding algorithm has to be reduced (in order to leave some redundancy in the subbands). Other work in [4] has also shown that real-valued BCH codes had the ability to detect and correct impulse errors. Other related works are the filterbanks over finite fields for channel coding [6] and the multiple description schemes [7]. Compared to these previous works, our intent is to provide a mechanism able to localize the impulse errors in the presence of background noise without impairing the coding efficiency.

In Sec. 2, after a brief recall on the structure of OFB, benefit will be taken from the redundancy between subbands to define syndromes. Sec. 3.1 presents an hypothesis test to determine whether impulse noise is present. Sec. 3.2 provides a technique for determining the characteristics of the detected impulse noise with examples in Sec. 4.

2. OVERSAMPLED FILTERBANKS

The block diagram of an OFB with L filters and decimation factor $M < L$ is shown in Fig. 1. The polyphase representations $\mathbf{X}(z)$ and $\mathbf{Y}(z)$ of the input and output signal of the analysis filterbank can be related by

$$\mathbf{Y}(z) = \mathbf{E}(z)\mathbf{X}(z), \quad (3)$$

where $\mathbf{E}(z) = \left(E_{ij}(z)\right)_{L\times M}$ is the polyphase matrix of the analysis filterbank. $E_{i,j}(z)$ is the z-transform of the polyphase component $e_{i,j}[n] = h_i[nM+j]$. $\mathbf{E}(z)$ admits a Smith-McMillan decomposition

$$\mathbf{E}(z) = \mathbf{U}(z)\begin{pmatrix}\mathbf{\Lambda}(z)\mathbf{W}(z)\\ \mathbf{0}\end{pmatrix}, \quad (4)$$

where $\mathbf{U}(z)$ and $\mathbf{W}(z)$ are unimodular matrices of sizes $L\times L$ and $M\times M$, respectively and $\mathbf{\Lambda}(z)$ is a diagonal matrix

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of size $M \times M$. Let us partition the inverse of $\mathbf{U}(z)$ as

$$\mathbf{U}^{-1}(z) = \begin{pmatrix} \mathbf{V}^0(z) \\ \mathbf{V}(z) \end{pmatrix}, \quad (5)$$

where $\mathbf{V}^0(z)$ is of size $M \times L$ and $\mathbf{V}(z)$ of size $(L-M) \times L$.

Using (4) and (5), one obtains $\mathbf{V}(z)\mathbf{E}(z) = \mathbf{0}$. When multiplying the noise-free output (3) of the filterbank by $\mathbf{V}(z)$, one gets

$$\mathbf{V}(z)\mathbf{Y}(z) = \mathbf{V}(z)\mathbf{E}(z)\mathbf{X}(z) = \mathbf{0}. \quad (6)$$

When transmission errors are considered, one gets

$$\mathbf{V}(z)\widetilde{\mathbf{Y}}(z) = \mathbf{V}(z)\mathbf{A}(z) + \mathbf{V}(z)\mathbf{B}(z) \quad (7)$$

where $\mathbf{A}(z)$ and $\mathbf{B}(z)$ are the polyphase representations of impulse and gaussian noises respectively.

$\mathbf{V}(z)$ can therefore be considered as a *parity-check polynomial matrix* and $\mathbf{S}(z) = \mathbf{V}(z)\widetilde{\mathbf{Y}}(z)$ can be considered as a *syndrome*, equal to zero when no transmission error is encountered, and non-zero if there are some errors. This property is used in the next section, to characterize the presence and location of the impulse errors.

3. IMPULSE NOISE CANCELLATION

The proposed technique for detecting and correcting impulse noise in OFB involves the following steps. First, one has to test whether impulses are present and optionally to estimate the number of impulse errors. Then one has to find out their location and amplitude before correcting them.

3.1. Testing whether impulses are present

The test to decide whether impulse errors are present is based on the norm of the syndrome and is similar to that presented in [8]. $\mathbf{V}(z)$ is assumed to be of order N_V, thus

$$\mathbf{V}(z) = \sum_{i=0}^{N_V} \mathbf{V}_i z^{-i}, \text{ with } \mathbf{V}_i \in \mathbb{R}^{(L-M)\times L}. \quad (8)$$

The syndrome expression in the time domain over a window of size N in the subbands with impulse errors is

$$\underline{\mathbf{S}}(n) = \begin{pmatrix} \underline{\mathbf{V}} & \underline{\mathbf{V}} \end{pmatrix} \begin{pmatrix} \underline{\mathbf{b}}(n) \\ \underline{\mathbf{a}}(n) \end{pmatrix}, \quad (9)$$

where

$$\begin{aligned} \underline{\mathbf{S}}(n) &= \left(\mathbf{s}^T(n), \ldots, \mathbf{s}^T(n-N+1)\right)^T, \\ \underline{\mathbf{b}}(n) &= \left(\mathbf{b}^T(n), \ldots, \mathbf{b}^T(n-N+1-N_V)\right)^T, \\ \underline{\mathbf{a}}(n) &= \left(\mathbf{a}^T(n), \ldots, \mathbf{a}^T(n-N+1-N_V)\right)^T \end{aligned}$$

and $\underline{\mathbf{V}}$ is an $(L-M)N \times L(N+N_v)$ matrix

$$\underline{\mathbf{V}} = \begin{pmatrix} \mathbf{V}_0 & \mathbf{V}_1 & \cdots & \mathbf{V}_{N_V} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{V}_0 & \mathbf{V}_1 & \cdots & \mathbf{V}_{N_V} & \ddots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{V}_0 & \mathbf{V}_1 & \cdots & \mathbf{V}_{N_V} \end{pmatrix}.$$

When ν impulse errors are present, $\underline{\mathbf{a}}(n)$ contains thus only ν non-zero values. Let $\mathbf{k}(\nu) = (k_0, \ldots, k_{\nu-1})^T$ and $\boldsymbol{\ell}(\nu) = (\ell_0, \ldots, \ell_{\nu-1})^T$ be the vectors containing the subband and time locations of the ν impulses. Then, (9) can be rewritten as

$$\underline{\mathbf{S}}(n) = \begin{pmatrix} \underline{\mathbf{V}} & \underline{\underline{\mathbf{V}}}(\mathbf{k}(\nu), \boldsymbol{\ell}(\nu)) \end{pmatrix} \begin{pmatrix} \underline{\mathbf{b}}(n) \\ \underline{\underline{\mathbf{a}}}(n) \end{pmatrix}, \quad (10)$$

where $\underline{\underline{\mathbf{a}}}(n) = (a_0, \ldots, a_{\nu-1})^T$, and $\underline{\underline{\mathbf{V}}}(\mathbf{k}(\nu), \boldsymbol{\ell}(\nu))$ is a size $(L-M)N \times \nu$ matrix obtained by selecting the columns of $\underline{\mathbf{V}}$ indexed by $k_i + L(n-\ell_i)$, $i = 0, \ldots, \nu-1$.

Since the gaussian and impulse noises do not have the same variance, after multiplying $\left(\underline{\mathbf{b}}^T(n), \underline{\underline{\mathbf{a}}}^T(n)\right)^T$ by a diagonal matrix $\mathbf{D}(\nu) = \text{diag}\left(\sigma_g^{-1}, \ldots, \sigma_g^{-1}, \sigma_i^{-1}, \ldots, \sigma_i^{-1}\right)$ of size $(L(N+N_V)+\nu)^2$ to normalize it, one gets

$$\begin{aligned} \mathbf{R}(\nu) &= \mathbf{D}(\nu) \begin{pmatrix} \underline{\mathbf{b}}(n) \\ \underline{\underline{\mathbf{a}}}(n) \end{pmatrix}, \\ \underline{\mathbf{S}}(n) &= \mathbf{R}(\nu)\mathbf{Q}(\mathbf{k}, \boldsymbol{\ell}), \\ \mathbf{Q}(\mathbf{k}, \boldsymbol{\ell}) &= \begin{pmatrix} \underline{\mathbf{V}} & \underline{\underline{\mathbf{V}}}(\mathbf{k}, \boldsymbol{\ell}) \end{pmatrix} \mathbf{D}(\nu)^{-1}, \end{aligned}$$

Now, $\mathbf{Q}^H(\mathbf{k}, \boldsymbol{\ell})\mathbf{Q}(\mathbf{k}, \boldsymbol{\ell})$ can be diagonalized as

$$\mathbf{Q}(\mathbf{k}, \boldsymbol{\ell})^H \mathbf{Q}(\mathbf{k}, \boldsymbol{\ell}) = \mathbf{K}^H(\mathbf{k}, \boldsymbol{\ell})\mathbf{G}(\mathbf{k}, \boldsymbol{\ell})\mathbf{K}(\mathbf{k}, \boldsymbol{\ell})$$

Hence, $\|\underline{\mathbf{S}}(n)\|^2$ can be written as a quadratic form of the normalized gaussian vector $\mathbf{u} = \mathbf{K}(\mathbf{k}, \boldsymbol{\ell})\mathbf{R}(\nu)$

$$\|\underline{\mathbf{S}}(n)\|^2 = \mathbf{u}^H \mathbf{G}(\mathbf{k}, \boldsymbol{\ell})\mathbf{u} = \sum_{k=0}^{\text{rank}(\mathbf{G}(\mathbf{k}(\nu), \boldsymbol{\ell}(\nu)))-1} (g_k u_k)^2$$

where the g_k^2 are the diagonal entries of $\mathbf{G}$. Hence, the pdf of y is a convolution of chi-square distributions with one degree of freedom

$$p(y|\nu, \mathbf{k}(\nu), \boldsymbol{\ell}(\nu)) = \bigotimes_{k=0}^{\text{rank}(\mathbf{G}(\mathbf{k}(\nu), \boldsymbol{\ell}(\nu)))-1} \chi_1^2(g_k, y), \quad (11)$$

where

$$\chi_1^2(\sigma, y) = \frac{1}{\sqrt{2\pi}\sigma} y^{-1/2} \exp\left(-\frac{y}{2\sigma^2}\right).$$

The pdf of the norm of the syndrome in presence of impulse errors is then obtained by averaging (11) for all possible $\nu > 0$, subband and time locations of the errors

$$p(y|\nu > 0) = \sum_{\nu>0} \sum_{\mathbf{k}(\nu)} \sum_{\boldsymbol{\ell}(\nu)} p(\nu, \mathbf{k}, \boldsymbol{\ell}) . p(y|\nu, \mathbf{k}, \boldsymbol{\ell}), \quad (12)$$

where $p(\nu, \mathbf{k}, \boldsymbol{\ell})$ is the *a priori* probability of having ν impulses in subbands $\mathbf{k}(\nu)$ and at times $\boldsymbol{\ell}(\nu)$.

When there is no impulse noise in the observation window, (9) simplifies to

$$\underline{\mathbf{S}}(n) = \underline{\mathbf{V}}\mathbf{b}(n). \tag{13}$$

Using computations similar to those developed previously, one gets in this case

$$p(y|\nu = 0) = \bigotimes_{k=0}^{\text{rank}(\mathbf{G})-1} \chi_1^2(g_k, y). \tag{14}$$

Now, two hypotheses can be formulated: (**i**) H_0 (there is no impulse error), with *a priori* probability $P(H_0) = p_0 = (1-p)^{L(N+N_v)}$, (**ii**) H_1 (there are impulse errors), with *a priori* probability $P(H_1) = p_1 = 1 - (1-p)^{L(N+N_v)}$, where p is defined in Sec. 1.

The likelihood ratio $\Lambda(\|\mathbf{S}(n)\|^2)$ for deciding which of H_0 or H_1 is true can then be built using p_0, p_1, (14) and (12). Finally, $\|\mathbf{S}(n)\|^2$ has simply to be compared to a threshold α for deciding which of H_0 or H_1 is true

$$\|\mathbf{S}(n)\|^2 \lessgtr_{H_1}^{H_0} \alpha. \tag{15}$$

A characterization of the test is obtained through the ROC-curve (probability of detection P_d versus false alarm P_f).

Extension to multiple decision tests to determine the number of impulse errors is presented in [8].

3.2. Characteristics of impulse errors

For each impulse error, $\overline{k}$ and $\overline{\ell}$, the subband and time where it accurs and $\overline{a}$, its amplitude have to be estimated. For the sake of brevity, only the case of a single error occurring during the observation of N samples of a syndrome in the subbands is presented. In this case, (10) can be written as

$$\underline{\mathbf{S}}(n) = \underline{\underline{\mathbf{V}}}\left(\overline{k}, \overline{\ell}\right)\overline{a} + \underline{\mathbf{V}}\mathbf{b}(n). \tag{16}$$

Thus, for each subband $m \in \{0, \ldots, L-M-1\}$, one gets

$$\mathbf{s}_m(n) = \overline{a}\mathbf{v}_{mk}\left(n - \overline{\ell}\right) + \widetilde{\mathbf{b}}_m(n), \tag{17}$$

where

$$\begin{aligned}
\mathbf{s}_m(n) &= (s_m(n), \ldots, s_m(n-N+1))^T, \\
\widetilde{\mathbf{b}}_m(n) &= \left(\widetilde{b}_m(n) \ldots . \widetilde{b}_m(n-N+1)\right)^T, \\
\mathbf{v}_{mk}^{\mathrm{T}}(n-\ell) &= (v_{mk}(n-\ell), \ldots . . v_{mk}(n-N+1-\ell)).
\end{aligned}$$

As $\widetilde{\mathbf{b}}_m(n) = \underline{\mathbf{V}}\mathbf{b}(n)$, its covariance matrix $\boldsymbol{\Gamma}_m$ is easily obtained from $\underline{\mathbf{V}}$ and $E\left(\underline{\mathbf{b}}(n)\underline{\mathbf{b}}^T(n)\right) = \sigma_g^2\mathbf{I}$.

First, compute the joint *a posteriori* pdf of k, a and ℓ. This is easily obtained from $f\left(\widetilde{\mathbf{b}}_m(n)|I\right)$. Then the *a posteriori* pdfs for k and ℓ are obtained by marginalizing the joint pdf with respect to the other parameters. The *a priori* pdfs for k and ℓ are uniform ones over the subbands and over the observation windows. Thus, one obtains

$$\begin{aligned}
&f(k|\mathbf{s}_0, \mathbf{s}_1, \ldots, \mathbf{s}_{L-M-1}, \boldsymbol{\Gamma}_0, \ldots, \boldsymbol{\Gamma}_{L-M-1}, I) \\
&\quad = \sum_{\ell=n}^{n-N-N_V+1} \frac{K_1}{\sqrt{R_k(n,\ell)}\sigma_i} \exp\left(\frac{Q_k^2(n,\ell)}{2R_k(n,\ell)}\right),
\end{aligned} \tag{18}$$

$$\begin{aligned}
&f(\ell|\mathbf{s}_0, \mathbf{s}_1, \ldots, \mathbf{s}_{L-M-1}, k, \boldsymbol{\Gamma}_0, \ldots, \boldsymbol{\Gamma}_{L-M-1}, I) \\
&\quad = \frac{K_2}{\sqrt{R_k(n,\ell)}} \exp\left(\frac{Q_k^2(n,\ell)}{2R_k(n,\ell)}\right),
\end{aligned} \tag{19}$$

where

$$Q_k(n,\ell) = \sum_{m=0}^{L-M-1} \mathbf{s}_m^{\mathrm{T}}(n)\boldsymbol{\Gamma}_m^{-1}\mathbf{v}_{mk}(n-\ell),$$

$$R_k(n,\ell) = \frac{1}{\sigma_i^2} + \sum_{m=0}^{L-M-1} \mathbf{v}_{mk}^{\mathrm{T}}(n-\ell)\boldsymbol{\Gamma}_m^{-1}\mathbf{v}_{mk}(n-\ell)$$

and K_1 and K_2 are constants independent with k and ℓ.

Estimates for $\overline{k}$ and $\overline{\ell}$ are obtained by computing the argument of the maximum of (18) and (19). Once the subband and time location of the error have been estimated, its amplitude may be obtained using (10), by computing an estimate of $\underline{\underline{\mathbf{a}}}(n)$ in the least-squares sense.

Note that despite the apparent complexity of the computation, the practical complexity of the test is reasonable, since the test to decide whether impulses are present amounts to compute only scalar product and to compare it to a threshold. The more complex calculations (also scalar products), require only to be realized once errors have been detected.

3.3. Algorithm

Fig. 2 summarizes the impulse error detection and correction algorithm. At each iteration, a syndrome is computed. If the test on the norm of the syndrome described in Sec. 3.1 concludes that there are errors, an estimation of their characteristics is performed (Sec. 3.2) before correction. The test on the syndrome can be applied again in order to detect other impulses or to verify *a posteriori* that the correction has been correctly performed. Once there is no more detected impulses, the buffer is flushed to the synthesis filterbank and new subband samples are collected.

4. EXAMPLE

An OFB oversampled twice with $L = 4$ subbands and a decimation factor $M = 2$ has been built from a critically sampled TDAC filterbank. In this case, a parity-check polynomial matrix can be defined without using any Smith-McMillan decomposition by

$$\mathbf{V}(z) = z^{-2}\left(z^{-1}\mathbf{I}_M, -\mathbf{I}_M\right)\widetilde{\mathbf{E}}_0(z^2), \tag{20}$$

where $\mathbf{E}_0(z)$ is the analysis polyphase matrix of the TDAC filterbank. The order of $\mathbf{V}(z)$ is $N_V = 3$, and an observation window of size $N = 4$ has been chosen. The filterbank input is a correlated noise sequence of unit variance. All subbands have been corrupted with gaussian and impulse noise with $INR = 30$dB and $p = 10^{-3}$, and the impulse error detection and correction algorithm has been applied.

For the test defined in Sec. 3.1, the *a priori* probabilities are $p_0 = 0.9724$ and $p_1 = 0.0276$. Given these numbers, the probability of having more than two errors is lower than 2.10^{-4}, thus, to compute $p(y|\nu > 0)$, only the cases $\nu = 1$ and $\nu = 2$ have been considered. The obtained theoretical and simulated ROC-curve have been represented in Fig. 3, showing a very good accordance. Table 1 presents the SNR values before and after impulse noise cancellation for different values of the threshold α. The reference number $SNR_0 = 30$dB is the gaussian channel case, while when Bernoulli gaussian noise is added, a $SNR_1 = 27$dB is observed. The best SNR is obtained when $\alpha = 0.18$. This corresponds to $P_d = 0.47$, meaning that only the impulse errors with high amplitudes have been detected. Missing the low amplitude impulse errors that could not be distinguished from gaussian noise does not impair the SNR.

5. CONCLUSION

This paper has presented an analogy between OFB and channel codes. We have shown that OFB have the ability to correct impulse noise in presence of background noise. Current work is applying this technique to image coding.

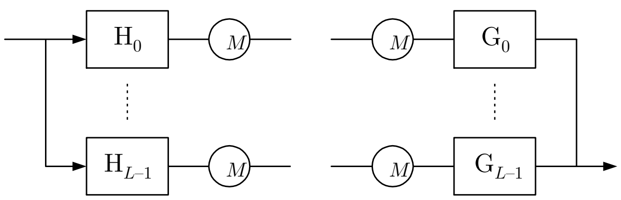

Fig. 1. Block diagram of an oversampled filter bank

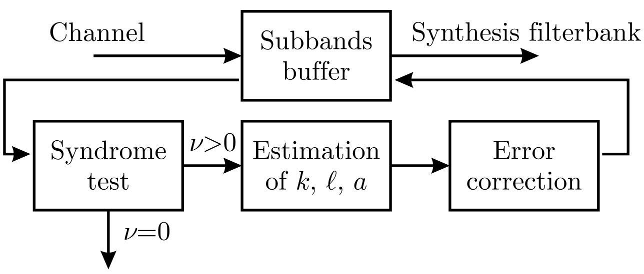

Fig. 2. Error correction scheme

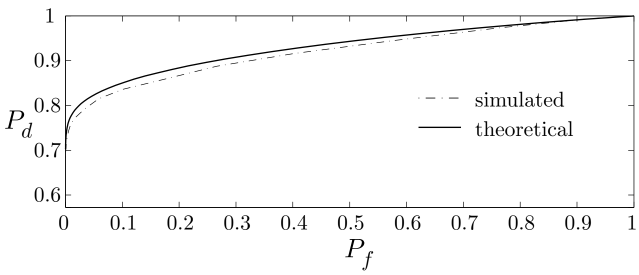

Fig. 3. Theoretical and simulated ROC curves

α	0.02	0.05	0.1	0.15	0.18	0.22
SNR	23.5	29.5	29.7	29.82	29.83	29.81
Sim. P_d	0.89	0.70	0.58	0.51	0.47	0.44
Sim. P_f	0.27	0.01	0.007	0.006	0.005	0.005

Table 1. Performances for various values of the threshold α

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